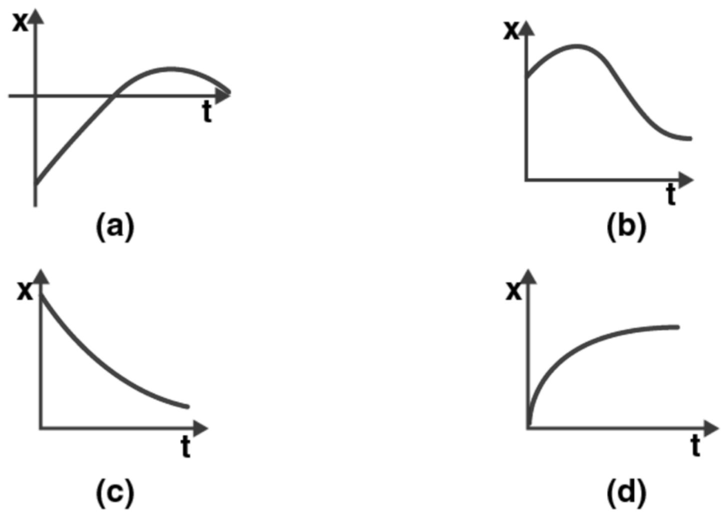
Very Short Answer Questions

Q. Refer to the graphs below and match the following:



Graph (a) → (iii): Has a point with zero displacement for t > 0 The curve crosses the time axis, indicating the particle returns to its starting position

Graph (b) → (ii): Has x > 0 throughout and has points with v = 0 and a = 0

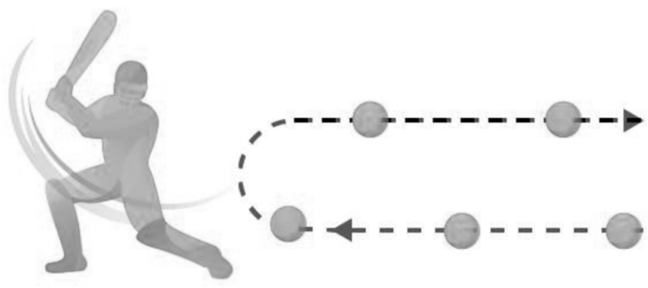
Always positive displacement, with a maximum (v = 0) and inflection point (a = 0)

Graph (c) → (iv): Has v < 0 and a > 0 Decreasing function (negative slope = negative velocity) with upward curvature (positive acceleration)

Graph (d) → (i): Has v > 0 and a < 0 throughout Increasing function (positive velocity) with downward curvature (negative acceleration)

3.13 Cricket Ball Collision

Question. A uniformly moving cricket ball is turned back by hitting it with a bat for a very short time interval.



Show the variation of its acceleration with taking acceleration in the backward direction as positive.

When a cricket ball collides with a bat:

Before collision: Ball moves with constant velocity ($a = 0$) **During collision:** Large impulsive force creates large acceleration **After collision:** Ball moves with constant velocity in opposite direction ($a = 0$)

The acceleration-time graph shows:

- Zero acceleration before and after collision
- Large positive acceleration during the brief collision time
- Rectangular pulse shape

14. Periodic Motion Examples

Enhanced Solutions:

(a) Particle moving forward, coming to rest periodically: $x(t) = t - \sin t$ $v(t) = 1 - \cos t$

- Particle moves forward overall (t term dominates)
- Velocity becomes zero when $\cos t = 1$ (at $t = 2\pi n$)
- Particle never moves backward since $v(t) \geq 0$

(b) Particle moving forward, coming to rest and moving backward: $x(t) = \sin t$ $v(t) = \cos t$

- At $t = \pi/2$: $x = 1$, $v = 0$ (momentarily at rest)
- At $t = \pi$: $x = 0$, $v = -1$ (moving backward)
- Demonstrates oscillatory motion with direction changes

15. Motion with x>0, v<0, a>0

Enhanced Example: Consider: $x(t) = A + Be^{-\gamma t}$ where $A > B > 0$ and $\gamma > 0$

Position: $x(t) = A + Be^{-\gamma t} > 0$ (since exponential term decreases from B to 0) **Velocity:** $v(t) = -B\gamma e^{-\gamma t} < 0$ (negative due to minus sign)

Acceleration: $a(t) = B\gamma^2 e^{-\gamma t} > 0$ (positive since all terms are positive)

Physical interpretation: Object moving leftward while slowing down due to rightward acceleration.

16. Terminal Velocity

Enhanced Solution: Given: $a = g - bv$

At terminal velocity, acceleration = 0: $0 = g - bv_0$ $v_0 = g/b$

Physical explanation:

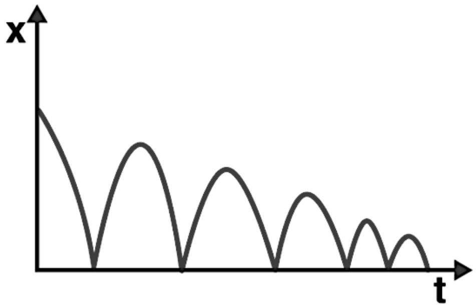
- Initially: gravitational force > drag force → net downward acceleration

- As speed increases: drag force increases
- At terminal velocity: gravitational force = drag force → zero acceleration
- Object continues falling at constant speed $v_0 = g/b$

Short Answer Questions

17. Bouncing Ball Graphs

A ball is dropped and its displacement vs time graph is as shown in the figure where displacement x is from the ground and all quantities are positive upwards.



