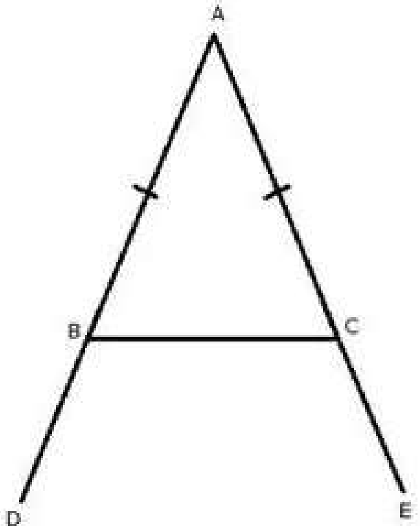
Exercise 10(B)

Solution 1:



Const: AB is produced to D and AC is produced to E so that exterior angles $\angle DBC$ and $\angle ECB$ is formed.

In ΔABC,

$AB = AC$ [Given]

$\therefore \angle C = \angle B$.......(i) [angles opp. to equal sides are equal]

Since angle B and angle C are acute they cannot be right angles or obtuse angles.

$\angle ABC + \angle DBC = 180^\circ$ [ABD is a st. line]

$\Rightarrow \angle DBC = 180^\circ - \angle ABC$

$\Rightarrow \angle DBC = 180^\circ - \angle B$........(ii)

Similarly,

$\angle ACB + \angle ECB = 180^\circ$ [ABD is a st. line]

$\Rightarrow \angle ECB = 180^\circ - \angle ACB$

$\Rightarrow \angle ECB = 180^\circ - \angle C$.........(iii)

$\Rightarrow \angle ECB = 180^\circ - \angle B$.........(iv) [from (i) and (iii)]

$\Rightarrow \angle DBC = \angle ECB$ [from (ii) and (iv)]

Now,

$\angle DBC = 180^\circ - \angle B$

But $\angle B$ = Acute angle

$\therefore \angle DBC = 180^\circ$ – Acute angle = obtuse angle

Similarly,

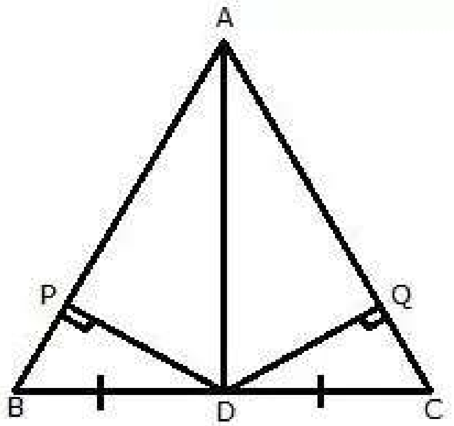
$\angle ECB = 180^\circ - \angle C$.

But $\angle C$ = Acute angle

$\therefore \angle ECB = 180^\circ$ – Acute angle = obtuse angle

Therefore, exterior angles formed are obtuse and equal.

Solution 2:



Const: Join AD.

In $\triangle ABC$,

$AB = AC$ [Given]

$\therefore \angle C = \angle B$.....(i) [angles opp. to equal sides are equal]

(i)

In $\triangle BPD$ and $\triangle CQD$,

$\angle BPD = \angle CQD$ [Each = 90°]

$\angle B = \angle C$ [proved]

$BD = DC$ [Given]

$\therefore \triangle BPD \cong \triangle CQD$ [AAS criterion]

$\therefore DP = DQ$ [cpct]

(ii) We have already proved that $\triangle BPD \cong \triangle CQD$

Therefore, $BP = CQ$ [cpct]

Now,

$AB = AC$ [Given]

$\Rightarrow AB - BP = AC - CQ$

$\Rightarrow AP = AQ$

(iii)

In ΔAPD and ΔAQD,

$DP = DQ$ [proved]

$AD = AD$ [common]

$AP = AQ$ [Proved]

$\therefore \Delta APD \cong \Delta AQD$ [SSS]

$\Rightarrow \angle PAD = \angle QAD$ [cpct]

Hence, AD bisects angle A.

Solution 3:

(i)

In ΔAEB and ΔAFC,

$\angle A = \angle A$ [Common]

$\angle AEB = \angle AFC = 90°$ [Given: $BE \perp AC$]

[Given: $CF \perp AB$]

$AB = AC$ [Given]

$\Rightarrow \Delta AEB \cong \Delta AFC$ [AAS]

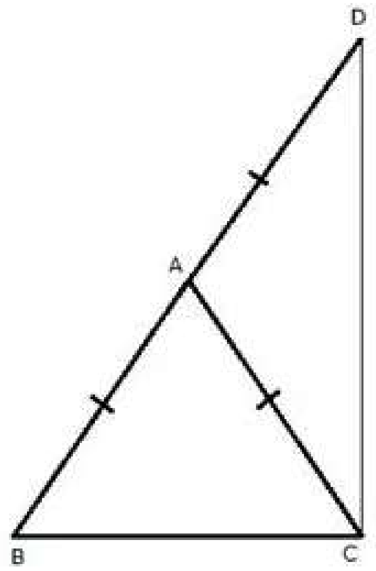
$\therefore BE = CF$ [cpct]

(ii) Since $\Delta AEB \cong \Delta AFC$

$\angle ABE = \angle AFC$

$\therefore AF = AE$ [congruent angles of congruent triangles]

Solution 4:



Const: Join CD.

