Exercise 14(B)

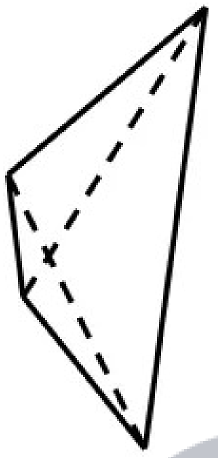
Solution 1:

(i) True.

This is true, because we know that a rectangle is a parallelogram. So, all the properties of a parallelogram are true for a rectangle. Since the diagonals of a parallelogram bisect each other, the same holds true for a rectangle.

(ii) False

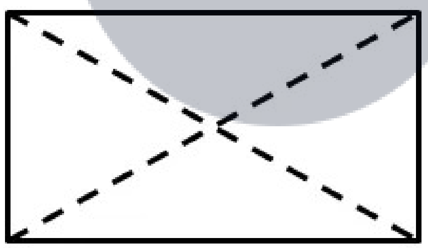
This is not true for any random quadrilateral. Observe the quadrilateral shown below.



Clearly the diagonals of the given quadrilateral do not bisect each other. However, if the quadrilateral was a special quadrilateral like a parallelogram, this would hold true.

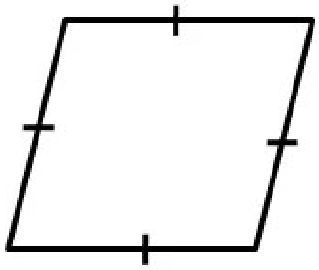
(iii) False

Consider a rectangle as shown below.

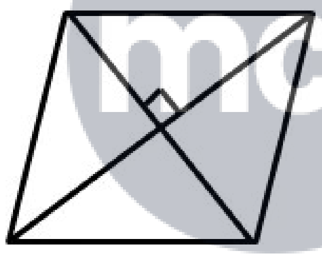


It is a parallelogram. However, the diagonals of a rectangle do not intersect at right angles, even though they bisect each other.

(iv) True
Since a rhombus is a parallelogram, and we know that the diagonals of a parallelogram bisect each other, hence the diagonals of a rhombus too, bisect other.
(v) False
This need not be true, since if the angles of the quadrilateral are not right angles, the quadrilateral would be a rhombus rather than a square.
(vi) True



A parallelogram is a quadrilateral with opposite sides parallel and equal.
Since opposite sides of a rhombus are parallel, and all the sides of the rhombus are equal, a rhombus is a parallelogram.
(vii) False
This is false, since a parallelogram in general does not have all its sides equal. Only opposite sides of a parallelogram are equal. However, a rhombus has all its sides equal. So, every parallelogram cannot be a rhombus, except those parallelograms that have all equal sides.
(viii) False
This is a property of a rhombus. The diagonals of a rhombus need not be equal.
(ix)True
A parallelogram is a quadrilateral with opposite sides parallel and equal.
A rhombus is a quadrilateral with opposite sides parallel, and all sides equal.
If in a parallelogram the adjacent sides are equal, it means all the sides of the parallelogram are equal, thus forming a rhombus.
(x)False



Observe the above figure. The diagonals of the quadrilateral shown above bisect each other at right angles, however the quadrilateral need not be a square, since the angles of the quadrilateral are clearly not right angles.

Solution 2:

From the given figure we conclude that

$$\angle A + \angle D = 180° \left[\text{since consecutive angles are supplementary}\right]$$

$$\frac{\angle A}{2} + \frac{\angle D}{2} = 90°$$

Again from the ΔADM

$$\frac{\angle A}{2} + \frac{\angle D}{2} + \angle M = 180°$$

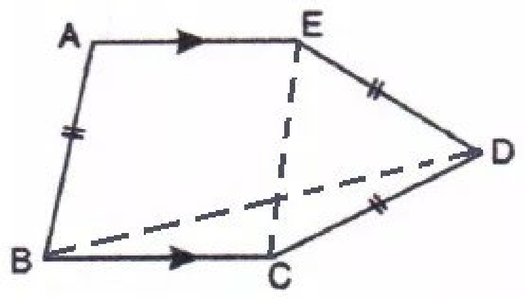
$$\Rightarrow 90° + \angle M = 180° \qquad \left[\sin ce \frac{\angle A}{2} + \frac{\angle D}{2} = 90°\right]$$

$$\Rightarrow \angle M = 90°$$

Hence $\angle AMD = 90°$

Solution 3:

In the given figure



Given that AE = BC

We have to find $\angle AEC$ $\angle BCD$

Let us join EC and BD.

