

# Limits

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## Exercise 27A

### Q. 1. Evaluate

$$\lim_{x \rightarrow 2} (5 - x)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 2} (5 - x)$$

**Formula used:**

We have,

$$\lim_{x \rightarrow a} f(x) = f(a)$$

As  $x \rightarrow 2$ , we have

$$\lim_{x \rightarrow 2} (5 - x) = 5 - 2$$

$$\lim_{x \rightarrow 2} (5 - x) = 3$$

Thus, the value of  $\lim_{x \rightarrow 2} (5 - x)$  is 3.

### Q. 2. Evaluate

$$\lim_{x \rightarrow 1} (6x^2 - 4x + 3)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 1} (x^2 - 4x + 3)$$

**Formula used:** We have,

$$\lim_{x \rightarrow a} f(x) = f(a)$$

As  $x \rightarrow 1$ , we have

$$\lim_{x \rightarrow 1} (x^2 - 4x + 3) = 1^2 - 4(1) + 3$$

$$\lim_{x \rightarrow 1} (x^2 - 4x + 3) = 0$$

Thus, the value of  $\lim_{x \rightarrow 1} (x^2 - 4x + 3)$  is 0.

### Q. 3. Evaluate

$$\lim_{x \rightarrow 3} \left( \frac{x^2 + 9}{x + 3} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 3} \frac{x^2 + 9}{x + 3}$$

Formula used:

We have,

$$\lim_{x \rightarrow a} f(x) = f(a)$$

As  $x \rightarrow 3$ , we have

$$\lim_{x \rightarrow 3} \frac{x^2 + 9}{x + 3} = \frac{3^2 + 9}{3 + 3}$$

$$\lim_{x \rightarrow 3} \frac{x^2 + 9}{x + 3} = \frac{18}{6}$$



$$\lim_{x \rightarrow 3} \frac{x^2 + 9}{x + 3} = 3$$

Thus, the value of  $\lim_{x \rightarrow 3} \frac{x^2 + 9}{x + 3}$  is 3.

#### Q. 4. Evaluate

$$\lim_{x \rightarrow 3} \left( \frac{x^2 - 4x}{x - 2} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 3} \frac{x^2 - 4x}{x - 2}$$

Formula used: We have,

$$\lim_{x \rightarrow a} f(x) = f(a)$$

As  $x \rightarrow 3$ , we have

$$\lim_{x \rightarrow 3} \frac{x^2 - 4x}{x - 2} = \frac{3^2 - 4(3)}{3 - 2}$$

$$\lim_{x \rightarrow 3} \frac{x^2 - 4x}{x - 2} = \frac{3^2 - 4(3)}{3 - 2}$$

$$\lim_{x \rightarrow 3} \frac{x^2 - 4x}{x - 2} = -3$$

Thus, the value of  $\lim_{x \rightarrow 3} \frac{x^2 - 4x}{x - 2}$  is -3.

#### Q. 5. Evaluate

$$\lim_{x \rightarrow 5} \left( \frac{x^2 - 25}{x - 5} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5}$$

Formula used:

We have,

$$\lim_{x \rightarrow a} f(x) = f(a)$$

As  $x \rightarrow 5$ , we have

$$\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5} = \lim_{x \rightarrow 5} \frac{(x + 5)(x - 5)}{x - 5}$$

$$\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5} = x + 5$$

$$\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5} = 5 + 5$$

$$\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5} = 10$$

Thus, the value of  $\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5}$  is -10.

**Q. 6. Evaluate**

$$\lim_{x \rightarrow 1} \left( \frac{x^3 - 1}{x - 1} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$$

**Formula used:** We have,

$$\lim_{x \rightarrow a} f(x) = f(a) \quad \text{and}$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

As  $x \rightarrow 1$ , we have

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{(x - 1)(x^2 + x + 1)}{x - 1}$$

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} = \lim_{x \rightarrow 1} (x^2 + x + 1)$$

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} = 1 + 1 + 1$$

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} = 3$$

Thus, the value of  $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$  is 3.

**Q. 7. Evaluate**

$$\lim_{x \rightarrow -2} \left( \frac{x^3 + 8}{x + 2} \right)$$

**Answer :**



To evaluate:  $\lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2}$

Formula used:

We have,

$$\lim_{x \rightarrow a} f(x) = f(a) \text{ and}$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

As  $x \rightarrow -2$ , we have

$$\lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2} = \lim_{x \rightarrow -2} \frac{(x + 2)(x^2 - 2x + 4)}{x + 2}$$

$$\lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2} = \lim_{x \rightarrow -2} (x^2 - 2x + 4)$$

$$\lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2} = 4 - 4 + 4$$

$$\lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2} = 4$$

Thus, the value of  $\lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2}$  is 4

**Q. 8. Evaluate**  $\lim_{x \rightarrow 3} \left( \frac{x^4 - 81}{x - 3} \right)$

**Answer :**

**Answer :**

To evaluate:  $\lim_{x \rightarrow 3} \frac{x^4 - 81}{x - 3}$

Formula used:

We have,

$\lim_{x \rightarrow a} f(x) = f(a)$  and



As  $x \rightarrow 3$ , we have

$$\lim_{x \rightarrow 3} \frac{x^4 - 81}{x - 3} = \lim_{x \rightarrow 3} \frac{(x^2 + 9)(x^2 - 9)}{x - 3}$$

$$\lim_{x \rightarrow 3} \frac{x^4 - 81}{x - 3} = \lim_{x \rightarrow 3} \frac{(x^2 + 9)(x + 3)(x - 3)}{x - 3}$$

$$\lim_{x \rightarrow 3} \frac{x^4 - 81}{x - 3} = \lim_{x \rightarrow 3} (x^2 + 9)(x + 3)$$

$$\lim_{x \rightarrow 3} \frac{x^4 - 81}{x - 3} = (9 + 9)(3 + 3)$$

$$\lim_{x \rightarrow 3} \frac{x^4 - 81}{x - 3} = 486$$

Thus, the value of  $\lim_{x \rightarrow 3} \frac{x^4 - 81}{x - 3}$  is 486.

### Q. 9. Evaluate

$$\lim_{x \rightarrow 3} \left( \frac{x^2 - 4x + 3}{x^2 - 2x - 3} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3}$$

**Formula used:**

We have,

$$\lim_{x \rightarrow a} f(x) = f(a) \quad \text{and}$$

As  $x \rightarrow 3$ , we have

$$\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3} = \lim_{x \rightarrow 3} \frac{(x - 3)(x - 1)}{(x - 3)(x + 2)}$$

$$\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3} = \lim_{x \rightarrow 3} \frac{(x - 1)}{(x + 2)}$$

$$\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3} = \frac{2}{5}$$

Thus, the value of



$$\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3} \text{ is } \frac{2}{5}.$$

**Q. 10. Evaluate**

$$\lim_{x \rightarrow \frac{1}{2}} \left( \frac{4x^2 - 1}{2x - 1} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow \frac{1}{2}} \frac{4x^2 - 1}{2x - 1}$$

Formula used:

We have,

$$\lim_{x \rightarrow a} f(x) = f(a)$$

As  $x \rightarrow \frac{1}{2}$ , we have

$$\lim_{x \rightarrow \frac{1}{2}} \frac{4x^2 - 1}{2x - 1} = \lim_{x \rightarrow \frac{1}{2}} \frac{(2x + 1)(2x - 1)}{2x - 1}$$

$$\lim_{x \rightarrow \frac{1}{2}} \frac{4x^2 - 1}{2x - 1} = \lim_{x \rightarrow \frac{1}{2}} (2x + 1)$$

$$\lim_{x \rightarrow \frac{1}{2}} \frac{4x^2 - 1}{2x - 1} = 2$$

Thus, the value of  $\lim_{x \rightarrow \frac{1}{2}} \frac{4x^2 - 1}{2x - 1}$  is 2.

**Q. 11. Evaluate**

$$\lim_{x \rightarrow 4} \left( \frac{x^3 - 64}{x^2 - 16} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 4} \frac{x^3 - 64}{x^2 - 16}$$

**Formula used:** We have,

$$\lim_{x \rightarrow a} f(x) = f(a)$$

and

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

As  $x \rightarrow 4$ , we have

$$\lim_{x \rightarrow 4} \frac{x^3 - 64}{x^2 - 16} = \lim_{x \rightarrow 4} \frac{(x - 4)(x^2 + 4x + 16)}{(x + 4)(x - 4)}$$

$$\lim_{x \rightarrow 4} \frac{x^3 - 64}{x^2 - 16} = \lim_{x \rightarrow 4} \frac{(x^2 + 4x + 16)}{(x + 4)}$$

$$\lim_{x \rightarrow 4} \frac{x^3 - 64}{x^2 - 16} = \frac{48}{8}$$

$$\lim_{x \rightarrow 4} \frac{x^3 - 64}{x^2 - 16} = 6$$

Thus, the value of

$$\lim_{x \rightarrow 4} \frac{x^3 - 64}{x^2 - 16} \text{ is } 6.$$

**Q. 12. Evaluate**

$$\lim_{x \rightarrow 2} \left( \frac{x^5 - 32}{x^3 - 8} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 2} \frac{x^5 - 32}{x^3 - 8}$$

**Formula used:** We have,

$$\frac{x^m - y^m}{x - y} = my^{m-1}$$

As  $x \rightarrow 2$ , we have

$$\lim_{x \rightarrow 2} \frac{x^5 - 32}{x^3 - 8} = \lim_{x \rightarrow 2} \frac{x^5 - 2^5}{x^3 - 2^3}$$

$$\lim_{x \rightarrow 2} \frac{x^5 - 32}{x^3 - 8} = \lim_{x \rightarrow 2} \frac{\frac{x^5 - 2^5}{x - 2}}{\frac{x^3 - 2^3}{x - 2}}$$

$$\lim_{x \rightarrow 2} \frac{x^5 - 32}{x^3 - 8} = \lim_{x \rightarrow 2} \frac{5(2)^4}{3(2)^2}$$

$$\lim_{x \rightarrow 2} \frac{x^5 - 32}{x^3 - 8} = \frac{20}{3}$$

Thus, the value of  $\lim_{x \rightarrow 2} \frac{x^5 - 32}{x^3 - 8}$  is  $\frac{20}{3}$

**Q. 13. Evaluate**

$$\lim_{x \rightarrow a} \left( \frac{x^{5/2} - a^{5/2}}{x - a} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow a} \frac{x^{\frac{5}{2}} - a^{\frac{5}{2}}}{x - a}$$

**Formula used:** We have,

$$\frac{x^m - y^m}{x - y} = my^{m-1}$$

As  $x \rightarrow a$ , we have

$$\lim_{x \rightarrow a} \frac{x^{\frac{5}{2}} - a^{\frac{5}{2}}}{x - a} = \frac{5}{2} a^{\frac{5}{2}-1}$$

$$\lim_{x \rightarrow a} \frac{x^{\frac{5}{2}} - a^{\frac{5}{2}}}{x - a} = \frac{5}{2} a^{\frac{3}{2}}$$

Thus, the value of  $\lim_{x \rightarrow a} \frac{x^{\frac{5}{2}} - a^{\frac{5}{2}}}{x - a}$  is  $\frac{5}{2} a^{\frac{3}{2}}$



**Q. 14. Evaluate**

$$\lim_{x \rightarrow a} \left\{ \frac{(x+2)^{5/3} - (a+2)^{5/3}}{x - a} \right\}$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow a} \left\{ \frac{(x+2)^{\frac{5}{3}} - (a+2)^{\frac{5}{3}}}{x - a} \right\}$$

**Formula used:** We have,

$$\frac{x^m - y^m}{x - y} = my^{m-1}$$

As  $x \rightarrow a$ , we have

$$\lim_{x \rightarrow a} \left\{ \frac{(x+2)^{\frac{5}{3}} - (a+2)^{\frac{5}{3}}}{x-a} \right\} = \lim_{x \rightarrow a} \left\{ \frac{(x+2)^{\frac{5}{3}} - (a+2)^{\frac{5}{3}}}{(x+2) - (a+2)} \right\}$$

$$\lim_{x \rightarrow a} \left\{ \frac{(x+2)^{\frac{5}{3}} - (a+2)^{\frac{5}{3}}}{x-a} \right\} = \frac{5}{3} (a+2)^{\frac{5}{3}-1}$$

$$\lim_{x \rightarrow a} \left\{ \frac{(x+2)^{\frac{5}{3}} - (a+2)^{\frac{5}{3}}}{x-a} \right\} = \frac{5}{3} (a+2)^{\frac{2}{3}}$$

Thus, the value of  $\lim_{x \rightarrow a} \left\{ \frac{(x+2)^{\frac{5}{3}} - (a+2)^{\frac{5}{3}}}{x-a} \right\}$  is  $\frac{5}{3} (a+2)^{\frac{2}{3}}$

**Q. 15. Evaluate**

$$\lim_{x \rightarrow 1} \left( \frac{x^n - 1}{x - 1} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 1} \frac{x^n - 1}{x - 1}$$

**Formula used:** We have,

$$\frac{x^m - y^m}{x - y} = my^{m-1}$$

As  $x \rightarrow a$ , we have

$$\frac{x^m - y^m}{x - y} = my^{m-1}$$

As  $x \rightarrow 1$ , we have

$$\lim_{x \rightarrow a} \frac{x^n - 1}{x - 1} = n$$

Thus, the value of  $\lim_{x \rightarrow a} \frac{x^n - 1}{x - 1}$  is  $n$ .