In ΔABC,

AB = AC [Given]

∴ ∠C = ∠B......(i) [angles opp. to equal sides are equal]

In ΔACD,

AC = AD [Given]

∴ ∠ADC = ∠ACD..........(ii)

Adding (i) and (ii)

∠B + ∠ADC = ∠C + ∠ACD

∠B + ∠ADC = ∠BCD........(iii)

In ΔBCD,

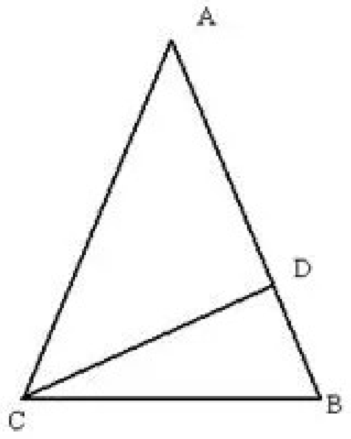
∠B + ∠ADC + ∠BCD = 180°

∠BCD + ∠BCD = 180° [From (iii)]

2∠BCD = 180°

∠BCD = 90°

Solution 5:



AB = AC

ΔABC is an isosceles triangle.

$\angle A = 36°$

$\angle B = \angle C = \dfrac{180° - 36°}{2} = 72°$

$\angle ACD = \angle BCD = 36°$ [$\because$ CD is the angle bisector of $\angle C$]

ΔADC is an isosceles triangle since $\angle DAC = \angle DCA = 36°$

$\therefore$ AD = CD......(i)

In ΔDCB,

$\angle CDB = 180° - (\angle DCB + \angle DBC)$

$= 180° - (36° + 72°)$

$= 180° - 108°$

$= 72°$

ΔDCB is an isosceles triangle since $\angle CDB = \angle CBD = 72°$

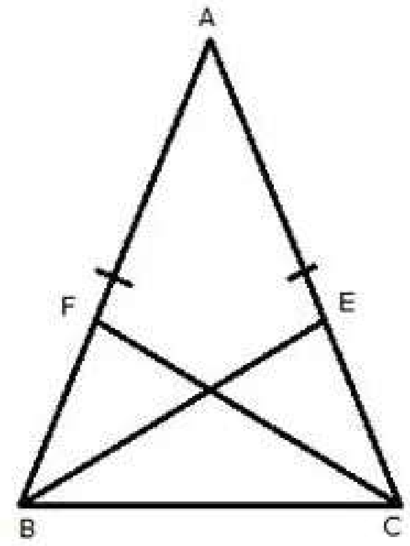
$\therefore$ DC = BC.......(ii)

From (i) and (ii), we get

AD = BC

Hence proved

Solution 6:



In ΔABC,

$AB = AC$ [Given]

$\therefore \angle C = \angle B$......(i) [angles opp. to equal sides are equal]

$\Rightarrow \frac{1}{2}\angle C = \frac{1}{2}\angle B$

$\Rightarrow \angle BCF = \angle CBE$........(ii)

In ΔBCE and ΔCBF,

$\angle C = \angle B$ [From (i)]

$\angle BCF = \angle CBE$ [From (ii)]

$BC = BC$ [Common]

$\therefore \Delta BCE \cong \Delta CBF$ [AAS]

$\Rightarrow BE = CF$ [cpct]

Solution 7:

In ΔABC,

$AB = AC$ [Given]

$\therefore \angle ACB = \angle ABC$ [angles opp. to equal sides are equal]

$\Rightarrow \angle ABC = \angle ACB$......(i)

$\angle DBC = \angle ECB = 90^o$[Given]

$\Rightarrow \angle DBC = \angle ECB$.......(ii)

Subtracting (i) from (ii)

$\angle DCB - \angle ABC = \angle ECB - \angle ACB$

$\Rightarrow \angle DBA = \angle ECA$.........(iii)

In ΔDBA and ΔECA,

$\angle DBA = \angle ECA$ [From (iii)]

$\angle DAB = \angle EAC$ [Vertically opposite angles]

$AB = AC$ [Given]

