Chapter 4. Expansions (Including Substitution)

Exercise 4(A)

Solution 1:

We know that

$(a+b)^2 = a^2 + b^2 + 2ab$

$(2a+b)^2 = 4a^2 + b^2 + 2 \times 2a \times b$

$= 4a^2 + b^2 + 4ab$

We know that

$(a+b)^2 = a^2 + b^2 + 2ab$

$(3a+7b)^2 = 9a^2 + 49b^2 + 2 \times 3a \times 7b$

$= 9a^2 + 49b^2 + 42ab$

We know that

$(a-b)^2 = a^2 + b^2 - 2ab$

$(3a-4b)^2 = 9a^2 + 16b^2 - 2 \times 3a \times 4b$

$= 9a^2 + 16b^2 - 24ab$

We know that

$(a-b)^2 = a^2 + b^2 - 2ab$

$\left(\frac{3a}{2b} - \frac{2b}{3a}\right)^2 = \left(\frac{3a}{2b}\right)^2 + \left(\frac{2b}{3a}\right)^2 - 2 \times \frac{3a}{2b} \times \frac{2b}{3a}$

$= \frac{9a^2}{4b^2} + \frac{4b^2}{9a^2} - 2$

Solution 2:

- $(101)^2$

$(101)^2 = (100+1)^2$

We know that

$(a+b)^2 = a^2 + b^2 + 2ab$

$\therefore\ (100+1)^2 = 100^2 + 1^2 + 2\times100\times1$

$= 10000 + 1 + 200$

$= 10,201$

- $(502)^2$

$(502)^2 = (500+2)^2$

We know that

$(a+b)^2 = a^2 + b^2 + 2ab$

$\therefore\ (500+2)^2 = 500^2 + 2^2 + 2\times500\times2$

$= 250000 + 4 + 2000$

$= 2,52,004$

- $(97)^2$

$(97)^2 = (100-3)^2$

We know that

$(a-b)^2 = a^2 + b^2 - 2ab$

$\therefore (100-3)^2 = 100^2 + 3^2 - 2 \times 100 \times 3$

$= 10000 + 9 - 600$

$= 9,409$

- $(998)^2$

$(998)^2 = (1000-2)^2$

We know that

$(a-b)^2 = a^2 + b^2 - 2ab$

$\therefore (1000-2)^2 = 1000^2 + 2^2 - 2 \times 1000 \times 2$

$= 1000000 + 4 - 4000$

$= 9,96,004$

Solution 3:

(i)

$$\left(\frac{7}{8}x+\frac{4}{5}y\right)^2$$

We know that

$(a+b)^2 = a^2+b^2+2ab$

$$\therefore \left(\frac{7}{8}x+\frac{4}{5}y\right)^2 = \left(\frac{7}{8}x\right)^2+\left(\frac{4}{5}y\right)^2+2\times\frac{7}{8}x\times\frac{4}{5}y$$

$$=\frac{49x^2}{64}+\frac{16y^2}{25}+\frac{7xy}{5}$$

(ii)

$$\left(\frac{2x}{7}-\frac{7y}{4}\right)^2$$

We know that

$(a-b)^2 = a^2+b^2-2ab$

$$\therefore \left(\frac{2x}{7}-\frac{7y}{4}\right)^2 = \left(\frac{2}{7}x\right)^2+\left(\frac{7}{4}y\right)^2-2\times\frac{2}{7}x\times\frac{7}{4}y$$

$$=\frac{4x^2}{49}+\frac{49y^2}{16}-xy$$

Solution 4:

(i) Consider the given expression:

Let us expand the first term: $\left(\frac{a}{2b}+\frac{2b}{a}\right)^2$

We know that

$(a+b)^2 = a^2+b^2+2ab$

$$\therefore \left(\frac{a}{2b}+\frac{2b}{a}\right)^2 = \left(\frac{a}{2b}\right)^2+\left(\frac{2b}{a}\right)^2+2\times\frac{a}{2b}\times\frac{2b}{a}$$

$$= \frac{a^2}{4b^2}+\frac{4b^2}{a^2}+2 \ldots(1)$$

Let us expand the second term: $\left(\frac{a}{2b}-\frac{2b}{a}\right)^2$

We know that

$(a-b)^2 = a^2+b^2-2ab$

$$\therefore \left(\frac{a}{2b}-\frac{2b}{a}\right)^2 = \left(\frac{a}{2b}\right)^2+\left(\frac{2b}{a}\right)^2-2\times\frac{a}{2b}\times\frac{2b}{a}$$

$$= \frac{a^2}{4b^2}+\frac{4b^2}{a^2}-2 \ldots(2)$$

Thus from (1) and (2), the given expression is

$$\left(\frac{a}{2b}+\frac{2b}{a}\right)^2-\left(\frac{a}{2b}-\frac{2b}{a}\right)^2-4 = \frac{a^2}{4b^2}+\frac{4b^2}{a^2}+2-\frac{a^2}{4b^2}-\frac{4b^2}{a^2}+2-4$$

$$= 0$$

(ii) Consider the given expression:

Let us expand the first term: $(4a+3b)^2$

We know that

$(a+b)^2 = a^2 + b^2 + 2ab$

$\therefore (4a+3b)^2 = (4a)^2 + (3b)^2 + 2 \times 4a \times 3b$

$= 16a^2 + 9b^2 + 24ab \ldots(1)$

Let us expand the second term: $(4a-3b)^2$

We know that

$(a-b)^2 = a^2 + b^2 - 2ab$

$\therefore (4a-3b)^2 = (4a)^2 + (3b)^2 - 2 \times 4a \times 3b$

$= 16a^2 + 9b^2 - 24ab \ldots(2)$

Thus from (1) and (2), the given expression is

$(4a+3b)^2 - (4a-3b)^2 + 48ab$

$= 16a^2 + 9b^2 + 24ab - 16a^2 - 9b^2 + 24ab + 48ab$

$= 96ab$

Solution 5:

We know that

$(a+b)^2 = a^2 + b^2 + 2ab$

and

$(a-b)^2 = a^2 + b^2 - 2ab$

Rewrite the above equation, we have

$(a-b)^2 = a^2 + b^2 + 2ab - 4ab$

$= (a+b)^2 - 4ab \ldots(1)$

Given that a+b = 7; ab=10

Substitute the values of (a+b) and (ab)

in equation (1), we have

$(a-b)^2 = (7)^2 - 4(10)$

$= 49 - 40 = 9$

$\Rightarrow a - b = \pm\sqrt{9}$

$\Rightarrow a - b = \pm 3$

Solution 6:

We know that

$(a-b)^2 = a^2 + b^2 - 2ab$

and

$(a+b)^2 = a^2 + b^2 + 2ab$

Rewrite the above equation, we have

$(a+b)^2 = a^2 + b^2 - 2ab + 4ab$

$= (a-b)^2 + 4ab....(1)$

Given that $a-b = 7$; $ab=18$

Substitute the values of $(a-b)$ and (ab) in equation (1), we have

$(a+b)^2 = (7)^2 + 4(18)$

$= 49 + 72 = 121$

$\Rightarrow a+b = \pm\sqrt{121}$

$\Rightarrow a+b = \pm 11$

Solution 7:

(i)

We know that

$(x+y)^2 = x^2 + y^2 + 2xy$

and

$(x-y)^2 = x^2 + y^2 - 2xy$

Rewrite the above equation, we have

$(x-y)^2 = x^2 + y^2 + 2xy - 4xy$

$= (x+y)^2 - 4xy....(1)$

Given that $x+y = \frac{7}{2}$; $xy=\frac{5}{2}$

Substitute the values of $(x+y)$ and (xy) in equation (1), we have

$(x-y)^2 = \left(\frac{7}{2}\right)^2 - 4\left(\frac{5}{2}\right)$

$= \frac{49}{4} - 10 = \frac{9}{4}$

$\Rightarrow x-y = \pm\sqrt{\frac{9}{4}}$

$\Rightarrow x-y = \pm\frac{3}{2}....(2)$

(ii)

We know that

$x^2 - y^2 = (x + y)(x - y) \ldots.(3)$

From equation (2) we have,

$x - y = \pm\frac{3}{2}$

Thus equation (3) becomes,

$x^2 - y^2 = \left(\frac{7}{2}\right)\left(\pm\frac{3}{2}\right)$ [given $x + y = \frac{7}{2}$]

$\Rightarrow x^2 - y^2 = \pm\frac{21}{4}$

Solution 8:

(i)

We know that

$(a-b)^2 = a^2 + b^2 - 2ab$

and

$(a+b)^2 = a^2 + b^2 + 2ab$

Rewrite the above equation, we have

$(a+b)^2 = a^2 + b^2 - 2ab + 4ab$

$= (a-b)^2 + 4ab....(1)$

Given that $a-b = 0.9$; $ab=0.36$

Substitute the values of $(a-b)$ and (ab) in equation (1), we have

$(a+b)^2 = (0.9)^2 + 4(0.36)$

$= 0.81 + 1.44 = 2.25$

$\Rightarrow a+b = \pm\sqrt{2.25}$

$\Rightarrow a+b = \pm 1.5....(2)$

(ii)