**Q. 16. Evaluate**

$$\lim_{x \rightarrow a} \left( \frac{\sqrt{x} - \sqrt{a}}{x - a} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow a} \frac{\sqrt{x} - \sqrt{a}}{x - a}$$

**Formula used:**

We have,

$$\frac{x^m - y^m}{x - y} = my^{m-1}$$

As  $x \rightarrow a$ , we have



$$\lim_{x \rightarrow a} \frac{x^{\frac{1}{2}} - a^{\frac{1}{2}}}{x - a} = \frac{1}{2} a^{\frac{1}{2}-1}$$

$$\lim_{x \rightarrow a} \frac{x^{\frac{1}{2}} - a^{\frac{1}{2}}}{x - a} = \frac{1}{2\sqrt{a}}$$

Thus, the value of  $\lim_{x \rightarrow a} \frac{x^{\frac{1}{2}} - a^{\frac{1}{2}}}{x - a}$  is  $\frac{1}{2\sqrt{a}}$

**Q. 17. Evaluate**

$$\lim_{h \rightarrow 0} \left( \frac{\sqrt{x+h} - \sqrt{x}}{h} \right)$$

**Answer :** To evaluate:

$$\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

**Formula used:**

L'Hospital's rule

Let  $f(x)$  and  $g(x)$  be two functions which are differentiable on an open interval  $I$  except at a point  $a$  where

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0 \text{ or } \pm \infty \text{ then}$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

As  $x \rightarrow 0$ , we have

$$\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h \rightarrow 0} \frac{\frac{d}{dh}(\sqrt{x+h} - \sqrt{x})}{\frac{d}{dh}(h)}$$

$$\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h \rightarrow 0} \frac{1}{2\sqrt{x+h}}$$

$$\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{1}{2\sqrt{x}}$$

Thus, the value of  $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$  is  $\frac{1}{2\sqrt{x}}$

**Q. 18. Evaluate**

$$\lim_{h \rightarrow 0} \frac{1}{h} \left\{ \frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}} \right\}$$

**Answer :** To evaluate:

$$\lim_{h \rightarrow 0} \frac{1}{h} \left\{ \frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}} \right\}$$

**Formula used:** L'Hospital's rule

Let  $f(x)$  and  $g(x)$  be two functions which are differentiable on an open interval  $I$  except at a point  $a$  where

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0 \text{ or } \pm \infty \text{ then}$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

As  $x \rightarrow 0$ , we have

$$\lim_{h \rightarrow 0} \frac{1}{h} \left\{ \frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}} \right\} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{h \rightarrow 0} \frac{1}{h} \left\{ \frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}} \right\} = \lim_{h \rightarrow 0} \frac{\frac{d}{dh} \left( \frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}} \right)}{\frac{d}{dh} (h)}$$

$$\lim_{h \rightarrow 0} \frac{1}{h} \left\{ \frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}} \right\} = \lim_{h \rightarrow 0} \frac{\frac{-1}{2\sqrt{x+h}} + \frac{1}{2\sqrt{x}}}{1}$$

$$\lim_{h \rightarrow 0} \frac{1}{h} \left\{ \frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}} \right\} = 0$$

Thus, the value of  $\lim_{h \rightarrow 0} \frac{1}{h} \left\{ \frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}} \right\}$  is 0.

**Q. 19. Evaluate**

$$\lim_{x \rightarrow 0} \left( \frac{\sqrt{1+x} - 1}{x} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$$

**Formula used:** L'Hospital's rule

Let  $f(x)$  and  $g(x)$  be two functions which are differentiable on an open interval  $I$  except at a point  $a$  where

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0 \text{ or } \pm \infty \text{ then}$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

As  $x \rightarrow 0$ , we have


$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(\sqrt{1+x} - 1)}{\frac{d}{dx}(x)}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} = \lim_{x \rightarrow 0} \frac{1}{2\sqrt{x+1}}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} = \frac{1}{2}$$

Thus, the value of  $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$  is  $\frac{1}{2}$ . 

**Q. 20. Evaluate**

$$\lim_{x \rightarrow 0} \left( \frac{\sqrt{2-x} - \sqrt{2+x}}{x} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 0} \frac{\sqrt{2-x} - \sqrt{2+x}}{x}$$

**Formula used:** L'Hospital's rule

Let  $f(x)$  and  $g(x)$  be two functions which are differentiable on an open interval  $I$  except at a point  $a$  where

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0 \text{ or } \pm \infty \text{ then}$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

As  $x \rightarrow 0$ , we have

$$\lim_{x \rightarrow 0} \frac{\sqrt{2-x} - \sqrt{2+x}}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{\sqrt{2-x} - \sqrt{2+x}}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(\sqrt{2-x} - \sqrt{2+x})}{\frac{d}{dx}(x)}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{2-x} - \sqrt{2+x}}{x} = \lim_{x \rightarrow 0} \frac{-\frac{1}{2\sqrt{2-x}} - \frac{1}{2\sqrt{2+x}}}{1}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{2-x} - \sqrt{2+x}}{x} = \frac{-2}{2\sqrt{2}}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{2-x} - \sqrt{2+x}}{x} = \frac{-1}{\sqrt{2}}$$

Thus, the value of  $\lim_{x \rightarrow 0} \frac{\sqrt{2-x} - \sqrt{2+x}}{x}$  is  $\frac{-1}{\sqrt{2}}$

**Q. 21. Evaluate**

$$\lim_{x \rightarrow 0} \left( \frac{\sqrt{1+x+x^2} - 1}{x} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2} - 1}{x}$$

**Formula used:** L'Hospital's rule

Let  $f(x)$  and  $g(x)$  be two functions which are differentiable on an open interval  $I$  except at a point  $a$  where

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0 \text{ or } \pm \infty \text{ then}$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

As  $x \rightarrow 0$ , we have

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2} - 1}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2} - 1}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(\sqrt{1+x+x^2} - 1)}{\frac{d}{dx}(x)}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2} - 1}{x} = \lim_{x \rightarrow 0} \frac{1 + 2x}{2\sqrt{1+x+x^2}}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2} - 1}{x} = \frac{1}{2}$$

Thus, the value of  $\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2} - 1}{x}$  is  $\frac{1}{2}$

**Q. 22. Evaluate**

$$\lim_{x \rightarrow 0} \left( \frac{\sqrt{3-x} - 1}{2-x} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 0} \frac{\sqrt{3-x} - 1}{2-x}$$

**Formula used:** We have,

$$\lim_{x \rightarrow a} f(x) = f(a)$$

As  $x \rightarrow 0$ , we have

$$\lim_{x \rightarrow 0} \frac{\sqrt{3-x} - 1}{2-x} = \frac{\sqrt{3} - 1}{2}$$

Thus, the value of  $\lim_{x \rightarrow 0} \frac{\sqrt{3-x} - 1}{2-x}$  is  $\frac{\sqrt{3}-1}{2}$

**Q. 23. Evaluate**

$$\lim_{x \rightarrow 0} \left( \frac{2x}{\sqrt{a+x} - \sqrt{a-x}} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}}$$

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**Formula used:**

Multiplying numerator and denominator by

$$\sqrt{a+x} + \sqrt{a-x}$$

$$\lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}} = \lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}} \left( \frac{\sqrt{a+x} + \sqrt{a-x}}{\sqrt{a+x} + \sqrt{a-x}} \right)$$

$$\lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}} = \lim_{x \rightarrow 0} \frac{2x(\sqrt{a+x} + \sqrt{a-x})}{a+x - a+x}$$

$$\lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}} = \lim_{x \rightarrow 0} \frac{2x(\sqrt{a+x} + \sqrt{a-x})}{2x}$$

$$\lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}} = \lim_{x \rightarrow 0} \sqrt{a+x} + \sqrt{a-x}$$

$$\lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}} = 2\sqrt{a}$$

Thus, the value of  $\lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}}$  is  $2\sqrt{a}$ .

**Q. 24. Evaluate**

$$\lim_{x \rightarrow 1} \left( \frac{\sqrt{3+x} - \sqrt{5-x}}{x^2 - 1} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 1} \frac{\sqrt{3+x} - \sqrt{5-x}}{x^2 - 1}$$

**Formula used:**

L'Hospital's rule

Let  $f(x)$  and  $g(x)$  be two functions which are differentiable on an open interval  $I$  except at a point  $a$  where

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0 \text{ or } \pm \infty \text{ then}$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

As  $x \rightarrow 0$ , we have

$$\lim_{x \rightarrow 0} \frac{\sqrt{3+x} - \sqrt{5-x}}{x^2 - 1} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 1} \frac{\sqrt{3+x} - \sqrt{5-x}}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{\frac{d}{dx}(\sqrt{3+x} - \sqrt{5-x})}{\frac{d}{dx}(x^2 - 1)}$$

$$\lim_{x \rightarrow 1} \frac{\sqrt{3+x} - \sqrt{5-x}}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{\frac{1}{2\sqrt{3+x}} + \frac{1}{2\sqrt{5-x}}}{2x}$$

$$\lim_{x \rightarrow 1} \frac{\sqrt{3+x} - \sqrt{5-x}}{x^2 - 1} = \frac{1}{4}$$

Thus, the value of  $\lim_{x \rightarrow 1} \frac{\sqrt{3+x} - \sqrt{5-x}}{x^2 - 1}$  is  $\frac{1}{4}$

**Q. 25. Evaluate**

$$\lim_{x \rightarrow 2} \left( \frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}}$$

**Formula used:** L'Hospital's rule

Let  $f(x)$  and  $g(x)$  be two functions which are differentiable on an open interval  $I$  except at a point  $a$  where

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0 \text{ or } \pm \infty \text{ then}$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

As  $x \rightarrow 0$ , we have

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

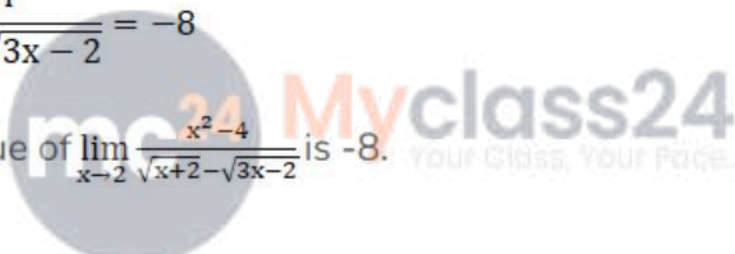
$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}} = \lim_{x \rightarrow 2} \frac{\frac{d}{dx}(x^2 - 4)}{\frac{d}{dx}(\sqrt{x+2} - \sqrt{3x-2})}$$

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}} = \lim_{x \rightarrow 2} \frac{2x}{\frac{1}{2\sqrt{x+2}} - \frac{3}{2\sqrt{3x-2}}}$$

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}} = \frac{4}{\frac{1}{2\sqrt{2+2}} - \frac{3}{2\sqrt{6-2}}}$$

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}} = \frac{8}{\frac{1}{2} - \frac{3}{2}}$$

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}} = -8$$

Thus, the value of  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}}$  is -8.  Myclass24 Your Class, Your Pace.

**Q. 26. Evaluate**

$$\lim_{x \rightarrow 4} \left( \frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 4} \left( \frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}} \right)$$

**Formula used:**

Multiplying numerator and denominator with conjugates of numerator and denominator  
i.e

$$(1 + \sqrt{5-x})(3 + \sqrt{5+x})$$

$$\lim_{x \rightarrow 4} \left( \frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}} \right) = \lim_{x \rightarrow 4} \left( \frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}} \right) \left( \frac{1 + \sqrt{5-x}}{1 + \sqrt{5-x}} \right) \left( \frac{3 + \sqrt{5+x}}{3 + \sqrt{5+x}} \right)$$

$$\lim_{x \rightarrow 4} \left( \frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}} \right) = \lim_{x \rightarrow 4} \left( \frac{4-x}{x-4} \right) \left( \frac{1 + \sqrt{5-x}}{3 + \sqrt{5+x}} \right)$$

$$\lim_{x \rightarrow 4} \left( \frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}} \right) = \lim_{x \rightarrow 4} - \left( \frac{1 + \sqrt{5-x}}{3 + \sqrt{5+x}} \right)$$

$$\lim_{x \rightarrow 4} \left( \frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}} \right) = -\frac{1}{3}$$

Thus, the value of  $\lim_{x \rightarrow 4} \left( \frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}} \right)$  is  $-\frac{1}{3}$

**Q. 27. Evaluate**

$$\lim_{x \rightarrow 0} \left( \frac{\sqrt{a+x} - \sqrt{a}}{x\sqrt{a(a+x)}} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 0} \frac{\sqrt{a+x} - \sqrt{a}}{x\sqrt{a(a+x)}}$$

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**Formula used:** L'Hospital's rule

Let  $f(x)$  and  $g(x)$  be two functions which are differentiable on an open interval  $I$  except at a point  $a$  where

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0 \text{ or } \pm \infty \text{ then}$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

As  $x \rightarrow 0$ , we have

$$\lim_{x \rightarrow 0} \frac{\sqrt{a+x} - \sqrt{a}}{x\sqrt{a(a+x)}} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{\sqrt{a+x} - \sqrt{a}}{x\sqrt{a(a+x)}} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(\sqrt{a+x} - \sqrt{a})}{\frac{d}{dx}(x\sqrt{a(a+x)})}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{a+x} - \sqrt{a}}{x\sqrt{a(a+x)}} = \lim_{x \rightarrow 0} \frac{\frac{1}{2\sqrt{a+x}}}{x\left(\frac{a}{2\sqrt{a(a+x)}}\right) + \sqrt{a(a+x)}}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{a+x} - \sqrt{a}}{x\sqrt{a(a+x)}} = \frac{1}{2\sqrt{a} \cdot a}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{a+x} - \sqrt{a}}{x\sqrt{a(a+x)}} = \frac{1}{2a\sqrt{a}}$$

Thus, the value of  $\lim_{x \rightarrow 0} \frac{\sqrt{a+x} - \sqrt{a}}{x\sqrt{a(a+x)}}$  is  $\frac{1}{2a\sqrt{a}}$

Thus, the value of  $\lim_{x \rightarrow 0} \frac{\sqrt{a+x} - \sqrt{a}}{x\sqrt{a(a+x)}}$  is  $\frac{1}{2a\sqrt{a}}$

**Q. 28. Evaluate**

$$\lim_{x \rightarrow 0} \left( \frac{\sqrt{1+x^2} - \sqrt{1+x}}{\sqrt{1+x^3} - \sqrt{1+x}} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - \sqrt{1+x}}{\sqrt{1+x^3} - \sqrt{1+x}}$$

**Formula used:** L'Hospital's rule

Let  $f(x)$  and  $g(x)$  be two functions which are differentiable on an open interval  $I$  except at a point  $a$  where

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0 \text{ or } \pm \infty \text{ then}$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

As  $x \rightarrow 0$ , we have

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - \sqrt{1+x}}{\sqrt{1+x^3} - \sqrt{1+x}} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - \sqrt{1+x}}{\sqrt{1+x^3} - \sqrt{1+x}} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(\sqrt{1+x^2} - \sqrt{1+x})}{\frac{d}{dx}(\sqrt{1+x^3} - \sqrt{1+x})}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - \sqrt{1+x}}{\sqrt{1+x^3} - \sqrt{1+x}} = \lim_{x \rightarrow 0} \frac{\frac{2x}{2\sqrt{1+x^2}} - \frac{1}{2\sqrt{1+x}}}{\frac{3x^2}{2\sqrt{1+x^3}} - \frac{1}{2\sqrt{1+x}}}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - \sqrt{1+x}}{\sqrt{1+x^3} - \sqrt{1+x}} = \lim_{x \rightarrow 0} \frac{-\frac{1}{2}}{-\frac{1}{2}}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - \sqrt{1+x}}{\sqrt{1+x^3} - \sqrt{1+x}} = -1$$



Thus, the value of  $\lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - \sqrt{1+x}}{\sqrt{1+x^3} - \sqrt{1+x}}$  is -1.