Enhanced Analysis: Given displacement vs time graph for bouncing ball:

(a) Velocity-time graph:

- $v = dx/dt$ = slope of displacement curve
- Positive slope (upward motion) → positive velocity
- Negative slope (downward motion) → negative velocity
- At bounce points: instantaneous velocity change from negative to positive
- Creates sawtooth pattern with discontinuities at bounce times

(b) Acceleration-time graph:

- During free fall: $a = -g$ (constant)
- At bounce instants: infinite acceleration (impulse)
- Between bounces: constant negative acceleration
- Creates rectangular pulses at bounce times superimposed on constant negative level

18. Exponential Motion Analysis

Enhanced Solution: Given: $x(t) = x_0(1 - e^{-\gamma t})$ where $t \geq 0$, $x_0 > 0$

Derivatives:

- $v(t) = dx/dt = x_0\gamma e^{-\gamma t}$
- $a(t) = dv/dt = -x_0\gamma^2 e^{-\gamma t}$

(a) Starting conditions:

- $x(0) = x_0(1 - 1) = 0$ (starts at origin)
- $v(0) = x_0\gamma$ (starts with positive velocity)

(b) Extrema analysis:

- $x(t)$: minimum at $t = 0$ ($x = 0$), maximum as $t \to \infty$ ($x \to x_0$)
- $v(t)$: maximum at $t = 0$ ($v = x_0\gamma$), minimum as $t \to \infty$ ($v \to 0$)
- $a(t)$: minimum at $t = 0$ ($a = -x_0\gamma^2$), maximum as $t \to \infty$ ($a \to 0$)

Behavior verification:

- $x(t)$ increases monotonically from 0 to x_0
- $v(t)$ decreases monotonically from $x_0\gamma$ to 0
- $a(t)$ increases monotonically from $-x_0\gamma^2$ to 0

19. Bird Between Cars

Enhanced Solution: Given data:

- Car 1 speed: 18 km/h = 5 m/s
- Car 2 speed: 27 km/h = 7.5 m/s
- Bird speed: 36 km/h = 10 m/s
- Initial separation: 36 km

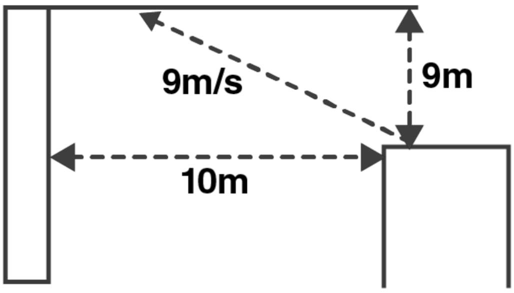
Solution approach: Time until cars meet = Distance/Relative speed = 36 km/(18 + 27) km/h = 36/45 = 0.8 h

Bird's motion: The bird flies continuously at 36 km/h for 0.8 hours Distance covered by bird = 36 × 0.8 = 28.8 km

Displacement analysis: The bird's displacement equals the distance traveled by Car 1 in 0.8 hours: Displacement = 18 × 0.8 = 14.4 km

20. Building Jump Problem

A man runs across the roof-top of a tall building and jumps horizontally with the hope of landing on the roof of the next building which is of a lower height than the first. If his speed is 9 m/s, the distance between the two buildings is 10 m and the height difference is 9 m, will he be able to land on the next building?



Solution: Given:

- Horizontal speed: 9 m/s
- Horizontal gap: 10 m
- Vertical drop: 9 m

Vertical motion analysis: Using $s = ut + \frac{1}{2}gt^2$: $9 = 0 + \frac{1}{2}(10)t^2$ $t = \sqrt{18/10} = \sqrt{1.8} \approx 1.34$ s

Horizontal distance covered: $d = v_x t = 9 \times 1.34 = 12.07$ m

Conclusion: Since 12.07 m > 10 m, the man will successfully land on the next building with 2.07 m to spare.

21. Relative Motion of Balls

Enhanced Solution: Ball 1 (dropped):

- Initial velocity: $u_1 = 0$
- Velocity at time t: $v_1 = gt$ (downward)

Ball 2 (thrown up):

- Initial velocity: $u_2 = 40$ m/s (upward)
- Velocity at time t: $v_2 = 40 - gt$

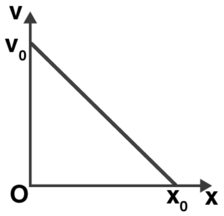
Relative velocity: $v_{rel} = v_1 - v_2 = gt - (40 - gt) = 2gt - 40$

Analysis:

- At t = 0: $v_{rel} = -40$ m/s (Ball 2 moving away from Ball 1)
- At t = 20s: $v_{rel} = 0$ (Same velocity)
- At t > 20s: $v_{rel} > 0$ (Ball 1 moving away from Ball 2)

22. Velocity-Displacement Relationship

The velocity-displacement graph of a particle is shown in the figure.



Enhanced Solution: (a) Relationship between v and x: From the linear graph: $v = v_0(1 - x/x_0)$

Verification:

- At $x = 0$: $v = v_0$ ✓
- At $x = x_0$: $v = 0$ ✓

(b) Acceleration-displacement relationship: Using $a = v(dv/dx)$: $a = v_0(1 - x/x_0) \times d/dx[v_0(1 - x/x_0)]$ $a = v_0(1 - x/x_0) \times (-v_0/x_0)$ $a = -(v_0^2/x_0)(1 - x/x_0)$

Graph characteristics:

- At $x = 0$: $a = -v_0^2/x_0$ (maximum negative acceleration)
- At $x = x_0$: $a = 0$ (zero acceleration)
- Linear relationship between a and x