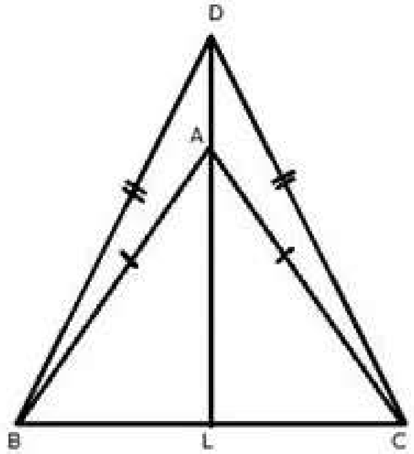
$\therefore \Delta DBA \cong \Delta ECA$ [ASA]

$\Rightarrow BD = CE$ [cpct]

Also,

$AD = AE$ [cpct]

Solution 8:



DA is produced to meet BC in L.

In ΔABC,

$AB = AC$ [Given]

$\therefore \angle ACB = \angle ABC$.......(i) [angles opposite to equal sides are equal]

In ΔDBC,

$DB = DC$ [Given]

$\therefore \angle DCB = \angle DBC$......(ii) [angles opposite to equal sides are equal]

Subtracting (i) from (ii)

$\angle DCB - \angle ACB = \angle DBC - \angle ABC$

$\Rightarrow \angle DCA = \angle DBA$.......(iii)

In ΔDBA and ΔDCA,

$DB = DC$ [Given]

$\angle DBA = \angle DCA$ [From (iii)]

$AB = AC$ [Given]

$\therefore \Delta DBA \cong \Delta DCA$ [SAS]

$\Rightarrow \angle BDA = \angle CDA$.........(iv) [cpct]

In ΔDBA,

$\angle BAL = \angle DBA + \angle BDA$..........(v)

[Ext. angle = sum of opp. int. angles]

From (iii), (iv) and (v)

$\angle BAL = \angle DCA + \angle CDA$.......(vi)

In ΔDCA,

$\angle CAL = \angle DCA + \angle CDA$..........(vii)

[Ext. angle = sum of opp. int. angles]

From (vi) and (vii)

$\angle BAL = \angle CAL$.......(viii)

In ΔBAL and ΔCAL,

$\angle BAL = \angle CAL$ [From (viii)]

$\angle ABL = \angle ACL$ [From (i)]

$AB = AC$ [Given]

$\therefore \Delta BAL \cong \Delta CAL$ [ASA]

$\Rightarrow \angle ALB = \angle ALC$ [cpct]

and $BL = LC$..........(ix) [cpct]

Now,

$\angle ALB + \angle ALC = 180^\circ$

$\Rightarrow \angle ALB + \angle ALB = 180^\circ$

$\Rightarrow 2\angle ALB = 180^\circ$

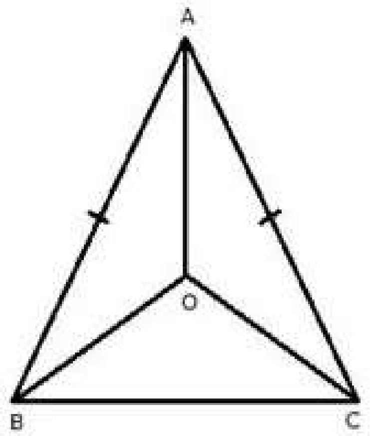
$\Rightarrow \angle ALB = 90^\circ$

$\therefore AL \perp BC$

or $DL \perp BC$ and $BL = LC$

$\therefore$ DA produced bisects BC at right angle.

Solution 9:



In $\triangle$ ABC, we have AB = AC

$\Rightarrow \angle B = \angle C$ [angles opposite to equal sides are equal]

$\Rightarrow \frac{1}{2}\angle B = \frac{1}{2}\angle C$

$\Rightarrow \angle OBC = \angle OCB$............(i)

$\Rightarrow OB = OC$..............(ii)

[angles opposite to equal sides are equal]

Now,

In $\triangle$ ABO and $\triangle$ ACO,

AB = AC [Given]

$\angle OBC = \angle OCB$ [From (i)]

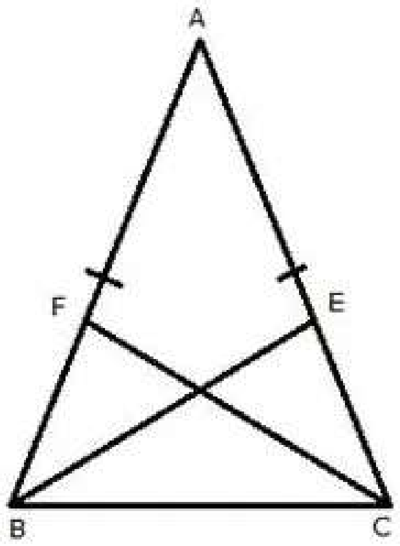
OB = OC [From (ii)]

$\Delta ABO \cong \Delta ACO$ [SAS criterion]

$\Rightarrow \angle BAO = \angle CAO$ [cpct]

Therefore, AO bisects $\angle$ BAC.