**Q. 29. Evaluate**

$$\lim_{x \rightarrow 1} \left( \frac{x^4 - 3x^2 + 2}{x^3 - 5x^2 + 3x + 1} \right)$$

**Answer :** To Evaluate:

$$\lim_{x \rightarrow 1} \left( \frac{x^4 - 3x^2 + 2}{x^3 - 5x^2 + 3x + 1} \right)$$

L'Hospital's rule

Let  $f(x)$  and  $g(x)$  be two functions which are differentiable on an open interval  $I$  except at a point  $a$  where

$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$  or  $\pm \infty$  then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

As  $x \rightarrow 0$ , we have

$$\lim_{x \rightarrow 1} \left( \frac{x^4 - 3x^2 + 2}{x^3 - 5x^2 + 3x + 1} \right) = \frac{0}{0}$$

Therefore,

$$\lim_{x \rightarrow 1} \left( \frac{x^4 - 3x^2 + 2}{x^3 - 5x^2 + 3x + 1} \right) = \lim_{x \rightarrow 1} \frac{4x^3 - 6x}{3x^2 - 10x + 3} = \frac{4 - 6}{3 - 10 + 3} = -\frac{2}{-4} = -\frac{1}{2}$$

Hence,

$$\lim_{x \rightarrow 1} \left( \frac{x^4 - 3x^2 + 2}{x^3 - 5x^2 + 3x + 1} \right) = -\frac{1}{2}$$

**Q. 30. Evaluate**

$$\lim_{x \rightarrow 2} \left( \frac{3^x - 3^{3-x} - 12}{3^{3-x} - 3^{x/2}} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 2} \left( \frac{3^x - 3^{3-x} - 12}{3^{3-x} - 3^{x/2}} \right)$$

Formula used:

L'Hospital's rule

Let  $f(x)$  and  $g(x)$  be two functions which are differentiable on an open interval  $I$  except at a point  $a$  where

$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$  or  $\pm \infty$  then



$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

As  $x \rightarrow 0$ , we have

$$\lim_{x \rightarrow 0} \frac{2^x - 1}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 2} \left( \frac{3^x - 3^{3-x} - 12}{3^{3-x} - 3^{\frac{x}{2}}} \right) = \lim_{x \rightarrow 2} \frac{\frac{d}{dx}(3^x - 3^{3-x} - 12)}{\frac{d}{dx}(3^{3-x} - 3^{\frac{x}{2}})}$$

$$\lim_{x \rightarrow 2} \left( \frac{3^x - 3^{3-x} - 12}{3^{3-x} - 3^{\frac{x}{2}}} \right) = \lim_{x \rightarrow 2} \frac{3^x \ln 3 + 3^{3-x} \ln 3}{-3^{3-x} \ln 3 + 3^{\frac{x}{2}} \left( \frac{1}{2} \right) \ln 3}$$

$$\lim_{x \rightarrow 2} \left( \frac{3^x - 3^{3-x} - 12}{3^{3-x} - 3^{\frac{x}{2}}} \right) = \frac{\ln 3 + 27 \ln 3}{-27 \ln 3 + \left( \frac{1}{2} \right) \ln 3}$$

$$\lim_{x \rightarrow 2} \left( \frac{3^x - 3^{3-x} - 12}{3^{3-x} - 3^{\frac{x}{2}}} \right) = \frac{28 \ln 3}{-26.5 \ln 3}$$

Thus, the value of  $\lim_{x \rightarrow 2} \left( \frac{3^x - 3^{3-x} - 12}{3^{3-x} - 3^{\frac{x}{2}}} \right)$  is  $\frac{28 \ln 3}{-26.5 \ln 3}$

### Q. 31. Evaluate

$$\lim_{x \rightarrow 0} \left( \frac{e^{4x} - 1}{x} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 0} \frac{e^{4x} - 1}{x}$$

**Formula used:**

L'Hospital's rule

Let  $f(x)$  and  $g(x)$  be two functions which are differentiable on an open interval  $I$  except at a point  $a$  where


$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0 \text{ or } \pm \infty \text{ then}$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

As  $x \rightarrow 0$ , we have

$$\lim_{x \rightarrow 0} \frac{e^{4x} - 1}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{e^{4x} - 1}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(e^{4x} - 1)}{\frac{d}{dx}(x)}$$


$$\lim_{x \rightarrow 0} \frac{e^{4x} - 1}{x} = \lim_{x \rightarrow 0} \frac{4e^{4x}}{1}$$

$$\lim_{x \rightarrow 0} \frac{e^{4x} - 1}{x} = 4$$

Thus, the value of  $\lim_{x \rightarrow 0} \frac{e^{4x} - 1}{x}$  is 4.

**Q. 32. Evaluate**

$$\lim_{x \rightarrow 0} \left( \frac{e^{2+x} - e^2}{x} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 0} \frac{e^{2+x} - e^2}{x}$$

**Formula used:** L'Hospital's rule

Let  $f(x)$  and  $g(x)$  be two functions which are differentiable on an open interval  $I$  except at a point  $a$  where

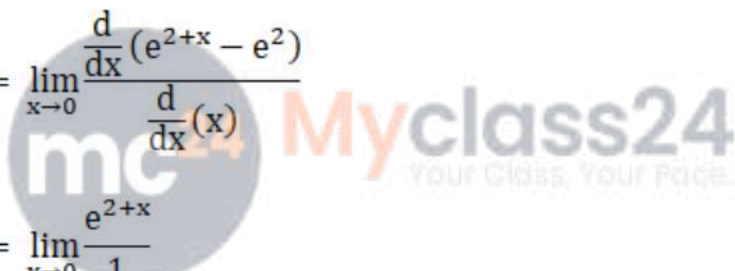
$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0 \text{ or } \pm \infty \text{ then}$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

As  $x \rightarrow 0$ , we have

$$\lim_{x \rightarrow 0} \frac{e^{2+x} - e^2}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{e^{2+x} - e^2}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(e^{2+x} - e^2)}{\frac{d}{dx}(x)}$$

$$\lim_{x \rightarrow 0} \frac{e^{2+x} - e^2}{x} = \lim_{x \rightarrow 0} \frac{e^{2+x}}{1}$$

$$\lim_{x \rightarrow 0} \frac{e^{2+x} - e^2}{x} = e^2$$

Thus, the value of  $\lim_{x \rightarrow 0} \frac{e^{2+x} - e^2}{x}$  is  $e^2$ .

**Q. 33. Evaluate**

$$\lim_{x \rightarrow 4} \left( \frac{e^x - e^4}{x - 4} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 4} \frac{e^x - e^4}{x - 4}$$

**Formula used:**

L'Hospital's rule

Let  $f(x)$  and  $g(x)$  be two functions which are differentiable on an open interval  $I$  except at a point  $a$  where

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0 \text{ or } \pm \infty \text{ then}$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

As  $x \rightarrow 0$ , we have

$$\lim_{x \rightarrow 0} \frac{e^{2+x} - e^2}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 4} \frac{e^x - e^4}{x - 4} = \lim_{x \rightarrow 4} \frac{\frac{d}{dx}(e^x - e^4)}{\frac{d}{dx}(x - 4)}$$

$$\lim_{x \rightarrow 4} \frac{e^x - e^4}{x - 4} = \lim_{x \rightarrow 4} \frac{e^x}{1}$$

$$\lim_{x \rightarrow 4} \frac{e^x - e^4}{x - 4} = e^4$$

Thus, the value of  $\lim_{x \rightarrow 4} \frac{e^x - e^4}{x - 4}$  is  $e^4$ .

**Q. 34. Evaluate**

$$\lim_{x \rightarrow 0} \left( \frac{e^{3x} - e^{2x}}{x} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 0} \frac{e^{3x} - e^{2x}}{x}$$

**Formula used:**

L'Hospital's rule

Let  $f(x)$  and  $g(x)$  be two functions which are differentiable on an open interval  $I$  except at a point  $a$  where

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0 \text{ or } \pm \infty \text{ then}$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

As  $x \rightarrow 0$ , we have

$$\lim_{x \rightarrow 0} \frac{e^{3x} - e^{2x}}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{e^{3x} - e^{2x}}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(e^{3x} - e^{2x})}{\frac{d}{dx}(x)}$$

$$\lim_{x \rightarrow 0} \frac{e^{3x} - e^{2x}}{x} = \lim_{x \rightarrow 0} \frac{3e^{3x} - 2e^{2x}}{1}$$

$$\lim_{x \rightarrow 0} \frac{e^{3x} - e^{2x}}{x} = 3 - 2$$

$$\lim_{x \rightarrow 0} \frac{e^{3x} - e^{2x}}{x} = 1$$

Thus, the value of  $\lim_{x \rightarrow 0} \frac{e^{3x} - e^{2x}}{x}$  is 1.

**Q. 35. Evaluate**

$$\lim_{x \rightarrow 0} \left( \frac{e^x - x - 1}{x} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 0} \frac{e^x - x - 1}{x}$$

**Formula used:**

L'Hospital's rule

Let  $f(x)$  and  $g(x)$  be two functions which are differentiable on an open interval  $I$  except at a point  $a$  where

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0 \text{ or } \pm \infty \text{ then}$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

As  $x \rightarrow 0$ , we have

$$\lim_{x \rightarrow 0} \frac{e^x - x - 1}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{e^x - x - 1}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(e^x - x - 1)}{\frac{d}{dx}(x)}$$

$$\lim_{x \rightarrow 0} \frac{e^x - x - 1}{x} = \lim_{x \rightarrow 0} \frac{e^x - 1}{1}$$

$$\lim_{x \rightarrow 0} \frac{e^x - x - 1}{x} = 1 - 1$$

$$\lim_{x \rightarrow 0} \frac{e^x - x - 1}{x} = 0$$

Thus, the value of  $\lim_{x \rightarrow 0} \lim_{x \rightarrow 0} \frac{e^x - x - 1}{x}$  is 0.

**Q. 36. Evaluate**

$$\lim_{x \rightarrow 0} \left( \frac{e^{bx} - e^{ax}}{x} \right), 0 < a < b$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 0} \frac{e^{bx} - e^{ax}}{x}$$



**Formula used:**

L'Hospital's rule

Let  $f(x)$  and  $g(x)$  be two functions which are differentiable on an open interval  $I$  except at a point  $a$  where

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0 \text{ or } \pm \infty \text{ then}$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

As  $x \rightarrow 0$ , we have

$$\lim_{x \rightarrow 0} \frac{e^{bx} - e^{ax}}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{e^{bx} - e^{ax}}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(e^{bx} - e^{ax})}{\frac{d}{dx}(x)}$$

$$\lim_{x \rightarrow 0} \frac{e^{bx} - e^{ax}}{x} = \lim_{x \rightarrow 0} \frac{be^{bx} - ae^{ax}}{1}$$

$$\lim_{x \rightarrow 0} \frac{e^{bx} - e^{ax}}{x} = b - a$$

Thus, the value of  $\lim_{x \rightarrow 0} \frac{e^{bx} - e^{ax}}{x}$  is  $b - a$ .

**Q. 37. Evaluate**

$$\lim_{x \rightarrow 0} \left( \frac{a^x - b^x}{x} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 0} \frac{a^x - b^x}{x}$$

**Formula used:**

L'Hospital's rule

Let  $f(x)$  and  $g(x)$  be two functions which are differentiable on an open interval  $I$  except at a point  $a$  where

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0 \text{ or } \pm \infty \text{ then}$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

As  $x \rightarrow 0$ , we have



$$\lim_{x \rightarrow 0} \frac{a^x - b^x}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{a^x - b^x}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(a^x - b^x)}{\frac{d}{dx}(x)}$$

$$\lim_{x \rightarrow 0} \frac{a^x - b^x}{x} = \lim_{x \rightarrow 0} \frac{a^x \ln a - b^x \ln b}{1}$$

$$\lim_{x \rightarrow 0} \frac{a^x - b^x}{x} = \ln a - \ln b$$

$$\lim_{x \rightarrow 0} \frac{a^x - b^x}{x} = \ln \frac{a}{b}$$

Thus, the value of  $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x}$  is  $\ln \frac{a}{b}$ .

**Q. 38. Evaluate**

$$\lim_{x \rightarrow 0} \left( \frac{a^x - a^{-x}}{x} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 0} \frac{a^x - a^{-x}}{x}$$

**Formula used:**

L'Hospital's rule

Let  $f(x)$  and  $g(x)$  be two functions which are differentiable on an open interval  $I$  except at a point  $a$  where

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0 \text{ or } \pm \infty \text{ then}$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

As  $x \rightarrow 0$ , we have

$$\lim_{x \rightarrow 0} \frac{a^x - a^{-x}}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{a^x - a^{-x}}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(a^x - a^{-x})}{\frac{d}{dx}(x)}$$

$$\lim_{x \rightarrow 0} \frac{a^x - a^{-x}}{x} = \lim_{x \rightarrow 0} \frac{a^x \ln a + a^{-x} \ln a}{1}$$

$$\lim_{x \rightarrow 0} \frac{a^x - a^{-x}}{x} = 2 \ln a$$

Thus, the value of  $\lim_{x \rightarrow 0} \frac{a^x - a^{-x}}{x}$  is  $2 \ln a$ .

**Q. 39. Evaluate**

$$\lim_{x \rightarrow 0} \left( \frac{2^x - 1}{x} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 0} \frac{2^x - 1}{x}$$

**Formula used:**

L'Hospital's rule

Let  $f(x)$  and  $g(x)$  be two functions which are differentiable on an open interval  $I$  except at a point  $a$  where

$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$  or  $\pm \infty$  then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

As  $x \rightarrow 0$ , we have

$$\lim_{x \rightarrow 0} \frac{2^x - 1}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{2^x - 1}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(2^x - 1)}{\frac{d}{dx}(x)}$$

$$\lim_{x \rightarrow 0} \frac{2^x - 1}{x} = \lim_{x \rightarrow 0} \frac{2^x \ln 2}{1}$$

$$\lim_{x \rightarrow 0} \frac{2^x - 1}{x} = \ln 2$$

Thus, the value of  $\lim_{x \rightarrow 0} \frac{2^x - 1}{x}$  is  $\ln 2$ .

**Q. 40. Evaluate**

$$\lim_{x \rightarrow 0} \left( \frac{3^{2+x} - 9}{x} \right)$$

**Answer :** To evaluate:

$$\lim_{x \rightarrow 0} \frac{3^{2+x} - 9}{x}$$

**Formula used:**

L'Hospital's rule

Let  $f(x)$  and  $g(x)$  be two functions which are differentiable on an open interval  $I$  except at a point  $a$  where

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0 \text{ or } \pm \infty \text{ then}$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

As  $x \rightarrow 0$ , we have

$$\lim_{x \rightarrow 0} \frac{3^{2+x} - 9}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{3^{2+x} - 9}{x} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(3^{2+x} - 9)}{\frac{d}{dx}(x)}$$

$$\lim_{x \rightarrow 0} \frac{3^{2+x} - 9}{x} = \lim_{x \rightarrow 0} \frac{(2+x) \ln 3}{1}$$

$$\lim_{x \rightarrow 0} \frac{3^{2+x} - 9}{x} = 2 \ln 3$$

Thus, the value of  $\lim_{x \rightarrow 0} \frac{3^{2+x} - 9}{x}$  is  $2 \ln 3$

## Exercise 27B

**Q. 1. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{6x}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfied any one from 7 indeterminate forms.