In the quadrilateral AECB

AE = BC and AB = EC

also AE || BC

$\Rightarrow$ AB || EC

So quadrilateral is a parallelogram.

In parallelogram consecutive angles are supplementary

$\Rightarrow \angle A + \angle B = 180^\circ$

$\Rightarrow 102^\circ + \angle B = 180^\circ$

$\Rightarrow \angle B = 78^\circ$

In parallelogram opposite angles are equal

$\Rightarrow \angle A = \angle BEC$ and $\angle B = \angle AEC$

$\Rightarrow \angle BEC = 102^\circ$ and $\angle AEC = 78^\circ$

Now consider ΔECD

EC = ED = CD [Since AB = EC]

Therefore ΔECD is an equilateral triangle.

$\Rightarrow \angle ECD = 60^\circ$

$\angle BCD = \angle BEC + \angle ECD$

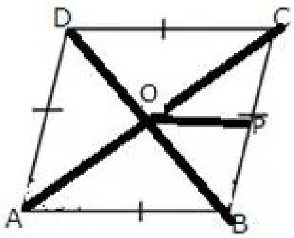
$\Rightarrow \angle BCD = 102^\circ + 60^\circ$

$\Rightarrow \angle BCD = 162^\circ$

Therefore $\angle AEC = 78^\circ$ and $\angle BCD = 162^\circ$

Solution 4:

Given ABCD is a square and diagonals meet at O.P is a point on BC such that OB=BP



In the

ΔBOC and ΔDOC

$\Rightarrow BD=BD$ [common side]

$\Rightarrow BO=CO$

ÞOD=OC [since diagonals cuts at O]

$\Delta BOC \cong \Delta DOC$ [by SSS]

Therefore

$\angle BOC = 90°$

NOW

$\angle POC = 22.5$

$\angle BOP = 67.5$ [since $\angle BOC=67.5°+22.5°$]

Again

ΔBDC

$\angle BDC = 45°$ [since $\angle B = 45°, \angle C = 90°$]

Therefore

$\angle BDC = 2\angle POC$

AGAIN

$\angle BOP = 67.5°$

$\Rightarrow \angle BOP = 2\angle POC$

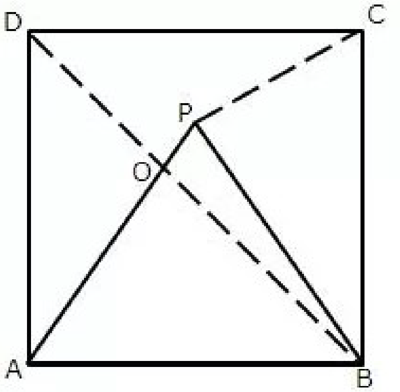
Hence proved that

i) $\angle PC = \left(22\frac{1}{2}^{\circ}\right)$

(ii) $\angle BDC = 2\angle POC$

(iii) $\angle BOP = 3\angle CPO$

Solution 5:



In the given figure ΔAPB is an equilateral triangle

Therefore all its angles are $60°$

Again in the

ΔADB
$\angle ABD = 45°$

$\angle AOB = 180° - 60° - 45°$
$= 75°$

Again

$\triangle BPC$
$\Rightarrow \angle BPC = 75°$ [Since BP = CB]

Now

$\angle C = \angle BCP + \angle PCD$
$\Rightarrow \angle PCD = 90° - 75°$
$\Rightarrow \angle PCD = 15°$

Therefore

$\angle APC = 60° + 75°$
$\Rightarrow \angle APC = 135°$
$\Rightarrow Reflex\ \angle APD = 360° - 135° = 225°$

(i) $\angle AOB = 75°$

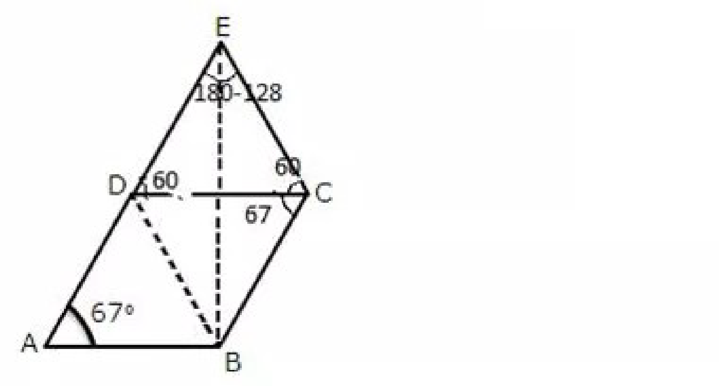
(ii) $\angle BPC = 75°$

(iii) $\angle PCD = 15°$

(iv) Reflex $\angle APD = 225°$

Solution 6:

Given that the figure ABCD is a rhombus with angle A = 67°



In the rhombus We have

$\angle A = 67^\circ = \angle C$ [Opposite angles]

$\angle A + \angle D = 180^\circ$[Consecutive angles are supplementary]

$\Rightarrow \angle D = 113^\circ$

$\Rightarrow \angle ABC = 113^\circ$

Consider $\triangle DBC$,

$DC = CB$ [Sides of rhombous]

So $\triangle DBC$ is an isoscales triangle

$\Rightarrow \angle CDB = \angle CBD$

Also,

$\angle CDB + \angle CDB + \angle BCD = 180^\circ$

$\Rightarrow 2\angle CBD = 113^\circ$

$\Rightarrow \angle CDB = \angle CBD = 56.5^\circ$............$(i)$

Consider $\triangle DCE$,

$EC = CB$

So $\triangle DCE$ is an isoscales triangle

$\Rightarrow \angle CBE = \angle CEB$

Also,

$\angle CBE + \angle CEB + \angle BCE = 180^\circ$

$\Rightarrow 2\angle CBE = 53^\circ$

$\Rightarrow \angle CDE = 26.5^\circ$

From (i)

$\angle CBD = 56.5^\circ$

$\Rightarrow \angle CBE + \angle DBE = 56.5^\circ$

$\Rightarrow 26.5^\circ + \angle DBE = 56.5^\circ$

$\Rightarrow \angle DBE = 30.5^\circ$

Solution 7:

(i)ABCD is a parallelogram

Therefore

AD=BC

AB=DC

Thus

$4y = 3x - 3$ [since AD=BC]

$\Rightarrow 3x - 4y = 3$ (i)

$6y + 2 = 4x$ [since AB=DC]

$4x - 6y = 2$ (ii)

Solving equations (i) and (ii) we have

x=5

y=3

(ii)

In the figure ABCD is a parallelogram

$\angle A = \angle C$

$\angle B = \angle D$ [since opposite angles are equal]

Therefore

$7y = 6y + 3y - 8^{\circ}$ (i) $[Since\ \angle A = \angle C]$

$4x + 20^{\circ} = 0$ (ii)

Solving (i), (ii) we have

$X = 12^{\circ}$

$Y = 16^{\circ}$

Solution 8:

Given that the angles of a quadrilateral are in the ratio $3:4:5:6$ Let the angles be $3x, 4x, 5x, 6x$

$$3x + 4x + 5x + 6x = 360°$$

$$\Rightarrow x = \frac{360°}{18}$$

$$\Rightarrow x = 20°$$

Therefore the angles are

$3 \times 20 = 60°$,

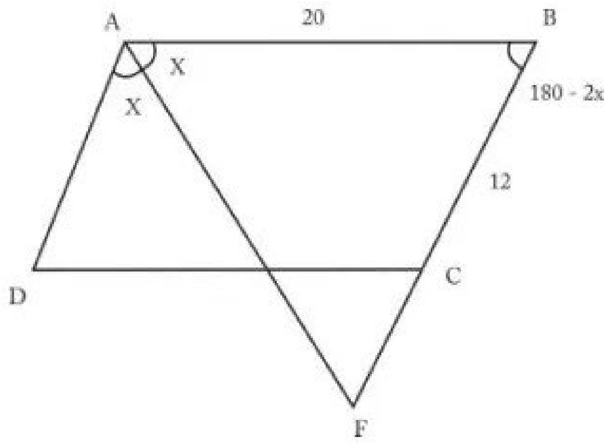
$4 \times 20 = 80°$,

$5 \times 20 = 100°$,

$6 \times 20 = 120°$

Since all the angles are of different degrees thus forms a trapezium

Solution 9:



Given AB = 20 cm and AD = 12 cm.

From the above figure, it's evident that ABF is an isosceles triangle with angle BAF = angle BFA = x

So AB = BF = 20

BF = 20

BC + CF = 20

CF = 20 - 12 = 8 cm

Solution 10:

We know that AQCP is a quadrilateral. So sum of all angles must be 360.
∴ x + y + 90 + 90 = 360
x + y = 180
Given x:y = 2:1
So substitute x = 2y
3y = 180
y = 60
x = 120
We know that angle C = angle A = x = 120
Angle D = Angle B = 180 – x = 180 – 120 = 60
Hence, angles of parallelogram are 120, 60, 120 and 60.