Solution 10:



In ΔABC,

$AB = AC$ [Given]

$\therefore \angle C = \angle B$......(i) [angles opp. to equal sides are equal]

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}AC$

$\Rightarrow BF = CE$........(ii)

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}AC$

$\Rightarrow BF = CE$........(ii)

In ΔBCE and ΔCBF,

$\angle C = \angle B$ [From (i)]

$BF = CE$ [From (ii)]

$BC = BC$ [Common]

$\therefore \Delta BCE \cong \Delta CBF$ [SAS]

$\Rightarrow BE = CF$ [cpct]

Solution 11:

In ΔAPQ,

$AP = AQ$ [Given]

$\therefore \angle APQ = \angle AQP$........(i)

[angles opposite to equal sides are equal]

In ΔABP,

$\angle APQ = \angle BAP + \angle ABP$.......(ii)

[Ext. angle is equal to sum of opp. int. angles]

In ΔAQC,

$\angle AQP = \angle CAQ + \angle ACQ$.......(iii)

[Ext. angle is equal to sum of opp. int. angles]

From (i), (ii) and (iii)

$\angle BAP + \angle ABP = \angle CAQ + \angle ACQ$

But, $\angle BAP = \angle CAQ$ [Given]

$\Rightarrow \angle CAQ + \angle ABP = \angle CAQ + \angle ACQ$

$\Rightarrow \angle ABP = \angle CAQ + \angle ACQ - \angle CAQ$

$\Rightarrow \angle ABP = \angle ACQ$

$\Rightarrow \angle B = \angle C$........(iv)

In ΔABC,

$\angle B = \angle C$

$\Rightarrow AB = AC$ [Sides opposite to equal angles are equal]

Solution 12:

Since AE || BC and DAB is the transversal

$\therefore \angle DAE = \angle ABC = \angle B$ [Corresponding angles]

Since AE || BC and AC is the transversal

$\angle CAE = \angle ACB = \angle C$ [Alternate Angles]

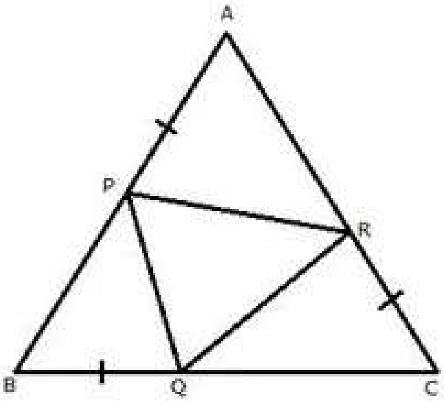
But AE bisects $\angle CAD$

$\therefore \angle DAE = \angle CAE$

$\Rightarrow \angle B = \angle C$

$\Rightarrow AB = AC$ [Sides opposite to equal angles are equal]

Solution 13:



AB = BC = CA.......(i) [Given]

AP = BQ = CR.......(ii) [Given]

Subtracting (ii) from (i)

AB - AP = BC - BQ = CA - CR

BP = CQ = AR ....(iii)

$\therefore \angle A = \angle B = \angle C$ ....(iv) [angles opp. to equal sides are equal]

In ΔBPQ and ΔCQR,

BP = CQ [From (iii)]

$\angle B = \angle C$ [From (iv)]

BQ = CR [Given]

$\therefore \Delta BPQ \cong \Delta CQR$ [SAS criterion]

$\Rightarrow$ PQ = QR........(v)

In ΔCQR and ΔAPR,

CQ = AR [From (iii)]

$\angle C = \angle A$ [From (iv)]

CR = AP [Given]

$\therefore \Delta CQR \cong \Delta APR$ [SAS criterion]

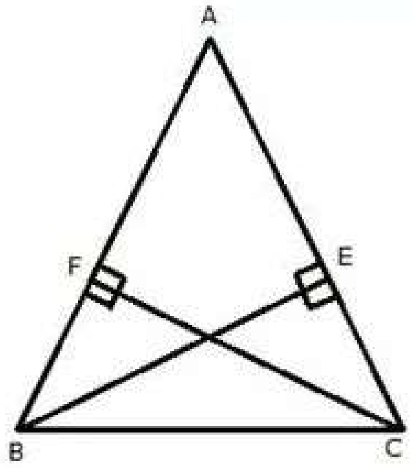
$\Rightarrow$ QR = PR........(vi)

From (v) and (vi)

PQ = QR = PR

Therefore, PQR is an equilateral triangle.