In this Case, indeterminate Form is  $\frac{0}{0}$

Formula used:  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

$$\text{So } \lim_{x \rightarrow 0} \frac{\sin 4x}{6x} = \lim_{x \rightarrow 0} \left( \frac{\sin 4x}{4x} \right) \times \frac{4}{6} = \frac{4}{6} = \frac{2}{3}$$

Therefore,  $\lim_{x \rightarrow 0} \frac{\sin 4x}{6x} = \frac{2}{3}$

**Q. 2. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\sin 5x}{\sin 8x}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfied any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

Formula used:  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

$$\text{So } \lim_{x \rightarrow 0} \frac{\sin 5x}{\sin 8x} = \lim_{x \rightarrow 0} \left( \frac{\sin 5x}{5x} \right) \times \frac{8x}{\sin 8x} \times \frac{5x}{8x} = \frac{5x}{8x} = \frac{5}{8}$$

Therefore,  $\lim_{x \rightarrow 0} \frac{\sin 5x}{\sin 8x} = \frac{5}{8}$

**Q. 3. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\tan 3x}{\tan 5x}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfied any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

Formula used:  $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$

$$\text{So } \lim_{x \rightarrow 0} \frac{\tan 3x}{\tan 5x} = \lim_{x \rightarrow 0} \left( \frac{\tan 3x}{3x} \right) \times \frac{5x}{\sin 5x} \times \frac{3x}{5x} = \frac{3x}{5x} = \frac{3}{5}$$

Therefore,  $\lim_{x \rightarrow 0} \frac{\tan 3x}{\tan 5x} = \frac{3}{5}$



**Q. 4. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\tan \alpha x}{\tan \beta x}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfied any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

Formula used:  $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$

$$\text{So } \lim_{x \rightarrow 0} \frac{\tan \alpha x}{\tan \beta x} = \lim_{x \rightarrow 0} \left( \frac{\tan \alpha x}{\alpha x} \right) \times \frac{\beta x}{\sin \beta x} \times \frac{\alpha x}{\beta x} = \frac{\alpha x}{\beta x} = \frac{\alpha}{\beta}$$

$$\text{Therefore, } \lim_{x \rightarrow 0} \frac{\tan \alpha x}{\tan \beta x} = \frac{\alpha}{\beta}$$

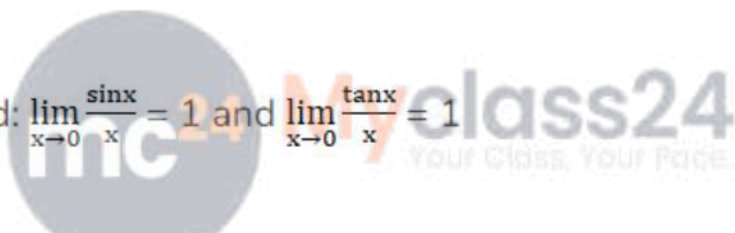
**Q. 5. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{\tan 7x}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

Formula used:  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$  and  $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$   Your Class, Your Pace.

$$\text{So } \lim_{x \rightarrow 0} \frac{\sin 4x}{\tan 7x} = \lim_{x \rightarrow 0} \left( \frac{\sin 4x}{4x} \right) \times \frac{7x}{\sin 7x} \times \frac{4x}{7x} = \frac{4x}{7x} = \frac{4}{7}$$

$$\text{Therefore, } \lim_{x \rightarrow 0} \frac{\sin 4x}{\tan 7x} = \frac{4}{7}$$

**Q. 6. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\tan 3x}{\sin 4x}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

Formula used:  $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$  and  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

$$\text{So } \lim_{x \rightarrow 0} \frac{\tan 3x}{\sin 4x} = \lim_{x \rightarrow 0} \left( \frac{\tan 3x}{3x} \right) \times \frac{4x}{\sin 4x} \times \frac{3x}{4x} = \frac{3x}{4x} = \frac{3}{4}$$

$$\text{Therefore, } \lim_{x \rightarrow 0} \frac{\tan 3x}{\sin 4x} = \frac{3}{4}$$

**Q. 7. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\sin mx}{\tan nx}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Use this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

Formula used:  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$  and  $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$

$$\text{So } \lim_{x \rightarrow 0} \frac{\sin mx}{\tan nx} = \lim_{x \rightarrow 0} \left( \frac{\sin mx}{mx} \right) \times \frac{nx}{\tan nx} \times \frac{mx}{nx} = \frac{mx}{nx} = \frac{m}{n}$$

$$\text{Therefore, } \lim_{x \rightarrow 0} \frac{\sin mx}{\tan nx} = \frac{m}{n}$$

**Q. 8. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\sin x - 2 \sin 3x + \sin 5x}{x}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

Formula used:  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

$$\text{So } \lim_{x \rightarrow 0} \frac{\sin x - 2 \sin 3x + \sin 5x}{x} = \lim_{x \rightarrow 0} \left( \frac{\sin x}{x} - \frac{2 \sin 3x}{x} + \frac{\sin 5x}{x} \right) = \lim_{x \rightarrow 0} \left( \frac{\sin x}{x} - \frac{2 \sin 3x}{3x} \times 3 + \frac{\sin 5x}{5x} \right)$$

By using the above formula, we have

$$\therefore \lim_{x \rightarrow 0} \left( \frac{\sin x}{x} - \frac{2 \sin 3x}{3x} \times 3 + \frac{\sin 5x}{5x} \right) = 1 - 2 \times 3 + 5 = 0$$

$$\text{Therefore, } \lim_{x \rightarrow 0} \frac{\sin x - 2 \sin 3x + \sin 5x}{x} = 0$$

**Q. 9. Evaluate the following limits:**

$$\lim_{x \rightarrow \pi/6} \frac{(2 \sin^2 x + \sin x - 1)}{(2 \sin^2 x - 3 \sin x + 1)}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

Formula used:  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$  or we can use L hospital Rule,

So, by using the rule, Differentiate numerator and denominator

$$\lim_{x \rightarrow \frac{\pi}{6}} \frac{4 \sin x \cos x + \cos x}{4 \sin x \cos x - 3 \cos x} = \lim_{x \rightarrow \frac{\pi}{6}} \frac{4 \sin x + 1}{4 \sin x - 3} = \frac{2+1}{2-3} = -3$$

$$\text{Therefore, } \lim_{x \rightarrow 0} \frac{2 \sin^2 x + \sin x - 1}{2 \sin^2 x - 3 \sin x + 1} = -3$$

**Q. 10. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{(\sin 2x + 3x)}{(2x + \sin 3x)}$$



**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

Formula used:  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$  or we can use L hospital Rule,

So, by using the above formula, we have

Divide numerator and denominator by x,

$$\lim_{x \rightarrow 0} \frac{\sin 2x + 3x}{2x + \sin 3x} = \lim_{x \rightarrow 0} \frac{\frac{\sin 2x + 3x}{x}}{\frac{2x + \sin 3x}{x}} = \lim_{x \rightarrow 0} \frac{\frac{\sin 2x}{x} + 3}{2 + \frac{\sin 3x}{x}} = \lim_{x \rightarrow 0} \frac{\frac{2 \sin 2x}{2x} + 3}{2 + \frac{2 \sin 3x}{3x}} = \frac{2+3}{2+3} = 1$$

ALTER: by using the rule, Differentiate numerator and denominator

$$\lim_{x \rightarrow 0} \frac{2 \cos 2x + 3}{2 + 3 \cos 3x} = \frac{5}{5} = 1$$

Therefore,  $\lim_{x \rightarrow 0} \frac{\sin 2x + 3x}{2x + \sin 3x} = 1$

**Q. 11. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{(\tan 2x - x)}{(3x - \tan x)}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

Formula used:  $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$  or we can use L hospital Rule,

So, by using the above formula, we have

Divide numerator and denominator by x,

$$\lim_{x \rightarrow 0} \frac{\tan 2x - x}{3x - \tan x} = \lim_{x \rightarrow 0} \frac{\frac{\tan 2x - x}{x}}{\frac{3x - \tan x}{x}} = \lim_{x \rightarrow 0} \frac{\frac{\tan 2x}{x} - 1}{3 - \frac{\tan x}{x}} = \lim_{x \rightarrow 0} \frac{\frac{2 \tan 2x}{2x} - 1}{3 - \frac{\tan x}{x}} = \frac{2-1}{3-1} = \frac{1}{2}$$

Therefore,  $\lim_{x \rightarrow 0} \frac{\tan 2x - x}{3x - \tan x} = \frac{1}{2}$

**Q. 12. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{(x^2 - \tan 2x)}{\tan x}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

Formula used:  $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$  or we can use L hospital Rule,

So, by using the above formula, we have

Divide numerator and denominator by x,

$$\lim_{x \rightarrow 0} \frac{x^2 - \tan 2x}{\tan x} = \lim_{x \rightarrow 0} \frac{\frac{x^2 - \tan 2x}{x}}{\frac{\tan x}{x}} = \lim_{x \rightarrow 0} \frac{x - \frac{\tan 2x}{x}}{\frac{\tan x}{x}} = \lim_{x \rightarrow 0} \frac{x - \frac{2 \tan 2x}{2x}}{\frac{\tan x}{x}} = \frac{0 - 2}{1} = -2$$

Therefore,  $\lim_{x \rightarrow 0} \frac{x^2 - \tan 2x}{\tan x} = -2$

**Q. 13. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{x \cos x + \sin x}{x^2 + \tan x}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

Formula used:  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$  and  $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$

So, by using the above formula, we have

Divide numerator and denominator by x,

$$\lim_{x \rightarrow 0} \frac{x \cos x + \sin x}{x^2 + \tan x} = \lim_{x \rightarrow 0} \frac{\frac{x \cos x + \sin x}{x}}{\frac{x^2 + \tan x}{x}} = \lim_{x \rightarrow 0} \frac{\cos x + \frac{\sin x}{x}}{x + \frac{\tan x}{x}} = \frac{1+1}{0+1} = 2$$

Therefore,  $\lim_{x \rightarrow 0} \frac{x \cos x + \sin x}{x^2 + \tan x} = 2$

**Q. 14. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{\sin^3 x}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

NOTE :  $\tan x - \sin x = \frac{\sin x}{\cos x} - \sin x = \frac{\sin x - \sin x \cos x}{\cos x} = \sin x \left( \frac{1 - \cos x}{\cos x} \right)$

$$\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{\sin^3 x} = \lim_{x \rightarrow 0} \frac{\frac{1 - \cos x}{\cos x}}{\sin^2 x} = \lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 x \cos x}$$

Divide numerator and denominator by  $x^2$ ,

$$= \lim_{x \rightarrow 0} \frac{\frac{1 - \cos x}{x^2}}{\frac{\sin^2 x \cos x}{x^2}}$$

Formula used:  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = 1/2$  and  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$  or we can use L hospital Rule,

So, by using the above formula, we have

$$\lim_{x \rightarrow 0} \frac{\frac{1 - \cos x}{x^2} \cos x}{\frac{\sin^2 x}{x^2}} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$$

Therefore,  $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{\sin^3 x} = \frac{1}{2}$

**Q. 15. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} x \operatorname{cosec} x$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form are  $0 \times \infty$

Formula used:  $\lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$

So, by using the above formula, we have

$$\lim_{x \rightarrow 0} x \operatorname{cosec} x = \lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$$

Therefore,  $\lim_{x \rightarrow 0} x \operatorname{cosec} x = 1$

**Q. 16. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} (x \cot 2x)$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $0 \times \infty$

Formula used:  $\lim_{x \rightarrow 0} \frac{x}{\tan x} = 1$

So, by using the above formula, we have

$$\lim_{x \rightarrow 0} x \cot 2x = \lim_{x \rightarrow 0} \frac{2x}{2 \tan 2x} = \frac{1}{2}$$

Therefore,  $\lim_{x \rightarrow 0} x \cot x = \frac{1}{2}$



**Q. 17. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\sin x \cos x}{3x}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

Formula used:  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

So, by using the above formula, we have

$$\lim_{x \rightarrow 0} \frac{\sin x \cos x}{3x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \times \frac{\cos x}{3} = \frac{1}{3}$$

$$\text{Therefore, } \lim_{x \rightarrow 0} \frac{\sin x \cos x}{3x} = \frac{1}{3}$$

**Q. 18. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\sin(x/4)}{x}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

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Formula used:  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

So, by using the above formula, we have

$$\lim_{x \rightarrow 0} \frac{\sin(x/4)}{x} = \lim_{x \rightarrow 0} \frac{\sin(x/4)}{4(x/4)} = \frac{1}{4}$$

$$\text{Therefore, } \lim_{x \rightarrow 0} \frac{\sin(x/4)}{x} = \frac{1}{4}$$

**Q. 19. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\tan(x/2)}{3x}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

Formula used:  $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$

So, by using the above formula, we have

$$\lim_{x \rightarrow 0} \frac{\tan(x/2)}{3x} = \lim_{x \rightarrow 0} \frac{\tan(x/2)}{6(x/2)} = \frac{1}{6} \text{ [Divide and multiply with 2 on denominator]}$$

Therefore,  $\lim_{x \rightarrow 0} \frac{\tan(x/2)}{3x} = \frac{1}{6}$



**Q. 20. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 x}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

[NOTE:  $1 - \cos x = 2 \sin^2(x/2)$ ]

Formula used:  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

So, by using the above formula, we have

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 x} = \lim_{x \rightarrow 0} \frac{2 \sin^2(x/2)}{\sin^2 x}$$

Divide numerator and denominator by  $x^2$ , we have

$$\lim_{x \rightarrow 0} \frac{2 \sin^2(x/2)}{\sin^2 x} = \lim_{x \rightarrow 0} \frac{\frac{2 \sin^2(\frac{x}{2})}{x^2}}{\frac{\sin^2 x}{x^2}} = \lim_{x \rightarrow 0} \frac{\frac{2 \sin^2(\frac{x}{2})}{4 \frac{x^2}{4}}}{\frac{\sin^2 x}{x^2}} = \frac{2}{4} = \frac{2}{4} = \frac{1}{2}$$

$$[\text{NOTE: } \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}]$$

$$\text{Therefore, } \lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 x} = \frac{1}{2}$$

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**Q. 21. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{1 - \cos 3x}{x^2}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

$$\text{Formula used: } \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$$

So, by using the above formula, we have

$$\lim_{x \rightarrow 0} \frac{1 - \cos 3x}{x^2} = \lim_{x \rightarrow 0} \frac{9[1 - \cos 3x]}{(3x)^2} = \frac{9}{2}$$

