

EXERCISE 8.1

Express each of the following as the sum or difference of sines and cosines:

(i) $2 \sin 3x \cos x$

(ii) $2 \cos 3x \sin 2x$

(iii) $2 \sin 4x \sin 3x$

(iv) $2 \cos 7x \cos 3x$

Solution:

(i) $2 \sin 3x \cos x$

By using the formula,

$$2 \sin A \cos B = \sin (A + B) + \sin (A - B)$$

$$\begin{aligned} 2 \sin 3x \cos x &= \sin (3x + x) + \sin (3x - x) \\ &= \sin (4x) + \sin (2x) \\ &= \sin 4x + \sin 2x \end{aligned}$$

(ii) $2 \cos 3x \sin 2x$

By using the formula,

$$2 \cos A \sin B = \sin (A + B) - \sin (A - B)$$

$$\begin{aligned} 2 \cos 3x \sin 2x &= \sin (3x + 2x) - \sin (3x - 2x) \\ &= \sin (5x) - \sin (x) \\ &= \sin 5x - \sin x \end{aligned}$$

(iii) $2 \sin 4x \sin 3x$

By using the formula,

$$2 \sin A \sin B = \cos (A - B) - \cos (A + B)$$

$$\begin{aligned} 2 \sin 4x \sin 3x &= \cos (4x - 3x) - \cos (4x + 3x) \\ &= \cos (x) - \cos (7x) \\ &= \cos x - \cos 7x \end{aligned}$$

(iv) $2 \cos 7x \cos 3x$

By using the formula,

$$2 \cos A \cos B = \cos (A + B) + \cos (A - B)$$

$$\begin{aligned} 2 \cos 7x \cos 3x &= \cos (7x + 3x) + \cos (7x - 3x) \\ &= \cos (10x) + \cos (4x) \\ &= \cos 10x + \cos 4x \end{aligned}$$

2. Prove that:

(i) $2 \sin 5\pi/12 \sin \pi/12 = 1/2$

(ii) $2 \cos 5\pi/12 \cos \pi/12 = 1/2$

(iii) $2 \sin 5\pi/12 \cos \pi/12 = (\sqrt{3} + 2)/2$

Solution:

(i) $2 \sin 5\pi/12 \sin \pi/12 = 1/2$

By using the formula,

$$2 \sin A \sin B = \cos (A - B) - \cos (A + B)$$

$$\begin{aligned} 2 \sin 5\pi/12 \sin \pi/12 &= \cos (5\pi/12 - \pi/12) - \cos (5\pi/12 + \pi/12) \\ &= \cos (4\pi/12) - \cos (6\pi/12) \\ &= \cos (\pi/3) - \cos (\pi/2) \\ &= \cos (180^\circ/3) - \cos (180^\circ/2) \\ &= \cos 60^\circ - \cos 90^\circ \\ &= 1/2 - 0 \\ &= 1/2 \end{aligned}$$

Hence Proved.

(ii) $2 \cos 5\pi/12 \cos \pi/12 = 1/2$

By using the formula,

$$2 \cos A \cos B = \cos (A + B) + \cos (A - B)$$

$$\begin{aligned} 2 \cos 5\pi/12 \cos \pi/12 &= \cos (5\pi/12 + \pi/12) + \cos (5\pi/12 - \pi/12) \\ &= \cos (6\pi/12) + \cos (4\pi/12) \\ &= \cos (\pi/2) + \cos (\pi/3) \\ &= \cos (180^\circ/2) + \cos (180^\circ/3) \\ &= \cos 90^\circ + \cos 60^\circ \\ &= 0 + 1/2 \\ &= 1/2 \end{aligned}$$

Hence Proved.

(iii) $2 \sin 5\pi/12 \cos \pi/12 = (\sqrt{3} + 2)/2$

By using the formula,

$$2 \sin A \cos B = \sin (A + B) + \sin (A - B)$$

$$\begin{aligned} 2 \sin 5\pi/12 \cos \pi/12 &= \sin (5\pi/12 + \pi/12) + \sin (5\pi/12 - \pi/12) \\ &= \sin (6\pi/12) + \sin (4\pi/12) \\ &= \sin (\pi/2) + \sin (\pi/3) \\ &= \sin (180^\circ/2) + \sin (180^\circ/3) \\ &= \sin 90^\circ + \sin 60^\circ \\ &= 1 + \sqrt{3} \\ &= (2 + \sqrt{3})/2 \\ &= (\sqrt{3} + 2)/2 \end{aligned}$$

Hence Proved.