Solution 14:



In Δ ABE and Δ ACF,

$\angle A = \angle A$[Common]

$\angle AEB = \angle AFC = 90^0$[Given: BE $\perp$ AC; CF $\perp$ AB]

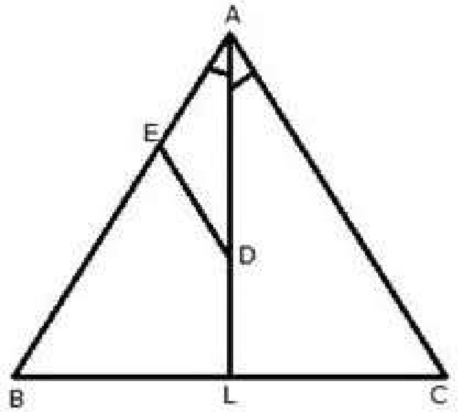
BE = CF[Given]

$\therefore \Delta ABE \cong \Delta ACF$ [AAS criterion]

$\Rightarrow AB = AC$

Therefore, ABC is an isosceles triangle.

Solution 15:



AL is bisector of angle A. Let D is any point on AL. From D, a straight line DE is drawn parallel to AC.

DE || AC [Given]

$\therefore \angle ADE = \angle DAC$....(i) [Alternate angles]

$\angle DAC = \angle DAE$.......(ii) [AL is bisector of $\angle A$]

From (i) and (ii)

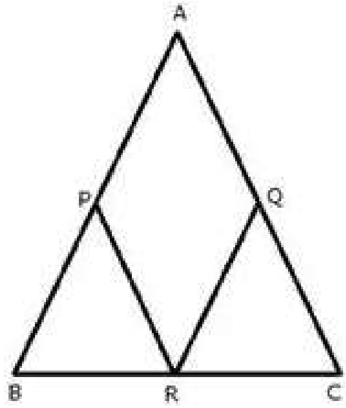
$\angle ADE = \angle DAE$

$\therefore$ AE = ED [Sides opposite to equal angles are equal]

Therefore, AED is an isosceles triangle.

Solution 16:

(i)



In Δ ABC,

AB = AC

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}AC$

$\Rightarrow$AP = AQ ....(i)[Since P and Q are mid - points]

In Δ BCA,

PR = $\frac{1}{2}$AC [PR is line joining the mid - points of AB and BC]

$\Rightarrow$PR = AQ........(ii)

In Δ CAB,

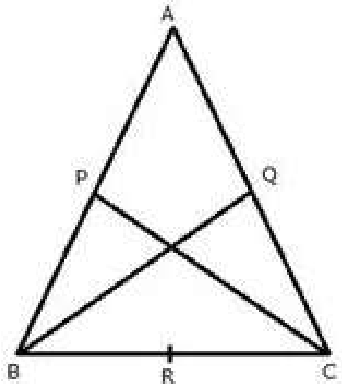
QR = $\frac{1}{2}$AB [QR is line joining the mid - points of AC and BC]

$\Rightarrow$QR = AP.......(iii)

From (i), (ii) and (iii)

PR = QR

(ii)



AB = AC

$\Rightarrow \angle B = \angle C$

Also,

$\frac{1}{2}AB = \frac{1}{2}AC$

$\Rightarrow BP = CQ$ [P and Q are mid-points of AB and AC]

In $\triangle BPC$ and $\triangle CQB$,

BP = CQ

$\angle B = \angle C$

BC = BC

Therefore, $\Delta BPC \cong \Delta CQB$ [SAS]

BP = CP

Solution 17:

(i) In $\triangle$ ACB,

AC = AC[Given]

$\therefore \angle ABC = \angle ACB$ ....(i)[angles opposite to equal sides are equal]

$\angle ACD + \angle ACB = 180^0$ ....(ii)[DCB is a straight line]

$\angle ABC + \angle CBE = 180^0$ ....(iii)[ABE is a straight line]

Equating (ii) and (iii)

$\angle ACD + \angle ACB = \angle ABC + \angle CBE$

$\Rightarrow \angle ACD + \angle ACB = \angle ACB + \angle CBE$[From (i)]

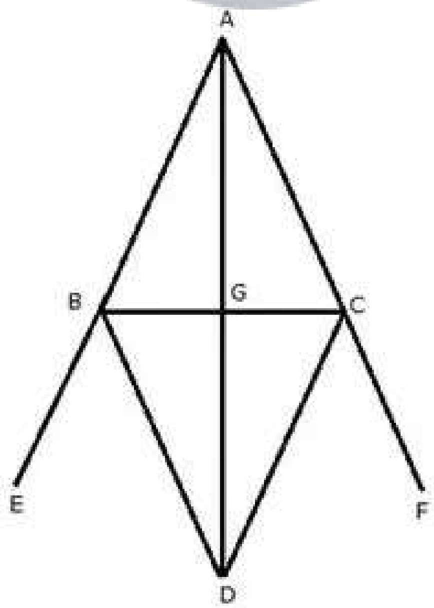
$\Rightarrow \angle ACD = \angle CBE$

(ii)

In ΔACD and ΔCBE,

DC = CB	[Given]
AC = BE	[Given]
$\angle ACD = \angle CBE$	[Proved Earlier]
$\therefore \Delta ACD \cong \Delta CBE$	[SAS criterion]
$\Rightarrow$ AD = CE	[cpct]

Solution 18:



AB is produced to E and AC is produced to F. BD is bisector of angle CBE and CD is bisector of angle BCF. BD and CD meet at D.