Therefore,  $\lim_{x \rightarrow 0} \frac{1 - \cos 3x}{x^2} = \frac{9}{2}$

**Q. 22. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 2x}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

Formula used:  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$  and  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

Divide numerator and denominator by  $x^2$ , we have

So, by using the above formula, we have

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 2x} = \lim_{x \rightarrow 0} \frac{\frac{[1 - \cos x]}{x^2}}{\frac{\sin^2 2x}{x^2}} = \frac{1}{2}$$

Therefore,  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 2x} = \frac{1}{2}$

**Q. 23. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{3 \tan^2 x}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

Formula used:  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$  and  $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$

Divide numerator and denominator by  $x^2$ , we have

So, by using the above formula, we have

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{3 \tan^2 x} = \lim_{x \rightarrow 0} \frac{\frac{4[1 - \cos 2x]}{(4)x^2}}{\frac{3 \tan^2 x}{x^2}} = \frac{4}{6} = \frac{2}{3}$$

Therefore,  $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{3 \tan^2 x} = \frac{1}{6}$

**Q. 24. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{(1 - \cos 4x)}{(1 - \cos 6x)}$$

**Answer :**

To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

Formula used:  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$

Divide numerator and denominator by  $x^2$ , we have

So, by using the above formula, we have

$$\lim_{x \rightarrow 0} \frac{1 - \cos 4x}{1 - \cos 6x} = \lim_{x \rightarrow 0} \frac{\frac{16[1 - \cos 4x]}{(4x)^2}}{\frac{36[1 - \cos 6x]}{(6x)^2}} = \frac{\frac{16}{2}}{\frac{36}{2}} = \frac{8}{18} = \frac{4}{9}$$

Therefore,  $\lim_{x \rightarrow 0} \frac{1 - \cos 4x}{1 - \cos 6x} = \frac{4}{9}$



**Q. 25. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{1 - \cos mx}{1 - \cos nx}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

Formula used:  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$

Divide numerator and denominator by  $m^2$  and  $n^2$ , we have

So, by using the above formula, we have

$$\lim_{x \rightarrow 0} \frac{1 - \cos mx}{1 - \cos nx} = \lim_{x \rightarrow 0} \frac{\frac{m^2[1 - \cos mx]}{(mx)^2}}{\frac{n^2[1 - \cos nx]}{(nx)^2}} = \frac{m^2}{n^2}$$

Therefore,  $\lim_{x \rightarrow 0} \frac{1 - \cos mx}{1 - \cos nx} = \frac{m^2}{n^2}$

**Q. 26. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{2 \sin x - \sin 2x}{x^3}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

We know that  $\sin 2x = 2 \sin x \cos x$

Formula used:  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$  and  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

So, by using the above formula, we have

$$\lim_{x \rightarrow 0} \frac{2 \sin x - \sin 2x}{x^3} = \lim_{x \rightarrow 0} \frac{2 \sin x - 2 \sin x \cos x}{x^3} = \lim_{x \rightarrow 0} \frac{2 \sin x [1 - \cos x]}{x^3} = \lim_{x \rightarrow 0} \frac{2 \sin x}{x} \times \frac{[1 - \cos x]}{x^2} = \frac{2}{2} = 1$$

Therefore,  $\lim_{x \rightarrow 0} \frac{2 \sin x - \sin 2x}{x^3} = 1$

**Q. 27. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{(\tan x - \sin x)}{x^3}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

$$\text{NOTE : } \tan x - \sin x = \frac{\sin x}{\cos x} - \sin x = \frac{\sin x - \sin x \cos x}{\cos x} = \sin x \left( \frac{1 - \cos x}{\cos x} \right)$$

$$\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3} = \lim_{x \rightarrow 0} \frac{\left( \frac{1 - \cos x}{\cos x} \right) \sin x}{x^3} = \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2 \cos x} \times \frac{\sin x}{x}$$

Formula used:  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = 1/2$  and  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$  or we can use L hospital Rule,

So, by using the above formula, we have

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2 \cos x} \times \frac{\sin x}{x} = \frac{1}{2}$$

$$\text{Therefore, } \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3} = \frac{1}{2}$$

**Q. 28. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\tan 2x - \sin 2x}{x^3}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

$$\lim_{x \rightarrow 0} \frac{\tan 2x - \sin 2x}{x^3} = \lim_{x \rightarrow 0} \frac{\sin 2x + \sin 2x \cos 2x}{x^3} = \lim_{x \rightarrow 0} \frac{\sin 2x(1 + \cos 2x)}{x^3} = \lim_{x \rightarrow 0} \frac{2 \sin 2x}{2x} \times \frac{4(1 + \cos 2x)}{(2x)^2} = 4$$

Therefore,  $\lim_{x \rightarrow 0} \frac{\tan 2x - \sin 2x}{x^3} = 4$

**Q. 29. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\operatorname{cosec} x - \cot x}{x}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\infty \times \infty$

$$\operatorname{cosec} x - \cot x = \frac{1 - \cos x}{\sin x}$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{cosec} x - \cot x}{x} = \lim_{x \rightarrow 0} \frac{1 - \cos x}{x \sin x} = \lim_{x \rightarrow 0} \frac{\frac{1 - \cos x}{x^2}}{\frac{\sin x}{x}} = \lim_{x \rightarrow 0} \frac{\frac{1 - \cos x}{x^2}}{\frac{\sin x}{x}}$$

Formula used:  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = 1/2$  and  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

$$\lim_{x \rightarrow 0} \frac{\operatorname{cosec} x - \cot x}{x} = \lim_{x \rightarrow 0} \frac{\frac{1 - \cos x}{x^2}}{\frac{\sin x}{x}} = \frac{1}{2}$$

Therefore,  $\lim_{x \rightarrow 0} \frac{\operatorname{cosec} x - \cot x}{x} = \frac{1}{2}$

**Q. 30. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\cot 2x - \operatorname{cosec} 2x}{x}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form are  $\infty \times \infty$

$$\operatorname{cosec} 2x - \cot 2x = (1 - \cos 2x)/\sin 2x$$

$$\lim_{x \rightarrow 0} \frac{\cot 2x - \operatorname{cosec} 2x}{x} = \lim_{x \rightarrow 0} \frac{\cos 2x - 1}{x \sin 2x} = \lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\frac{x^2}{x \sin 2x}} = \lim_{x \rightarrow 0} \frac{4[\cos 2x - 1]}{\frac{(2x)^2}{2 \sin 2x}}$$

Formula used:  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = 1/2$  and  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

$$\lim_{x \rightarrow 0} \frac{\cot 2x - \operatorname{cosec} 2x}{x} = \lim_{x \rightarrow 0} \frac{4[\cos 2x - 1]}{\frac{(2x)^2}{2 \sin 2x}} = \frac{-4}{2} = -2$$

Therefore,  $\lim_{x \rightarrow 0} \frac{\cot 2x - \operatorname{cosec} 2x}{x} = -2$

**Q. 31. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\sin 2x(1 - \cos 2x)}{x^3}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

$$\lim_{x \rightarrow 0} \frac{\sin 2x(1 - \cos 2x)}{x^3} = \lim_{x \rightarrow 0} \frac{\sin 2x}{x} \times \frac{(1 - \cos 2x)}{x^2} = \lim_{x \rightarrow 0} \frac{2\sin 2x}{2x} \times \frac{4(1 - \cos 2x)}{(2x)^2}$$

Formula used:  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = 1/2$  and  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

$$\lim_{x \rightarrow 0} \frac{\sin 2x(1 - \cos 2x)}{x^3} = 4$$

Therefore,  $\lim_{x \rightarrow 0} \frac{\sin 2x(1 - \cos 2x)}{x^3} = 4$

**Q. 32 . Evaluate the following limits:**

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sec^2 x - 2}{\tan x - 1}$$



**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

By using L hospital Rule,

Differtiate both sides w.r.t x

$$\text{So } \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sec^2 x - 2}{\tan x - 1} = \text{So } \lim_{x \rightarrow \frac{\pi}{4}} \frac{2\sec x(\sec x \tan x) - 0}{\sec^2 x - 0} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{2\sec x(\sec x \tan x)}{\sec^2 x} = \lim_{x \rightarrow \frac{\pi}{4}} 2 \tan x = 2$$

Therefore,  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sec^2 x - 2}{\tan x - 1} = 2$

**Q. 33. Evaluate the following limits:**

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\operatorname{cosec}^2 x - 2}{\cot x - 1}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

By using L hospital Rule,

Differtiate both sides w.r.t x 

$$\text{So } \lim_{x \rightarrow \frac{\pi}{4}} \frac{\operatorname{cosec}^2 x - 2}{\cot x - 1} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{2 \operatorname{cosec} x (-\operatorname{cosec} x \cot x) - 0}{-\operatorname{cosec}^2 x - 0} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{2 \operatorname{cosec} x (-\operatorname{cosec} x \cot x)}{-\operatorname{cosec}^2 x} = \lim_{x \rightarrow \frac{\pi}{4}} 2 \cot x = 2$$

Therefore,  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\operatorname{cosec}^2 x - 2}{\cot x - 1} = 2$

**Q. 34. Evaluate the following limits:**

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \tan x}{x - \frac{\pi}{4}}$$

**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

By using L hospital Rule,

Differtiate both sides w.r.t x

$$\text{So } \lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \tan x}{x - \frac{\pi}{4}} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{0 - \sec^2 x}{1 - 0} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{-\sec^2 x}{1} = -2$$

$$\text{Therefore, } \lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \tan x}{x - \frac{\pi}{4}} = -2$$

**Q. 35. Evaluate the following limits:**

$$\lim_{x \rightarrow \pi} \frac{\sin 3x - 3\sin x}{(\pi - x)^3}$$



**Answer :** To Find: Limits

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

By using L hospital Rule,

Differtiate both sides w.r.t x

$$\text{So } \lim_{x \rightarrow \pi} \frac{\sin 3x - 3\sin x}{(\pi - x)^3} = \lim_{x \rightarrow \pi} \frac{3\cos 3x - 3\cos x}{-3(\pi - x)^2}$$

Again, indeterminate Form is  $\frac{0}{0}$

So, Differentiate both sides w.r.t x again, we have

$$\lim_{x \rightarrow \pi} \frac{\sin 3x - 3\sin x}{(\pi - x)^2} = \lim_{x \rightarrow \pi} \frac{-9\sin 3x + 3\sin x}{6(\pi - x)}$$

Again, indeterminate Form is  $\frac{0}{0}$

So, Differentiate both sides w.r.t x again, we have

$$\lim_{x \rightarrow \pi} \frac{\sin 3x - 3\sin x}{(\pi - x)^2} = \lim_{x \rightarrow \pi} \frac{-27\cos 3x + 3\cos x}{-6} = \frac{-27\cos 3\pi + 3\cos \pi}{-6} = \frac{27 - 3}{-6} = -4$$

Therefore,  $\lim_{x \rightarrow \pi} \frac{\sin 3x - 3\sin x}{(\pi - x)^2} = -4$

**Q. 36. Evaluate the following limits:**

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 + \cos 2x}{(\pi - 2x)^2}$$

**Answer : To Find: Limits**

NOTE: First Check the form of limit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is  $\frac{0}{0}$

By using L hospital Rule,

Differentiate both sides w.r.t x

$$\text{So } \lim_{x \rightarrow \frac{\pi}{2}} \frac{1 + \cos 2x}{(\pi - 2x)^2} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{0 + (-2 \sin 2x)}{2(\pi - 2x)(-2)} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{-2 \sin 2x}{-4(\pi - 2x)} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{2 \sin 2x}{4(\pi - 2x)}$$

Again, indeterminate Form is  $\frac{0}{0}$

So, Differentiate both sides w.r.t x again, we have

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 + \cos 2x}{(\pi - 2x)^2} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{4 \cos 2x}{4(-2)} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos 2x}{(-2)} = \frac{\cos \pi}{(-2)} = \frac{-1}{(-2)} = \frac{1}{2}$$

$$\text{Therefore, } \lim_{x \rightarrow \frac{\pi}{2}} \frac{1 + \cos 2x}{(\pi - 2x)^2} = \frac{1}{2}$$

**Q. 37. Evaluate the following limits:**

$$\lim_{x \rightarrow a} \frac{(\cos x - \cos a)}{(x - a)}$$

**Answer :**

$$= \lim_{x \rightarrow a} \frac{(\cos x - \cos a)}{(x - a)}$$

$$= \lim_{x \rightarrow a} \frac{-2 \times \sin\left(\frac{x+a}{2}\right) \sin\left(\frac{x-a}{2}\right)}{x-a} \left[ \because \cos x - \cos a = -2 \times \sin \frac{x+a}{2} \sin \frac{x-a}{2} \right]$$

$$= \lim_{x \rightarrow a} \sin \frac{x+a}{2} \times -\frac{\sin\left(\frac{x-a}{2}\right)}{(x-a)}$$

$$= -1 \times \lim_{x \rightarrow a} \sin \frac{x+a}{2} \left[ \because \lim_{x \rightarrow a} \frac{\sin \theta}{\theta} = 1 \right]$$

$$= -1 \times \sin \frac{(a+a)}{2}$$

$$= -1 \times \sin \frac{2a}{2}$$

$$= -\sin(a)$$

$$\therefore \lim_{x \rightarrow a} \frac{(\cos x - \cos a)}{(x - a)} = -\sin a$$

**Q. 38. Evaluate the following limits:**

$$\lim_{x \rightarrow a} \frac{(\sin x - \sin a)}{(x - a)}$$

**Answer :**

$$= \lim_{x \rightarrow a} \frac{(\sin x - \sin a)}{(x - a)}$$

$$= \lim_{x \rightarrow a} \frac{\left(2 \times \cos \frac{x+a}{2} \sin \frac{x-a}{2}\right)}{(x-a)} \left[ \because \sin x - \sin a = 2 \times \cos \frac{x+a}{2} \sin \frac{x-a}{2} \right]$$

$$= 1 \times \lim_{x \rightarrow a} \cos \frac{x+a}{2} \left[ \because \lim_{x \rightarrow a} \frac{\sin \theta}{\theta} = 1 \right]$$

$$= \cos \frac{a+a}{2}$$

$$= \cos a$$

$$\therefore \lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a} = \cos a$$