We know that

$a^2 - b^2 = (a+b)(a-b)....(3)$

From equation (2) we have,

$a+b = \pm 1.5$

Thus equation (3) becomes,

$a^2 - b^2 = (\pm 1.5)(0.9)$ [given $a-b = 0.9$]

$\Rightarrow a^2 - b^2 = \pm 1.35$

Solution 9:

(i)

We know that

$(a-b)^2 = a^2 + b^2 - 2ab$

Rewrite the above identity as,

$a^2 + b^2 = (a-b)^2 + 2ab \ldots(1)$

Similarly, we know that,

$(a+b)^2 = a^2 + b^2 + 2ab$

Rewrite the above identity as,

$a^2 + b^2 = (a+b)^2 - 2ab \ldots(2)$

Adding the equations (1) and (2), we have

$2(a^2 + b^2) = (a-b)^2 + 2ab + (a+b)^2 - 2ab$

$\Rightarrow 2(a^2 + b^2) = (a-b)^2 + (a+b)^2$

$\Rightarrow (a^2 + b^2) = \frac{1}{2}\left[(a-b)^2 + (a+b)^2\right] \ldots(3)$

Given that a+b = 6; a − b=4

Substitute the values of (a+b) and (a − b)

in equation (3), we have

$(a^2 + b^2) = \frac{1}{2}\left[(4)^2 + (6)^2\right]$

$= \frac{1}{2}[16 + 36]$

$= \frac{52}{2}$

$\Rightarrow a^2 + b^2 = 26 \ldots(4)$

(ii)

From equation (4), we have

$a^2 + b^2 = 26$

Consider the identity

$(a-b)^2 = a^2 + b^2 - 2ab \ldots(5)$

Subsitute the value $a - b = 4$ and $a^2 + b^2 = 26$

in the above equation, we have

$(4)^2 = 26 - 2ab$

$\Rightarrow 2ab = 26 - 16$

$\Rightarrow 2ab = 10$

$\Rightarrow ab = 5$

Solution 10:

(i)

We know that

$(a+b)^2 = a^2 + b^2 + 2ab$

and

$(a-b)^2 = a^2 + b^2 - 2ab$

Thus,

$$\left(a+\frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2 \times a \times \frac{1}{a}$$

$$= a^2 + \frac{1}{a^2} + 2 .....(1)$$

Given that $a+\frac{1}{a} = 6$; Substitute in equation (1), we have

$$(6)^2 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 36 - 2$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 34....(2)$$

Similarly, consider

$$\left(a-\frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2 \times a \times \frac{1}{a}$$

$$= a^2 + \frac{1}{a^2} - 2$$

$$= 34 - 2 \ \text{[from (2)]}$$

$$\Rightarrow \left(a-\frac{1}{a}\right)^2 = 32$$

$$\Rightarrow a - \frac{1}{a} = \pm\sqrt{32}$$

$$\Rightarrow a - \frac{1}{a} = \pm 4\sqrt{2} \quad .....(3)$$

(ii)

We need to find $a^2 - \frac{1}{a^2}$:

We know that, $a^2 - \frac{1}{a^2} = \left(a-\frac{1}{a}\right)\left(a+\frac{1}{a}\right)$

$a - \frac{1}{a} = \pm 4\sqrt{2}; a + \frac{1}{a} = 6$

Thus,

$$a^2 - \frac{1}{a^2} = \left(\pm 4\sqrt{2}\right)(6)$$

$$\Rightarrow a^2 - \frac{1}{a^2} = \pm 24\sqrt{2}$$

Solution 11:

(i)

We know that

$(a+b)^2 = a^2 + b^2 + 2ab$

and

$(a-b)^2 = a^2 + b^2 - 2ab$

Thus,

$$\left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2 \times a \times \frac{1}{a}$$

$$= a^2 + \frac{1}{a^2} - 2 \ldots\ldots(1)$$

Given that $a - \frac{1}{a} = 8$; Substitute in equation (1), we have

$$(8)^2 = a^2 + \frac{1}{a^2} - 2$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 64 + 2$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 66 \ldots\ldots(2)$$

Similarly, consider

$$\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2 \times a \times \frac{1}{a}$$

$$= a^2 + \frac{1}{a^2} + 2$$

$$= 66 + 2 \text{ [from (2)]}$$

$$\Rightarrow \left(a + \frac{1}{a}\right)^2 = 68$$

$$\Rightarrow a + \frac{1}{a} = \pm 2\sqrt{17}$$

$$\Rightarrow a + \frac{1}{a} = \pm 2\sqrt{17} \quad \ldots\ldots(3)$$

(ii)