In $\triangle ABC$,

$AB = AC$[Given]

$\therefore \angle C = \angle B$[angles opposite to equal sides are equal]

$\angle CBE = 180^0 - \angle B$[ABE is a straight line]

$\Rightarrow \angle CBD = \frac{180^\circ - \angle B}{2}$ [BD is bisector of $\angle CBE$]

$\Rightarrow \angle CBD = 90^\circ - \frac{\angle B}{2}$..............(i)

Similarly,

$\angle BCF = 180^0 - \angle C$[ACF is a straight line]

$\Rightarrow \angle BCD = \frac{180^\circ - \angle C}{2}$ [CD is bisector of $\angle BCF$]

$\Rightarrow \angle BCD = 90^\circ - \frac{\angle C}{2}$..............(ii)

Now,

$\Rightarrow \angle CBD = 90^\circ - \frac{\angle C}{2}$ [$\because \angle B = \angle C$]

$\Rightarrow \angle CBD = \angle BCD$

In $\triangle BCD$,

$\angle CBD = \angle BCD$

$\therefore BD = CD$

In $\triangle ABD$ and $\triangle ACD$,

$AB = AC$[Given]

$AD = AD$[Common]

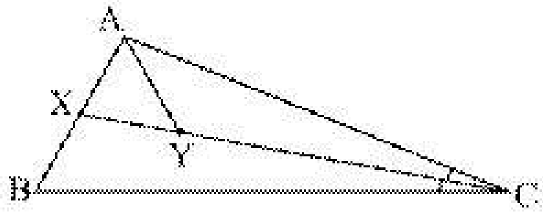
$BD = CD$[Proved]

$\therefore \Delta ABD \cong \Delta ACD$ [SSS criterion]

$\Rightarrow \angle BAD = \angle CAD$ [cpct]

Therefore, AD bisects $\angle A$.

Solution 19:



In $\triangle$ ABC,

CX is the angle bisector of $\angle$C

$\Rightarrow \angle$ACY = $\angle$BCX .... (i)

In $\triangle$ AXY,

AX = AY [Given]

$\angle$AXY = $\angle$AYX ....(ii) [angles opposite to equal sides are equal]

Now $\angle$XYC = $\angle$AXB = 180° [straight line]

$\Rightarrow \angle$AYX + $\angle$AYC = $\angle$AXY + $\angle$BXY

$\Rightarrow \angle$AYC = $\angle$BXY .... (iii) [From (ii)]

In $\triangle$ AYC and $\triangle$ BXC

$\angle$AYC + $\angle$ACY + $\angle$CAY = $\angle$BXC + $\angle$BCX + $\angle$XBC = 180°

$\Rightarrow \angle$CAY = $\angle$XBC [From (i) and (iii)]

$\Rightarrow \angle$CAY = $\angle$ABC

Solution 20:

Since IA || CP and CA is a transversal

$\therefore \angle CAI = \angle PCA$ [Alternate angles]

Also, IA || CP and AP is a transversal

$\therefore \angle IAB = \angle APC$ [Corresponding angles]

But $\therefore \angle CAI = \angle IAB$ [Given]

$\therefore \angle PCA = \angle APC$

$\Rightarrow$ AC = AP

Similarly,

BC = BQ

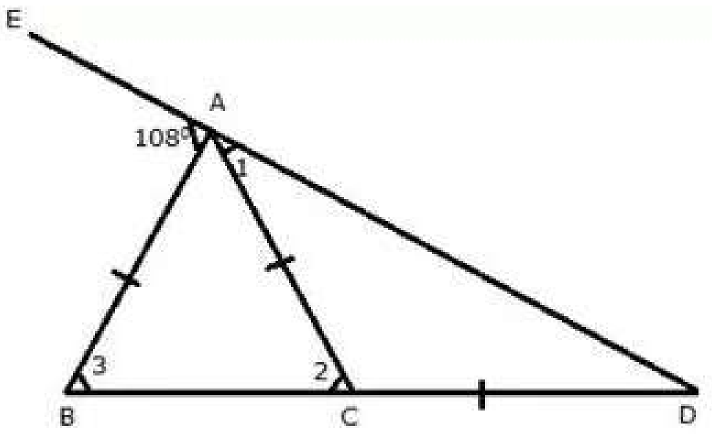
Now,

PQ = AP + AB + BQ

= AC + AB + BC

= Perimeter of $\triangle$ ABC

Solution 21:



In $\triangle$ ABD,

$\angle BAE = \angle 3 + \angle ADB$

$\Rightarrow 108^0 = \angle 3 + \angle ADB$

But AB = AC

$\Rightarrow \angle 3 = \angle 2$

$\Rightarrow 108^0 = \angle 2 + \angle ADB$ ......(i)

Now,

In $\triangle$ ACD,

$\angle 2 = \angle 1 + \angle ADB$

But AC = CD

$\Rightarrow \angle 1 = \angle ADB$

$\Rightarrow \angle 2 = \angle ADB + \angle ADB$

$\Rightarrow \angle 2 = 2\angle ADB$

Putting this value in (i)

$\Rightarrow 108^0 = 2\angle ADB + \angle ADB$

$\Rightarrow 3\angle ADB = 108^0$

$\Rightarrow \angle ADB = 36^0$

Solution 22:

ABC is an equilateral triangle.

Therefore, AB = BC = AC = 15 cm

$\angle A = \angle B = \angle C = 60^\circ$

In ΔADE, DE || BC[Given]

$\angle AED = 60^\circ$ [$\because \angle ACB = 60^\circ$]

$\angle ADE = 60^\circ$ [$\because \angle ABC = 60^\circ$]

$\angle DAE = 180^\circ - (60^\circ + 60^\circ) = 60^\circ$

Similarly, ΔBDF & ΔGEC are equilateral triangles.

$= 60^\circ$ [$\because \angle C = 60^\circ$]

Let AD = x, AE = x, DE = x [$\because \Delta ADE$ is an equilateral triangle]

Let BD = y, FD = y, FB = y [$\because \Delta BDF$ is an equilateral triangle]

Let EC = z, GC = z, GE = z [$\because \Delta GEC$ is an equilateral triangle]

Now, AD + DB = 15 $\Rightarrow x + y = 15$......(i)

AE + EC = 15 $\Rightarrow x + z = 15$.....(ii)

Given, DE + DF + EG = 20

$\Rightarrow x + y + z = 20$

$\Rightarrow 15 + z = 20$ [from (i)]

$\Rightarrow z = 5$

From (ii), we get $x = 10$

$\therefore y = 5$

Also, BC = 15

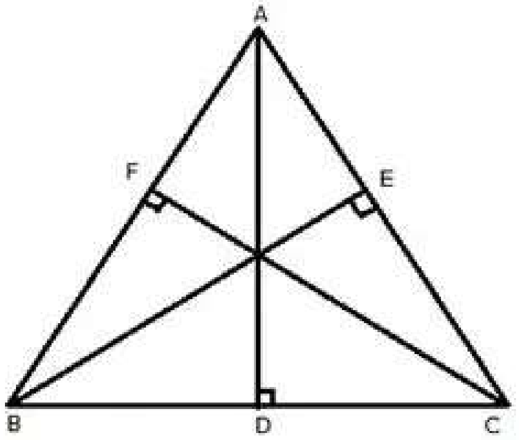
BF + FG + GC = 15

$\Rightarrow y$ + FG + z = 15

$\Rightarrow$ 5 + FG + 5 = 15

$\Rightarrow$ FG = 5

Solution 23:



In right Δ BEC and Δ BFC,

BE = CF[Given]

BC = BC[Common]

$\angle$BEC = $\angle$BFC[each = 90^0]

$\therefore \Delta BEC \cong \Delta BFC$ [RHS]

$\Rightarrow \angle B = \angle C$

Similarly,

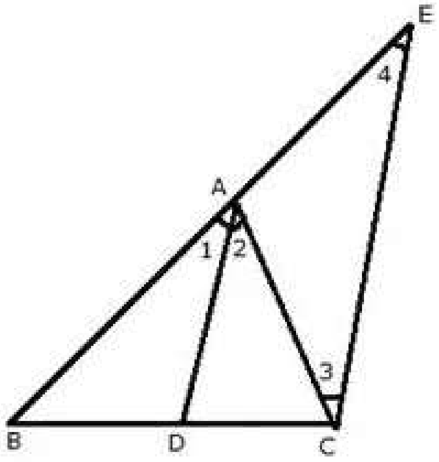
$\angle A = \angle B$

Hence, $\angle A = \angle B = \angle C$

$\Rightarrow AB = BC = AC$

Therefore, ABC is an equilateral triangle.