**Q. 39. Evaluate the following limits:**

$$\lim_{x \rightarrow a} \frac{(\sin x - \sin a)}{(\sqrt{x} - \sqrt{a})}$$

**Answer :**

$$= \lim_{x \rightarrow a} \frac{(\sin x - \sin a)}{(\sqrt{x} - \sqrt{a})}$$

$$= \lim_{x \rightarrow a} \frac{(\sin x - \sin a)}{(\sqrt{x} - \sqrt{a})} \times \frac{(\sqrt{x} + \sqrt{a})}{(\sqrt{x} + \sqrt{a})} \text{ [Multiply and divide by } \sqrt{x} - \sqrt{a}]$$

$$= \lim_{x \rightarrow a} \frac{(\sin x - \sin a) \times (\sqrt{x} + \sqrt{a})}{(x - a)}$$

$$= \cos a \times \lim_{x \rightarrow a} (\sqrt{x} + \sqrt{a}) \left[ \because \lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a} = \cos a \right]$$

$$= 2\sqrt{a} \times \cos a$$

$$= 2\sqrt{a} \cos a$$

$$\therefore \lim_{x \rightarrow a} \frac{(\sin x - \sin a)}{(\sqrt{x} - \sqrt{a})} = 2\sqrt{a} \cos a$$

**Q. 40. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\sin 5x - \sin 3x}{\sin x}$$

**Answer :**

$$= \lim_{x \rightarrow 0} \frac{\left( 2 \sin \frac{5x - 3x}{2} \cos \frac{5x + 3x}{2} \right)}{\sin x} \left[ \text{Applying } \sin C - \sin D \right]$$

$$= 2 \sin \frac{C - D}{2} \cos \frac{C + D}{2}$$

$$= \lim_{x \rightarrow 0} 2 \cos 4x$$

$$= 2 \times 1$$

$$\therefore \lim_{x \rightarrow 0} \frac{\sin 5x - \sin 3x}{\sin x} = 2$$

**Q. 41. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{(\cos 3x - \cos 5x)}{x^2}$$

**Answer :**

$$\begin{aligned}
&= \lim_{x \rightarrow 0} \frac{(1 - \cos 5x) - (1 - \cos 3x)}{x^2} \\
&= \lim_{x \rightarrow 0} \left( \frac{1 - \cos 5x}{x^2} - \frac{1 - \cos 3x}{x^2} \right) \\
&= \left( \lim_{x \rightarrow 0} \frac{1 - \cos 5x}{x^2} \times \frac{25}{25} \right) - \left( \lim_{x \rightarrow 0} \frac{1 - \cos 3x}{x^2} \times \frac{9}{9} \right) \\
&= \frac{25}{2} - \frac{9}{2} \left[ \because \lim_{x \rightarrow 0} \frac{1 - \cos ax}{(ax)^2} = \frac{1}{2} \right] \\
&= \frac{16}{2} \\
&= 8
\end{aligned}$$

$$\therefore \lim_{x \rightarrow 0} \frac{(\cos 3x - \cos 5x)}{x^2} = 8$$

**Q. 42. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{(\sin 3x + \sin 5x)}{(\sin 6x - \sin 4x)}$$

**Answer :**

$$\begin{aligned}
&= \lim_{x \rightarrow 0} \frac{\left( 2 \times \sin \frac{3x + 5x}{2} \times \cos \frac{3x - 5x}{2} \right)}{\left( 2 \times \cos \frac{6x + 4x}{2} \sin \frac{6x - 4x}{2} \right)} \\
&= \lim_{x \rightarrow 0} \frac{\sin 4x \cos x}{\cos 5x \sin x} \\
&= \lim_{x \rightarrow 0} \frac{\sin 4x}{\cos 5x} \times \frac{\sin x}{\cos x} \times \frac{4x}{4x} \\
&= 4 \times \lim_{x \rightarrow 0} \frac{\sin 4x}{4x} \times \frac{1}{\cos 5x} \times \frac{x}{\tan x} \left[ \because \lim_{x \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \right. \\
&\quad \left. \lim_{x \rightarrow 0} \frac{\theta}{\tan \theta} = 1 \right] \\
&= 4
\end{aligned}$$

$$\therefore \lim_{x \rightarrow 0} \frac{(\sin 3x + \sin 5x)}{(\sin 6x - \sin 4x)} = 4$$

**Q. 43. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{[\sin(2+x) - \sin(2-x)]}{x}$$

**Answer :**

$$= \lim_{x \rightarrow 0} \frac{[\sin(2+x) - \sin(2-x)]}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\left[ 2 \times \cos \frac{(2+x+2-x)}{2} \times \sin \frac{(2+x-2+x)}{2} \right]}{x}$$

$$= \lim_{x \rightarrow 0} \frac{(2 \times \cos 2 \times \sin x)}{x}$$

$$= 2 \cos 2 \lim_{x \rightarrow 0} \frac{\sin x}{x}$$

$$= 2 \cos 2$$

$$\lim_{x \rightarrow 0} \frac{[\sin(2+x) - \sin(2-x)]}{x} = 2 \cos 2$$

**Q. 44. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{(1 - \cos 2x)}{(\cos 2x - \cos 8x)}$$

**Answer :**

$$= \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{(\cos 2x - \cos 8x)}$$

$$= \lim_{x \rightarrow 0} \frac{2 \times \sin x \times \sin x}{2 \times \sin 3x \times \sin 5x} \times \frac{5x \times 3x}{x \times x} \times \frac{1}{15}$$

$$= \frac{1}{15} \times 1 \times 1 \times 1 \times 1 \left[ \because \lim_{x \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \right]$$

$$= \frac{1}{15}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{(\cos 2x - \cos 8x)} = \frac{1}{15}$$

**Q. 45. Evaluate the following limits:**

$$\lim_{x \rightarrow \frac{\pi}{2}} \left( \frac{\pi}{2} - x \right) \tan x$$

**Answer :**

$$= \lim_{x \rightarrow \frac{\pi}{2}} \left( \frac{\pi}{2} - x \right) \tan x$$

$$= -1 \times \lim_{y \rightarrow 0} y \tan \left( y + \frac{\pi}{2} \right) \left[ x - \frac{\pi}{2} = y \right]$$

$$= -1 \times \lim_{y \rightarrow 0} y \cot y \times -1$$

$$= \lim_{y \rightarrow 0} \frac{y}{\tan y}$$

$$= 1$$

$$\therefore \lim_{x \rightarrow \frac{\pi}{2}} \left( \frac{\pi}{2} - x \right) \tan x = 1$$

**Q. 46. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{(\sqrt{1+2x} - \sqrt{1-2x})}{\sin x}$$

**Answer :**

$$\begin{aligned}
&= \lim_{x \rightarrow 0} \frac{(\sqrt{1+2x} - \sqrt{1-2x})}{\sin x} \\
&= \lim_{x \rightarrow 0} \frac{(\sqrt{1+2x} - \sqrt{1-2x})}{\sin x} \times \frac{\sqrt{1+2x} + \sqrt{1-2x}}{(\sqrt{1+2x} + \sqrt{1-2x})} \\
&= \lim_{x \rightarrow 0} \frac{1+2x - 1+2x}{\sin x} \times \frac{1}{\sqrt{1+2x} + \sqrt{1-2x}} \\
&= 4 \times \lim_{x \rightarrow 0} \frac{x}{\sin x} \times \frac{1}{\sqrt{1+2x} + \sqrt{1-2x}} \\
&= 4 \times \frac{1}{2} \times 1 \\
&= 2
\end{aligned}$$

$$\therefore \lim_{x \rightarrow 0} \frac{(\sqrt{1+2x} - \sqrt{1-2x})}{\sin x} = 2$$

**Q. 47. Evaluate the following limits:**

$$\lim_{h \rightarrow 0} \frac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h}$$

**Answer :**

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \frac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h} \\
&= \lim_{h \rightarrow 0} \frac{a^2(\sin(a+h) - \sin a) + 2ah \sin(a+h) + h^2 \sin(a+h)}{h} \\
&= 2a \sin a + 0 + \lim_{h \rightarrow 0} \frac{a^2 \times 2 \times \cos\left(a + \frac{h}{2}\right) \times \sin h}{h} \\
&= 2a \sin a + 2a^2 \cos a
\end{aligned}$$

$$=2a^2\cos a+2a\sin a$$

$$\therefore \lim_{h \rightarrow 0} \frac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h} = 2a^2 \cos a + 2a \sin a$$

**Q. 48. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{(e^{3+x} - \sin x - e^3)}{x}$$

**Answer :**

$$= \lim_{x \rightarrow 0} \frac{(e^{3+x} - \sin x - e^3)}{x}$$

$$= -\lim_{x \rightarrow 0} \frac{\sin x}{x} + \lim_{x \rightarrow 0} \frac{e^{3+x} - e^3}{x}$$

$$= -1 + \lim_{x \rightarrow 0} \frac{e^3(e^x - 1)}{x}$$

$$= -1 + e^3$$

$$\therefore \lim_{x \rightarrow 0} \frac{(e^{3+x} - \sin x - e^3)}{x} = e^3 - 1$$

**Q. 49. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{(e^{\tan x} - 1)}{\tan x}$$

**Answer :**

$$= \lim_{x \rightarrow 0} \frac{(e^{\tan x} - 1)}{\tan x}$$

As  $x$  tends to 0,  $\tan(x)$  also tends to zero,

So,

$$\lim_{x \rightarrow 0} \frac{(e^{\tan x} - 1)}{\tan x} = \lim_{\tan x \rightarrow 0} \frac{(e^{\tan x} - 1)}{\tan x}$$

$$= 1$$

$$\therefore \lim_{x \rightarrow 0} \frac{(e^{\tan x} - 1)}{\tan x} = 1$$

**Q. 50. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{(e^{\tan x} - 1)}{x}$$

**Answer :**

$$= \lim_{x \rightarrow 0} \frac{e^{\tan x} - 1}{x}$$

$$= \lim_{x \rightarrow 0} \frac{e^{\tan x} - 1}{x} \times \frac{\tan x}{\tan x}$$

$$= \lim_{x \rightarrow 0} \frac{e^{\tan x} - 1}{\tan x} \times \frac{\tan x}{x}$$

$$= 1 \times 1$$

$$= 1$$

$$\therefore \lim_{x \rightarrow 0} \frac{e^{\tan x} - 1}{x} = 1$$

**Q. 51. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{ax + x \cos x}{b \sin x}$$

**Answer :**

$$= \lim_{x \rightarrow 0} \frac{ax + x \cos x}{b \sin x}$$

$$\begin{aligned}
&= \lim_{x \rightarrow 0} \frac{ax}{b \sin x} + \frac{x \cos x}{b \sin x} \\
&= \lim_{x \rightarrow 0} \frac{ax}{b \sin x} + \lim_{x \rightarrow 0} \frac{x \cos x}{b \sin x} \\
&= \frac{a}{b} + \frac{1}{b} \left[ \begin{array}{l} \because \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \\ \lim_{x \rightarrow 0} \cos \theta = 1 \end{array} \right] \\
&= \frac{a + 1}{b} \\
\therefore \lim_{x \rightarrow 0} \frac{ax + x \cos x}{b \sin x} &= \frac{a + 1}{b}
\end{aligned}$$

**Q. 52. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\sin ax + bx}{ax + \sin bx}, \text{ where } a, b, a + b \neq 0$$

**Answer :**

$$\begin{aligned}
&= \lim_{x \rightarrow 0} \frac{\sin(ax) + bx}{ax + \sin(bx)} \\
&= \lim_{x \rightarrow 0} \frac{\sin(ax) + bx}{ax + \sin(bx)} \times \frac{bx}{ax} \times \frac{a}{b} \\
&= \lim_{x \rightarrow 0} \frac{\frac{\sin ax + bx}{ax}}{\frac{ax + \sin bx}{bx}} \times \frac{a}{b} \\
&= \frac{a}{b} \times \frac{\lim_{x \rightarrow 0} \frac{\sin ax + bx}{ax}}{\lim_{x \rightarrow 0} \frac{ax + \sin bx}{bx}} \\
&= \frac{a}{b} \times \frac{1 + \frac{b}{a}}{1 + \frac{a}{b}}
\end{aligned}$$



$$= \frac{a}{b} \times \frac{b}{a}$$

$$= 1$$

$$\therefore \lim_{x \rightarrow 0} \frac{\sin(ax) + bx}{ax + \sin(bx)} = 1$$

**Q. 53. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\sin(\pi - x)}{\pi(\pi - x)}$$

**Answer :**

$$= \lim_{x \rightarrow 0} \frac{\sin(\pi - x)}{\pi(\pi - x)}$$

$$= \lim_{x \rightarrow 0} \frac{\sin x}{\pi(\pi - x)} \times \frac{x}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\sin x}{x} \times \lim_{x \rightarrow 0} \frac{x}{\pi - (\pi - x)}$$

$$= 1 \times \lim_{x \rightarrow 0} \left( \frac{1}{\pi - x} - \frac{1}{\pi} \right)$$

$$= \frac{1}{\pi} - \frac{1}{\pi}$$

$$= 0$$

$$\therefore \lim_{x \rightarrow 0} \frac{\sin(\pi - x)}{\pi(\pi - x)} = 0$$



**Q. 54. Evaluate the following limits:**

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$$

**Answer :**

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$$

As  $x$  tends to  $\pi/2$ ,  $x - \pi/2$  tends to zero.