We need to find $a^2 - \frac{1}{a^2}$:

We know that, $a^2 - \frac{1}{a^2} = \left(a - \frac{1}{a}\right)\left(a + \frac{1}{a}\right)$

$a - \frac{1}{a} = 8; a + \frac{1}{a} = \pm 2\sqrt{17}$

Thus,

$a^2 - \frac{1}{a^2} = \left(\pm 2\sqrt{17}\right)(8)$

$\Rightarrow a^2 - \frac{1}{a^2} = \pm 16\sqrt{17}$

Solution 12:

(i)

Consider the given equation

$a^2 - 3a + 1 = 0$

Rewrite the given equation, we have

$a^2 + 1 = 3a$

$\Rightarrow \frac{a^2 + 1}{a} = 3$

$\Rightarrow \frac{a^2}{a} + \frac{1}{a} = 3$

$\Rightarrow a + \frac{1}{a} = 3....(1)$

(ii)

We need to find $a^2 + \frac{1}{a^2}$:

We know the identity, $(a+b)^2 = a^2 + b^2 + 2ab$

$$\therefore \left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2 \ldots (2)$$

From equation (1), we have,

$$a + \frac{1}{a} = 3$$

Thus equation (2), becomes,

$$(3)^2 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow 9 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 7$$

Solution 13:

(i)

Consider the given equation

$$a^2 - 5a - 1 = 0$$

Rewrite the given equation, we have

$$a^2 - 1 = 5a$$

$$\Rightarrow \frac{a^2 - 1}{a} = 5$$

$$\Rightarrow \frac{a^2}{a} - \frac{1}{a} = 5$$

$$\Rightarrow a - \frac{1}{a} = 5 \ldots (1)$$

(ii)

We need to find $a+\dfrac{1}{a}$:

We know the identity, $(a-b)^2 = a^2 + b^2 - 2ab$

$\therefore \left(a-\dfrac{1}{a}\right)^2 = a^2 + \dfrac{1}{a^2} - 2$

$\Rightarrow (5)^2 = a^2 + \dfrac{1}{a^2} - 2 \quad [\text{from } (1)]$

$\Rightarrow 25 = a^2 + \dfrac{1}{a^2} - 2$

$\Rightarrow a^2 + \dfrac{1}{a^2} = 27 \ldots\ldots(2)$

Now consider the identity $(a+b)^2 = a^2 + b^2 + 2ab$

$\therefore \left(a+\dfrac{1}{a}\right)^2 = a^2 + \dfrac{1}{a^2} + 2$

$\Rightarrow \left(a+\dfrac{1}{a}\right)^2 = 27 + 2 \quad [\text{from } (2)]$

$\Rightarrow \left(a+\dfrac{1}{a}\right)^2 = 29$

$\Rightarrow a + \dfrac{1}{a} = \pm\sqrt{29} \quad \ldots\ldots(3)$

Solution 14:

Given that $(3x+4y) = 16$ and $xy=4$

We need to find $9x^2+16y^2$.

We know that

$(a+b)^2 = a^2+b^2+2ab$

Consider the square of $3x+4y$:

$\therefore (3x+4y)^2 = (3x)^2+(4y)^2+2\times 3x\times 4y$

$\Rightarrow (3x+4y)^2 = 9x^2+16y^2+24xy.....(1)$

Substitute the values of $(3x+4y)$ and xy

in the above equation (1), we have

$(3x+4y)^2 = 9x^2+16y^2+24xy$

$\Rightarrow (16)^2 = 9x^2+16y^2+24(4)$

$\Rightarrow 256 = 9x^2+16y^2+96$

$\Rightarrow 9x^2+16y^2 = 160$

Solution 15:

Given x is 2 more than y, so $x = y + 2$

Sum of squares of x and y is 34, so $x^2 + y^2 = 34$.

Replace $x = y + 2$ in the above equation and solve for y.

We get $(y + 2)^2 + y^2 = 34$

$2y^2 + 4y - 30 = 0$

$y^2 + 2y - 15 = 0$

$(y + 5)(y - 3) = 0$

So $y = -5$ or 3

For $y = -5$, $x = -3$

For $y = 3$, $x = 5$

Product of x and y is 15 in both cases.

Let the two positive numbers be a and b.
Given difference between them is 5 and sum of squares is 73.
So $a - b = 5$, $a^2 + b^2 = 73$
Squaring on both sides gives
$(a - b)^2 = 5^2$
$a^2 + b^2 - 2ab = 25$
but $a^2 + b^2 = 73$
so $2ab = 73 - 25 = 48$
$ab = 24$
So, the product of numbers is 24.