Solution 24:



DA || CE[Given]

$\Rightarrow \angle 1 = \angle 4$........(i) [Corresponding angles]

$\angle 2 = \angle 3$........(ii) [Alternate angles]

But $\angle 1 = \angle 2$.........(iii) [AD is the bisector of $\angle$ A]

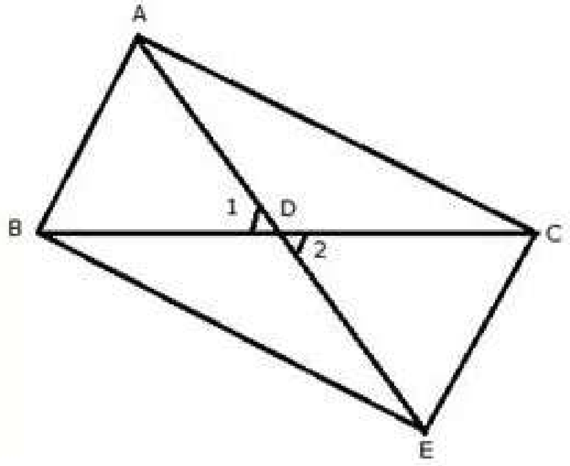
From (i), (ii) and (iii)

$\angle 3 = \angle 4$

$\Rightarrow$ AC = AE

$\Rightarrow \Delta$ ACE is an isosceles triangle.

Solution 25:



Produce AD upto E such that AD = DE.

In ΔABD and ΔEDC,

AD = DE [by construction]

BD = CD [Given]

∠1 = ∠2 [vertically opposite angles]

∴ ΔABD ≅ ΔEDC [SAS]

⇒ AB = CE.......(i)

and ∠BAD = ∠CED

But, ∠BAD = ∠CAD [AD is bisector of ∠BAC]

∴ ∠CED = ∠CAD

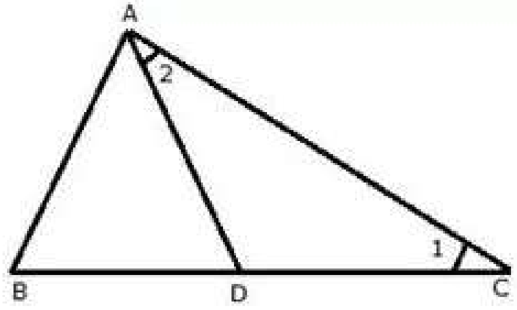
⇒ AC = CE......(ii)

From (i) and (ii)

AB = AC

Hence, ABC is an isosceles triangle.

Solution 26:



Since AB = AD = BD

$\therefore$ ΔABD is an equilateral triangle.

$\therefore \angle ADB = 60^\circ$

$\Rightarrow \angle ADC = 180^\circ - \angle ADB$

$= 180^\circ - 60^\circ$

$= 120^\circ$

Again in ΔADC,

AD = DC

$\therefore \angle 1 = \angle 2$

But,

$\angle 1 + \angle 2 + \angle ADC = 180^\circ$

$\Rightarrow 2\angle 1 + 120^\circ = 180^\circ$

$\Rightarrow 2\angle 1 = 60^\circ$

$\Rightarrow \angle 1 = 30^\circ$

$\Rightarrow \angle C = 30^\circ$

$\therefore \angle ADC : \angle C = 120^\circ : 30^\circ$

$\Rightarrow \angle ADC : \angle C = 4 : 1$

Solution 27:

(i)

In ΔCAE, $\angle CAE = \angle AEC = \dfrac{180^\circ - 68^\circ}{2} = 56^\circ$ [$\because$ CE=AC]

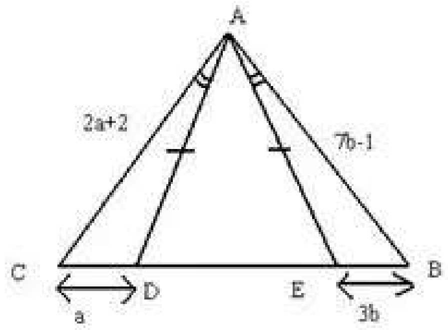
In $\angle BEA$, $a = 180^\circ - 56^\circ = 124^\circ$

In ΔABE, $\angle ABE = 180^\circ - (a + \angle BAE)$

$= 180^\circ - (124^\circ + 14^\circ)$

$= 180^\circ - 138^\circ = 42^\circ$

(ii)



In ΔAEB & ΔCAD,

$\angle EAB = \angle CAD$ [Given]

$\angle ADC = \angle AEB$ [$\because \angle ADE = \angle AED$ {AE=AD}

$180^\circ - \angle ADE = 180^\circ - \angle AED$

$\angle ADC = \angle AEB$]

AE=AD[Given]

$\therefore \Delta AEB \cong \Delta CAD$ [ASA]

AC=AB[By C.P.C.T.]

$2a+2=7b-1$

$\Rightarrow 2a-7b=-3$.....(i)

CD=EB

$\Rightarrow a=3b$.......(ii)

Solving (i) & (ii), we get

$a=9$, $b=3$