Let,

$$y = x - \frac{\pi}{2}$$

$$= \lim_{y \rightarrow 0} \frac{\tan\left(2y + \frac{\pi}{2} \times 2\right)}{y}$$

$$= \lim_{y \rightarrow 0} \frac{\tan(\pi + 2y)}{y}$$

$$= \lim_{y \rightarrow 0} \frac{\tan 2y}{2y} \times 2$$

$$= 2$$

$$\therefore \lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}} = 2$$

**Q. 55. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\cos x - 1}$$

**Answer :**

$$\begin{aligned}
&= \lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\cos x - 1} \\
&= \lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\cos x - 1} \times \frac{4 \times x \times x}{2x \times 2x} \\
&= 4 \times \lim_{x \rightarrow 0} \frac{\frac{\cos 2x - 1}{2x \times 2x}}{\frac{\cos x - 1}{x \times x}} \\
&= 4 \times \frac{\lim_{x \rightarrow 0} \frac{\cos 2x - 1}{(2x)^2}}{\lim_{x \rightarrow 0} \frac{\cos x - 1}{x^2}} \\
&= 4 \times \frac{\frac{1}{2}}{\frac{1}{2}} = 4
\end{aligned}$$

$$\therefore \lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\cos x - 1} = 4$$

**Q. 56. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} (\operatorname{cosec} x - \cot x)$$

**Answer :**

$$= \lim_{x \rightarrow 0} (\csc x - \cot x)$$

$$= \lim_{x \rightarrow 0} \left( \frac{1}{\sin x} - \frac{\cos x}{\sin x} \right)$$

$$= \lim_{x \rightarrow 0} \left( \frac{1 - \cos x}{\sin x} \right)$$

$$= \lim_{x \rightarrow 0} \left( \frac{2 \times \sin \frac{x}{2} \times \sin \frac{x}{2}}{2 \times \sin \frac{x}{2} \times \cos \frac{x}{2}} \right) \quad [\because 1 - \cos \theta = 2 \sin \theta \times \sin \theta]$$

$$= \lim_{x \rightarrow 0} \left( \tan \frac{x}{2} \right)$$

$$= 0$$

$$\therefore \lim_{x \rightarrow 0} (\csc x - \cot x) = 0$$

**Q. 57. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2mx}{1 - \cos 2nx}$$

**Answer :**

$$= \lim_{x \rightarrow 0} \frac{1 - \cos 2mx}{1 - \cos 2nx}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1 - \cos 2mx}{(2mx)^2} \times (2mx)^2}{\frac{1 - \cos 2nx}{(2nx)^2} \times (2nx)^2}$$

$$= \frac{1}{2} \times \frac{m \times m}{n \times n} \left[ \because \frac{1 - \cos \theta}{\theta \times \theta} = \frac{1}{2} \right]$$

$$= \frac{m^2}{n^2}$$

$$\therefore \lim_{x \rightarrow 0} \frac{1 - \cos 2mx}{1 - \cos 2nx} = \frac{m^2}{n^2}$$

**Q. 58. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{1 - \cos mx}{1 - \cos nx}$$

**Answer :**

$$= \lim_{x \rightarrow 0} \frac{1 - \cos mx}{1 - \cos nx}$$



$$= \lim_{x \rightarrow 0} \frac{1 - \cos mx}{\frac{mx \times mx}{1 - \cos nx}} \times \frac{m \times m}{n \times n}$$

$$= \frac{m^2}{n^2}$$

$$\therefore \lim_{x \rightarrow 0} \frac{1 - \cos mx}{1 - \cos nx} = \frac{m^2}{n^2}$$

**Q. 59. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\sin^2 mx}{\sin^2 nx}$$

**Answer :**

$$= \lim_{x \rightarrow 0} \frac{\sin mx \times \sin mx}{\sin nx \times \sin nx}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{\sin mx \times \sin mx}{mx \times mx}}{\frac{\sin nx \times \sin nx}{nx \times nx}} \times \frac{m^2}{n^2}$$

$$= \frac{m^2}{n^2}$$

$$\therefore \lim_{x \rightarrow 0} \frac{\sin mx \times \sin mx}{\sin nx \times \sin nx} = \frac{m^2}{n^2}$$

**Q. 60. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\sin 2x + \sin 3x}{2x + \sin 3x}$$

**Answer :**

$$= \lim_{x \rightarrow 0} \frac{\sin 2x + \sin 3x}{2x + \sin 3x} \times \frac{3x}{3x}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{\sin 2x + \sin 3x}{3x}}{\frac{2x + \sin 3x}{3x}}$$

$$= \frac{\frac{2}{3} + 1}{\frac{2}{3} + 1}$$

$$= 1$$

$$\therefore \lim_{x \rightarrow 0} \frac{\sin 2x + \sin 3x}{2x + \sin 3x} = 1$$

**Q. 61. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\sec 4x - \sec 2x}{\sec 3x - \sec x}$$

**Answer :**

$$= \lim_{x \rightarrow 0} \frac{\sec 4x - \sec 2x}{\sec 3x - \sec x}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{\cos 4x} - \frac{1}{\cos 2x}}{\frac{1}{\cos 3x} - \frac{1}{\cos x}}$$

$$= \lim_{x \rightarrow 0} \frac{(\cos 2x - \cos 4x) \times \cos x \times \cos 3x}{(\cos x - \cos 3x) \times \cos 2x \times \cos 4x}$$

$$= \lim_{x \rightarrow 0} \frac{2 \times \sin 3x \times \sin x \times \cos x \times \cos 3x}{2 \times \sin 2x \times \sin x \times \cos 2x \times \cos 4x}$$

$$= \lim_{x \rightarrow 0} \frac{\sin 6x \times \cos x}{\sin 4x \times \cos 4x} \times \frac{2}{2}$$

$$= 2 \times \lim_{x \rightarrow 0} \frac{\sin 6x \times \cos x}{\sin 8x} \times \frac{8x}{6x} \times \frac{6}{8}$$

$$= \frac{3}{2} \times \lim_{x \rightarrow 0} \frac{\frac{\sin 6x}{6x} \times \cos x}{\frac{\sin 8x}{8x}}$$

$$= \frac{3}{2} \times \frac{1 \times 1}{1}$$

$$= \frac{3}{2}$$

$$\therefore \lim_{x \rightarrow 0} \frac{\sec 4x - \sec 2x}{\sec 3x - \sec x} = \frac{3}{2}$$

**Q. 62. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}$$

**Answer :**

$$= \lim_{x \rightarrow 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin x \times \sin x}$$

$$= \lim_{x \rightarrow 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin x \times \sin x} \times \frac{\sqrt{2} + \sqrt{1 + \cos x}}{\sqrt{2} + \sqrt{1 + \cos x}}$$

$$= \lim_{x \rightarrow 0} \frac{2 - (1 - \cos x)}{\sin x \times \sin x \times \sqrt{2} + \sqrt{1 + \cos x}}$$

$$= \lim_{x \rightarrow 0} \frac{1 - \cos x}{2 \times \sin \frac{x}{2} \cos \frac{x}{2} \sin x (\sqrt{2} + \sqrt{1 + \cos x})}$$

$$= \lim_{x \rightarrow 0} \frac{2 \times \sin \frac{x}{2} \times \sin \frac{x}{2}}{2 \times \sin \frac{x}{2} \cos \frac{x}{2} \sin x (\sqrt{2} + \sqrt{1 + \cos x})}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{\sin \frac{x}{2}}{\frac{x}{2}} \times \frac{1}{2}}{\frac{\sin x}{x}} \times \lim_{x \rightarrow 0} \frac{1}{\cos \frac{x}{2} \times (\sqrt{2} + \sqrt{1 + \cos x})}$$

$$= \frac{1}{2} \times \frac{1}{(\sqrt{2} + \sqrt{2})}$$

$$= \frac{1}{4\sqrt{2}}$$

$$\therefore \lim_{x \rightarrow 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin x \times \sin x} = \frac{1}{4\sqrt{2}}$$

**Q. 63. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\sqrt{1 + \sin x} - \sqrt{1 - \cos x}}{x}$$

**Answer :**

$$= \lim_{x \rightarrow 0} \frac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{x} \times \frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}$$

$$= \lim_{x \rightarrow 0} \frac{1 + \sin x - (1 - \sin x)}{x(\sqrt{1 + \sin x} + \sqrt{1 - \sin x})}$$

$$= \lim_{x \rightarrow 0} \frac{2 \times \sin x}{x(\sqrt{1 + \sin x} + \sqrt{1 - \sin x})}$$

$$= 2 \times \lim_{x \rightarrow 0} \frac{\sin x}{x} \times \lim_{x \rightarrow 0} \frac{1}{(\sqrt{1 + \sin x} + \sqrt{1 - \sin x})}$$

$$= 2 \times 1 \times \frac{1}{2}$$

$$= 1$$

$$\therefore \lim_{x \rightarrow 0} \frac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{x} = 1$$

**Q. 64. Evaluate the following limits:**

$$\lim_{x \rightarrow \frac{\pi}{6}} \frac{2 - \sqrt{3} \cos x - \sin x}{(6x - \pi)^2}$$

**Answer :**

$$= \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 - \sqrt{3} \cos x - \sin x}{(6x - \pi)^2}$$

$$= \lim_{y \rightarrow 0} \frac{2 - \sqrt{3} \cos\left(y + \frac{\pi}{6}\right) - \sin\left(y + \frac{\pi}{6}\right)}{y^2 \times 36}$$

$$= \frac{1}{36} \times \lim_{y \rightarrow 0} \frac{2 - \frac{3}{2} \cos y + \frac{\sqrt{3}}{2} \sin y - \frac{1}{2} \cos y - \frac{\sqrt{3}}{2} \sin y}{y^2}$$

$$= \frac{1}{36} \times \lim_{y \rightarrow 0} \frac{2(1 - \cos y)}{y^2}$$

$$= 2 \times \frac{1}{2} \times \frac{1}{36}$$

$$= \frac{1}{36}$$

$$\therefore \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 - \sqrt{3} \cos x - \sin x}{(6x - \pi)^2} = \frac{1}{36}$$

**Q. 65. Evaluate the following limits:**

$$\lim_{x \rightarrow 0} \frac{\cos ax - \cos bx}{\cos cx - 1}$$

**Answer :**

$$\begin{aligned}
&= \lim_{x \rightarrow 0} \frac{\cos ax - \cos bx}{\cos cx - 1} \\
&= \lim_{x \rightarrow 0} \frac{1 - \cos bx - (1 - \cos ax)}{\cos cx - 1} \\
&= \lim_{x \rightarrow 0} \frac{\frac{(1 - \cos bx)}{(bx)^2} \times b^2 - \frac{(1 - \cos ax)}{(ax)^2} \times a^2}{\frac{-(1 - \cos cx)}{(cx)^2} \times c^2} \\
&= \frac{a^2 - b^2}{c^2} \\
\therefore \lim_{x \rightarrow 0} \frac{\cos ax - \cos bx}{\cos cx - 1} &= \frac{a^2 - b^2}{c^2}
\end{aligned}$$

**Q. 66. Evaluate the following limits:**

$$\lim_{x \rightarrow a} \frac{\cos x - \cos a}{\cot x - \cot a}$$

**Answer :**

$$= \lim_{x \rightarrow a} \frac{\cos x - \cos a}{\cot x - \cot a}$$

$$= \lim_{x \rightarrow a} \frac{(\cos x - \cos a)}{\frac{\sin(a - x)}{\sin x \sin a}}$$

$$= \sin a \times \lim_{x \rightarrow a} \frac{\sin\left(\frac{x+a}{2}\right) \times \sin x}{\cos\left(\frac{x-a}{2}\right)}$$

$$= \sin^3 a$$

$$\therefore \lim_{x \rightarrow a} \frac{\cos x - \cos a}{\cot x - \cot a} = \sin a \times \sin a \times \sin a$$



**Q. 67. Evaluate the following limits:**

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$$

**Answer :**

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\left(\frac{-\sin x (\cos 2x)}{\cos x \cos x \cos x}\right)}{\left(\frac{\cos x - \sin x}{\sqrt{2}}\right)}$$

$$= -\sqrt{2} \times \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x (\cos x + \sin x)}{\cos x \times \cos x \times \cos x}$$

$$= -\sqrt{2} \times \frac{\frac{1}{\sqrt{2}} \times \sqrt{2}}{\frac{1}{\sqrt{2}}^3}$$

$$= -4$$



**Q. 68. Evaluate the following limits:**

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\sqrt{2} \cos^2 x}$$

**Answer :**

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\sqrt{2} \cos x \cos x}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\sqrt{2} \cos x \cos x} \times \frac{\sqrt{2} + \sqrt{1 + \sin x}}{\sqrt{2} + \sqrt{1 + \sin x}}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{\sqrt{2} + \sqrt{1 + \sin x} (\sqrt{2} \cos x \cos x)}$$

Let,

$$y = x - \frac{\pi}{2}$$

$$= \lim_{y \rightarrow 0} \frac{1 - \cos y}{\sqrt{2} + \sqrt{1 + \cos y} (\sqrt{2} \sin y \sin y)}$$

$$= \frac{1}{2\sqrt{2}} \times \frac{1}{2\sqrt{2}}$$

$$= \frac{1}{8}$$

$$\therefore \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\sqrt{2} \cos x \cos x} = \frac{1}{8}$$

**Q. 69. Evaluate the following limits:**

$$\lim_{x \rightarrow \frac{\pi}{6}} \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2}$$

**Answer :**

$$= \lim_{x \rightarrow \frac{\pi}{6}} \frac{\cot x \times \cot x - 3}{\operatorname{csc} x - 2}$$

$$= \lim_{x \rightarrow \frac{\pi}{6}} \frac{(\cos x \times \cos x) - 3 \times \sin x \times \sin x}{\sin x (1 - 2 \sin x)}$$

$$= \lim_{x \rightarrow \frac{\pi}{6}} \frac{1 - 4 \times \sin x \times \sin x}{\sin x (1 - 2 \sin x)}$$

$$= \lim_{x \rightarrow \frac{\pi}{6}} \frac{(1 - 2 \sin x) \times (1 + 2 \sin x)}{\sin x (1 - 2 \sin x)}$$

$$= 4$$

$$\therefore \lim_{x \rightarrow \frac{\pi}{6}} \frac{\cot x \times \cot x - 3}{\csc x - 2} = 4$$

Q. 70. Evaluate the following limits:

$$\lim_{x \rightarrow \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2}$$

Answer :

$$= \lim_{x \rightarrow \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2}$$

$$= \lim_{x \rightarrow \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2} \times \frac{\sqrt{2 + \cos x} + 1}{\sqrt{2 + \cos x} + 1}$$

$$= \lim_{x \rightarrow \pi} \frac{1 + \cos x}{(\pi - x)^2} \times \frac{1}{\sqrt{2 + \cos x} + 1}$$

Let,

$$y = x - \pi$$

$$= \lim_{y \rightarrow 0} \frac{1 - \cos y}{x^2 \times \sqrt{2 - \cos y} + 1}$$

$$= \frac{1}{4}$$

$$\therefore \lim_{x \rightarrow \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2} = \frac{1}{4}$$

**Q. 71. Evaluate the following limits:**

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \tan x}{1 - \sqrt{2} \sin x}$$

**Answer :**

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \tan x}{1 - \sqrt{2} \sin x}$$

**Let,**

$$y = x - \frac{\pi}{4}$$

$$= \lim_{y \rightarrow 0} \frac{2 \tan x}{1 - \cos x + \sin x}$$

$$= \lim_{y \rightarrow 0} \frac{\frac{2 \cos \frac{x}{2}}{\cos x}}{\sin \frac{x}{2} + \cos \frac{x}{2}}$$

$$= 2$$

$$\therefore \lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \tan x}{1 - \sqrt{2} \sin x} = 2$$

**Q. 72. Evaluate the following limits:**

$$\lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin^2 x + \sin x - 1}{2 \sin^2 x - 3 \sin x + 1}$$

**Answer :**

$$= \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \times \sin x \times \sin x + \sin x - 1}{2 \times \sin x \times \sin x - 3 \sin x + 1}$$

$$\begin{aligned}
&= \lim_{x \rightarrow \frac{\pi}{6}} \frac{(2 \sin x - 1) \times (\sin x + 1)}{(2 \sin x - 1)(\sin x - 1)} \\
&= -3 \\
\therefore \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \times \sin x \times \sin x + \sin x - 1}{2 \times \sin x \times \sin x - 3 \sin x + 1} &= -3
\end{aligned}$$

### Exercise 27C

Q. 1.

If  $f(x) = |x| - 3$ , find  $\lim_{x \rightarrow 3} f(x)$

**Answer :**

Left Hand Limit(L.H.L.):

$$\begin{aligned}
&\lim_{x \rightarrow 3^-} f(x) \\
&= \lim_{x \rightarrow 3^-} |x| - 3
\end{aligned}$$

$$= \lim_{x \rightarrow 3^-} -(x - 3)$$

$$= -(3 - 3)$$

$$= 0$$

Right Hand Limit(R.H.L.):

$$\lim_{x \rightarrow 3^+} f(x)$$

$$= \lim_{x \rightarrow 3^+} |x| - 3$$

$$= \lim_{x \rightarrow 3^+} (x - 3)$$

$$= 3 - 3$$

$$= 0$$

Since,

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x)$$

We can say that the limit exists and

$$\lim_{x \rightarrow 3} f(x) = 0$$

**Q. 2.**

$$\text{Let } f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Show that  $\lim_{x \rightarrow 0} f(x)$  does not exist.