3. show that:

(i) $\sin 50^\circ \cos 85^\circ = (1 - \sqrt{2}\sin 35^\circ)/2\sqrt{2}$

(ii) $\sin 25^\circ \cos 115^\circ = 1/2 \{ \sin 140^\circ - 1 \}$

Solution:

(i) $\sin 50^\circ \cos 85^\circ = (1 - \sqrt{2}\sin 35^\circ)/2\sqrt{2}$

By using the formula,

$$2 \sin A \cos B = \sin (A + B) + \sin (A - B)$$

$$\sin A \cos B = [\sin (A + B) + \sin (A - B)] / 2$$

$$\sin 50^\circ \cos 85^\circ = [\sin(50^\circ + 85^\circ) + \sin (50^\circ - 85^\circ)] / 2$$

$$= [\sin (135^\circ) + \sin (-35^\circ)] / 2$$

$$= [\sin (135^\circ) - \sin (35^\circ)] / 2 \text{ (since, } \sin (-x) = -\sin x)$$

$$= [\sin (180^\circ - 45^\circ) - \sin 35^\circ] / 2$$

$$= [\sin 45^\circ - \sin 35^\circ] / 2$$

$$= [(1/\sqrt{2}) - \sin 35^\circ] / 2$$

$$= [(1 - \sin 35^\circ)/\sqrt{2}] / 2$$

$$= (1 - \sin 35^\circ) / 2\sqrt{2}$$

Hence proved.

(ii) $\sin 25^\circ \cos 115^\circ = 1/2 \{ \sin 140^\circ - 1 \}$

By using the formula,

$$2 \sin A \cos B = \sin (A + B) + \sin (A - B)$$

$$\sin A \cos B = [\sin (A + B) + \sin (A - B)] / 2$$

$$\sin 20^\circ \cos 115^\circ = [\sin(25^\circ + 115^\circ) + \sin (25^\circ - 115^\circ)] / 2$$

$$= [\sin (140^\circ) + \sin (-90^\circ)] / 2$$

$$= [\sin (140^\circ) - \sin (90^\circ)] / 2 \text{ (since, } \sin (-x) = -\sin x)$$

$$= 1/2 \{ \sin 140^\circ - 1 \}$$

Hence proved.

4. Prove that:

$4 \cos x \cos (\pi/3 + x) \cos (\pi/3 - x) = \cos 3x$

Solution:

Let us consider LHS:

$$4 \cos x \cos (\pi/3 + x) \cos (\pi/3 - x) = 2 \cos x (2 \cos (\pi/3 + x) \cos (\pi/3 - x))$$

By using the formula,

$$2 \cos A \cos B = \cos (A + B) + \cos (A - B)$$

$$2 \cos x (2 \cos (\pi/3+x) \cos (\pi/3 - x)) = 2 \cos x (\cos (\pi/3+x + \pi/3-x) + \cos (\pi/3+x - \pi/3+x))$$

$$= 2 \cos x (\cos (2\pi/3) + \cos (2x))$$

$$= 2 \cos x \{ \cos 120^\circ + \cos 2x \}$$

$$= 2 \cos x \{ \cos (180^\circ - 60^\circ) + \cos 2x \}$$

$$\begin{aligned} &= 2 \cos x (\cos 2x - \cos 60^\circ) \text{ (since, } \{\cos (180^\circ - A) = -\cos A\}) \\ &= 2 \cos 2x \cos x - 2 \cos x \cos 60^\circ \\ &= (\cos (x + 2x) + \cos (2x - x)) - (2\cos x)/2 \\ &= \cos 3x + \cos x - \cos x \\ &= \cos 3x \\ &= \text{RHS} \end{aligned}$$

Hence Proved.