**Answer :**

Left Hand Limit(L.H.L.):

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{x}{|x|}$$

$$= \lim_{x \rightarrow 0^-} \frac{x}{(-x)}$$

$$= \lim_{x \rightarrow 0^-} -1$$

$$= -1$$

Right Hand Limit(R.H.L.):

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{x}{|x|}$$

$$= \lim_{x \rightarrow 0^+} \frac{x}{(+x)}$$

$$= \lim_{x \rightarrow 0^+} 1$$

$$= 1$$

Since  $\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$ ,  $\lim_{x \rightarrow 0} f(x)$  does not exist



**Q. 3.**

$$\text{Let } f(x) = \begin{cases} \frac{|x-3|}{(x-3)}, & x \neq 3 \\ 0, & x = 3 \end{cases}$$

Show that  $\lim_{x \rightarrow 3} f(x)$  does not exist.

**Answer :**

Left Hand Limit(L.H.L.):

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} \frac{|x-3|}{x-3}$$

$$= \lim_{x \rightarrow 3^-} \frac{-(x-3)}{x-3}$$

$$= \lim_{x \rightarrow 3^-} -1$$

$$= -1$$

Right Hand Limit(R.H.L.):

$$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} \frac{|x-3|}{x-3}$$

$$= \lim_{x \rightarrow 3^+} \frac{(x-3)}{x-3}$$

$$= \lim_{x \rightarrow 3^+} 1$$

$$= 1$$

$$\lim_{x \rightarrow 3^-} f(x) \neq \lim_{x \rightarrow 3^+} f(x)$$

Thus,  $\lim_{x \rightarrow 3} f(x)$  does not exist.



**Q. 4.**

$$\text{Let } f(x) = \begin{cases} 1+x^2, & 0 \leq x \leq 1 \\ 2-x, & x > 1 \end{cases}$$

Show that  $\lim_{x \rightarrow 1} f(x)$  does not exist.

**Answer :** Left Hand Limit(L.H.L.):

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} 1 + x^2$$

$$= 1 + (1)^2$$

$$= 1 + 1$$

$$= 2$$

Right Hand Limit(R.H.L.):

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} 2 - x$$

$$= 2 - (1)$$

$$= 2 - 1$$

$$= 1$$

$$\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$$

Thus,  $\lim_{x \rightarrow 1} f(x)$  does not exist.

**Q. 5.**

$$\text{Let } f(x) = \begin{cases} \frac{x - |x|}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$$

Show that  $\lim_{x \rightarrow 0} f(x)$  does not exist

**Answer :**

**Left Hand Limit(L.H.L.):**

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{x - |x|}{x}$$

$$= \lim_{x \rightarrow 0^-} \frac{x - (-x)}{x}$$

$$= \lim_{x \rightarrow 0^-} \frac{x + x}{x}$$

$$= \lim_{x \rightarrow 0^-} \frac{2x}{x}$$

$$= \lim_{x \rightarrow 0^-} 2$$

$$= 2$$

**Right Hand Limit(R.H.L.):**

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{x - |x|}{x}$$

$$= \lim_{x \rightarrow 0^+} \frac{x - (x)}{x}$$

$$= \lim_{x \rightarrow 0^+} \frac{0}{x}$$

$$= \lim_{x \rightarrow 0^+} 0$$

$$= 0$$

$$\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$

Thus,  $\lim_{x \rightarrow 0} f(x)$  does not exist.



Q. 6

$$\text{Let } f(x) = \begin{cases} 5x - 4, & 0 < x \leq 1 \\ 4x^3 - 3x, & 1 < x < 2 \end{cases}$$

Find  $\lim_{x \rightarrow 1} f(x)$

Answer :

Left Hand Limit(L.H.L.):

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} 5x - 4$$

$$= 5(1) - 4$$

$$= 5 - 4$$

$$= 1$$

Right Hand Limit(R.H.L.):

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} 4x^3 - 3x$$

$$= 4(1)^3 - 3(1)$$

$$= 4 - 3$$

$$= 1$$

$$\therefore \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x)$$

$$\text{Thus, } \lim_{x \rightarrow 1} f(x) = 1$$



Q. 7

$$\text{Let } f(x) = \begin{cases} 4x - 5, & x \leq 2 \\ x - a, & x > 2 \end{cases}$$

If  $\lim_{x \rightarrow 2} f(x)$  exists then find the value of a.

**Answer :**

**Left Hand Limit(L.H.L.):**

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} 4x - 5$$

$$= 4(2) - 5$$

$$= 8 - 5$$

$$= 3$$

**Right Hand Limit(R.H.L.):**

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} x - a$$

$$= 2 - a$$

Since  $\lim_{x \rightarrow 2} f(x)$  it exists,

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x)$$

$$\rightarrow 3 = 2 - a$$

$$\rightarrow a = 2 - 3$$

$$\rightarrow a = -1$$



Q. 8

$$\text{Let } f(x) = \begin{cases} \frac{3x}{|x| + 2x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Show that  $\lim_{x \rightarrow 0} f(x)$  does not exist.

Answer :

Left Hand Limit(L.H.L.):

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{3x}{|x| + 2x}$$

$$= \lim_{x \rightarrow 0^-} \frac{3x}{(-x) + 2x}$$

$$= \lim_{x \rightarrow 0^-} \frac{3x}{x}$$

$$= \lim_{x \rightarrow 0^-} 3$$

$$= 3$$

Right Hand Limit(R.H.L.):

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{3x}{|x| + 2x}$$

$$= \lim_{x \rightarrow 0^+} \frac{3x}{(x) + 2x}$$

$$= \lim_{x \rightarrow 0^+} \frac{3x}{3x}$$

$$= \lim_{x \rightarrow 0^+} 1$$



$$= 1$$

Since

$$\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$

Thus,  $\lim_{x \rightarrow 0} f(x)$  does not exist.

**Q. 9**

$$\text{Let } f(x) = \begin{cases} \cos x, & x \geq 0 \\ x + k, & x < 0 \end{cases}$$

Find the value of  $k$  for which  $\lim_{x \rightarrow 0} f(x)$  exist.

**Answer :**

**Left Hand Limit(L.H.L.):**

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} x + k$$

$$= 0 + k$$

$$= k$$

**Right Hand Limit(R.H.L.):**

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \cos x$$

$$= \cos(0)$$

$$= 1$$

It is given that  $\lim_{x \rightarrow 0} f(x)$  exists. Therefore,

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)$$

$$\rightarrow k = 1$$



Q. 10

Show that  $\lim_{x \rightarrow 0} \frac{1}{x}$  does not exist.

**Answer :** Let  $x = 0+h$  for  $x$  tending to  $0^+$

Since  $x \rightarrow 0$ ,  $h$  also tends to 0

**Right Hand Limit(R.H.L.):**

$$\lim_{x \rightarrow 0^+} f(x)$$

$$= \lim_{x \rightarrow 0^+} \frac{1}{x}$$

$$= \lim_{h \rightarrow 0^+} \frac{1}{0+h}$$

$$= \lim_{h \rightarrow 0^+} \frac{1}{+h}$$

$$= +\frac{1}{0}$$

$$= +\infty$$

Let  $x=0-h$  for  $x$  tending to  $0^-$

Since  $x \rightarrow 0$ ,  $h$  also tends to 0.

**Left Hand Limit(L.H.L.):**

$$= \lim_{x \rightarrow 0^-} f(x)$$

$$= \lim_{x \rightarrow 0^-} \frac{1}{x}$$

$$= \lim_{h \rightarrow 0^-} \frac{1}{0-h}$$



$$= \lim_{h \rightarrow 0^-} \frac{1}{-h}$$

$$= -\frac{1}{0}$$

$$= -\infty$$

Since,

$$\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$

Thus,  $\lim_{x \rightarrow 0} \frac{1}{x}$  does not exist.

Q. 11

mc<sup>24</sup>

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Show that  $\lim_{x \rightarrow 0} \frac{1}{|x|} = \infty$ .

**Answer :** Let  $x = 0 + h$ , when  $x$  is tends to  $0^+$

Since  $x$  tends to  $0$ ,  $h$  will also tend to  $0$ .

**Right Hand Limit(R.H.L):**

$$\lim_{x \rightarrow 0^+} f(x)$$

$$= \lim_{x \rightarrow 0^+} \frac{1}{|x|}$$

$$= \lim_{x \rightarrow 0^+} \frac{1}{(x)}$$

$$= \lim_{h \rightarrow 0^+} \frac{1}{(0 + h)}$$

$$= \frac{1}{0}$$

$$= \infty$$

Let  $x = 0 - h$ , when  $x$  tends to  $0^-$

since  $x$  tends to  $0$ ,  $h$  will also tend to  $0$ .

**Left Hand Limit(L.H.L.):**

$$\lim_{x \rightarrow 0^-} f(x)$$

$$= \lim_{x \rightarrow 0^-} \frac{1}{|x|}$$

$$= \lim_{x \rightarrow 0^-} \frac{1}{(-x)}$$

$$= \lim_{h \rightarrow 0^-} \frac{1}{-(0 - h)}$$

$$= \lim_{h \rightarrow 0^-} \frac{1}{h}$$

$$= \frac{1}{0}$$

$$= \infty$$

Thus,

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)$$

$$\therefore \lim_{x \rightarrow 0} \frac{1}{|x|} = \infty.$$

**Q. 12.**



Show that  $\lim_{x \rightarrow 0} e^{-1/x}$  does not exist.

Answer : Left Hand Limit(L.H.L.):

$$\begin{aligned} & \lim_{x \rightarrow 0^-} f(x) \\ &= \lim_{x \rightarrow 0^-} e^{\frac{-1}{(-x)}} \\ &= \lim_{x \rightarrow 0^-} e^{\frac{1}{x}} \\ &= e^{\frac{1}{0}} \\ &= e^{\infty} \end{aligned}$$

$$\lim_{x \rightarrow 0^+} f(x)$$



Right Hand Limit(R.H.L.):

$$\begin{aligned} &= \lim_{x \rightarrow 0^+} e^{\frac{-1}{x}} \\ &= e^{\frac{-1}{0}} \\ &= e^{-\infty} \end{aligned}$$

$$= \frac{1}{e^\infty}$$

[ Formula  $\frac{1}{\infty} = 0$ , anything to the power infinity is also infinity. Thus  $\frac{1}{e^\infty} = \frac{1}{\infty} = 0$  ]


$$= 0$$

Since

$$\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$

$\therefore \lim_{x \rightarrow 0} e^{-1/x}$  does not exist.

**Q. 13.**

Show that  $\lim_{x \rightarrow 0} \sin \frac{1}{x}$  does not exist.  Your Class, Your Pace.

**Answer :** Let  $x = 0 + h$ , when  $x$  is tends to  $0^+$

Since  $x$  tends to  $0$ ,  $h$  will also tend to  $0$ .

**Right Hand Limit(R.H.L.):**

$$\lim_{x \rightarrow 0^+} f(x)$$

$$= \lim_{x \rightarrow 0^+} \sin \frac{1}{x}$$

$$= \lim_{h \rightarrow 0^+} \sin \frac{1}{0 + h}$$

$$= \sin \frac{1}{0}$$

$$= \sin \infty$$

$$= \infty$$

Let  $x = 0 - h$ , when  $x$  is tends to  $0^-$

Since  $x$  tends to  $0$ ,  $h$  will also tend to  $0$ .

**Left Hand Limit(L.H.L.):**

$$\lim_{x \rightarrow 0^-} f(x)$$

$$= \lim_{x \rightarrow 0^-} \sin \frac{1}{x}$$

$$= \lim_{h \rightarrow 0^-} \sin \frac{1}{0 - h}$$

$$= \sin \frac{1}{-0}$$

$$= -\sin \frac{1}{0}$$

$$= -\sin \infty$$

$$= -\infty$$

Since,

$$\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$

$\therefore \lim_{x \rightarrow 0} \sin \frac{1}{x}$  does not exist.



Q. 14

Show that  $\lim_{x \rightarrow 0} \frac{x}{|x|}$  does not exist.

Answer :

Left Hand Limit(L.H.L.):

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{x}{|x|}$$

$$= \lim_{x \rightarrow 0^-} \frac{x}{(-x)}$$

$$= \lim_{x \rightarrow 0^-} -1$$

$$= -1$$

Right Hand Limit(R.H.L.):

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{x}{|x|}$$

$$= \lim_{x \rightarrow 0^+} \frac{x}{(x)}$$

$$= \lim_{x \rightarrow 0^+} 1$$

$$= 1$$

Since

$$\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$

Thus,  $\lim_{x \rightarrow 0} \frac{x}{|x|}$  does not exist.



Q. 15

$$\text{Let } f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & x \neq \frac{\pi}{2} \\ 3, & x = \frac{\pi}{2} \end{cases}$$

If  $\lim_{x \rightarrow \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$ , find the value of k.

**Answer :**

$$\lim_{x \rightarrow \frac{\pi}{2}} f(x) = \lim_{x \rightarrow \frac{\pi}{2}} \frac{k \cos x}{\pi - 2x}$$

$$\text{Let } h = x - \frac{\pi}{2}$$

$$\rightarrow x = h + \frac{\pi}{2}$$

$$x \rightarrow \frac{\pi}{2}$$

$$\text{or, } h + \frac{\pi}{2} \rightarrow \frac{\pi}{2}$$

$$\text{or, } h \rightarrow 0$$

Putting this in the original sum,



$$= \lim_{h \rightarrow 0} \frac{k \cos\left(\frac{\pi}{2} + h\right)}{\pi - 2\left(\frac{\pi}{2} + h\right)}$$

$$= \lim_{h \rightarrow 0} \frac{-k \sin h}{\pi - \pi + h}$$

$$= \lim_{h \rightarrow 0} \frac{-k \sin h}{h}$$

$$= -k \lim_{h \rightarrow 0} \frac{\sin h}{h}$$

[ Applying formula  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$  ]

$$= -k \times 1$$

$$= -k$$

$$f\left(\frac{\pi}{2}\right) = 3$$



It is given that  $\lim_{x \rightarrow \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$

$$\therefore -k = 3$$

$$\rightarrow k = -3$$