NCERT Solutions for Class-XII Physics

Chapter-11

NCERT Physics Class 12

1. Find the
(a) maximum frequency, and
(b) minimum wavelength of X-rays produced by 30 kV electrons.

1. Potential of the electrons, V = 30 kV = 3×10^4 V
Hence, energy of the electrons, E = 3×10^4 eV Where,
e = Charge on an electron = 1.6×10^{-19} C
(a) Maximum frequency produced by the X-rays = ν
The energy of the electrons is given by the relation: E = hν
Where, h = Planck's constant = 6.626×10^{-34} Js

$$\therefore \nu = \frac{E}{h}$$

$$= \frac{1.6 \times 10^{-19} \times 3 \times 10^4}{6.626 \times 10^{-34}} = 7.24 \times 10^{18}\,\text{Hz}$$

Hence, the maximum frequency of X-rays produced is 7.24×10^{18} Hz.
(b) The minimum wavelength produced by the X-rays is given as:

$$\lambda = \frac{c}{\nu}$$

$$= \frac{3 \times 10^8}{7.24 \times 10^{18}} = 4.14 \times 10^{-11}\,\text{m} = 0.0414\,\text{nm}$$

Hence, the minimum wavelength of X-rays produced is 0.0414 nm.

2. The work function of caesium metal is 2.14 eV. When the light of frequency 6×10^{14} Hz is incident on the metal surface, photoemission of electrons occurs. What is the
(a) maximum kinetic energy of the emitted electrons,
(b) Stopping potential, and
(c) maximum speed of the emitted photoelectrons?

2. (a) maximum energy (K. E_{max}) of emitted electron is given by the following equation.
K. $E_{max} = h\nu - W_0$ .... equation no. 1
Where h = plank's constant = 6.63×10^{-34} J. sec
ν = max. frequency of X-ray

W_0 = work function (minimum energy required to emit an electron from neutral atom)

$W_0 = 2.14$ eV

And, $v = 6 \times 10^{14}$ Hz

From equation No. 1

$$\Rightarrow KE_{max} = \frac{6.63\times10^{-34}\,J-sec\times6\times10^{14}\,sec^{-1}}{1.6\times10^{-19}\,C} - 2.14\,eV$$

$\Rightarrow KE_{max} = 0.34$ eV

Or in Joules,

$KE_{max} = 0.34 \times 1.6 \times 10^{-19} = 0.546 \times 10^{-19}$ J

Maximum KE of emitted electron is 0.546×10^{-19} J.

(b) Stopping Potential is calculated by following relation.

$eV_0 = K.E_{max}$

Magnitude of charge of the electron 'e' = 1.6×10^{-19} C

V_0 = Stopping Potential

$$\Rightarrow V_0 = \frac{0.34eV\times1.6\times10^{-19}\,J}{1.6\times10^{-19}\,J}$$

$\Rightarrow V_0 = 0.34$ V

Stopping Potential of emitted electron is 0.34 V.

(c) $K.E_{max} = \frac{1}{2} mv^2_{max}$

m = mass of the electron = 9.1×10^{-31} kg

v_{max} = maximum velocity of electron

$v^2_{max} = 2\,K.E_{max} / m$

$v_{max} = (2 \times 0.546 \times 10^{-19}\,J / 9.1 \times 10^{-31})^{1/2}$

= 347 km /sec

Maximum velocity of emitted electron is 347 km / sec.

3. The photoelectric cut-off voltage in a certain experiment is 1.5 V. What is the maximum kinetic energy of photoelectrons emitted?

3. Photoelectric cut-off voltage, $V_0 = 1.5$ V

The maximum kinetic energy of the emitted photoelectrons is given as:

$K_e = eV_0$

Where, e = Charge on an electron = 1.6×10^{-19} C

$\therefore K_e = 1.6 \times 10^{-19} \times 1.5$

$= 2.4 \times 10^{-19}$ J

Therefore, the maximum kinetic energy of the photoelectrons emitted in the given experiment is 2.4×10^{-19} J.

4. Monochromatic light of wavelength 632.8 nm is produced by a helium-neon laser. The power emitted is 9.42 mW.

(a) Find the energy and momentum of each photon in the light beam,

(b) How many photons per second, on the average, arrive at a target irradiated by this beam? (Assume the beam to have uniform cross-section which is less than the target area), and

(c) How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon?

Find the energy and momentum of each photon in the light beam,

4. (a) Energy of photon E = hv = hc/λ

Where h = plank's constant = 6.63×10^{-34} J. sec

v = max. frequency of X-ray

c = speed of light = 3×10^8 m/s

λ = minimum wavelength of X-ray

$E = 6.63 \times 10^{-34} \times 3 \times 10^8 / 632.8 \times 10^{-9}$

$\therefore E = 3.14 \times 10^{-19}$ J

Now, p = h / λ

p = momentum of photon

$p = 6.63 \times 10^{-34}/632.8 \times 10^{-9}$

$\therefore p = 1.05 \times 10^{-27}$ kg-m/sec

Maximum energy and momentum of the electron are 3.14×10^{-19} J and 1.05×10^{-27} kg-m/sec respectively.

(b) No. of photons 'n' per sec. is equal to total power radiated divided by number of photons (it is assumed that power of each and every photon is same and all the photons are falling on the target).

'n' = P/E

Where P = radiated power of LASSER

$n = 9.42 \times 10^6/3.14 \times 10^{-19}$

$n = 3 \times 10^{16}$ photons/sec.

No. of photons falling per second is equal to 3×10^{16} photons/sec.

(c) mv = p

m = mass of Hydrogen = mass of proton = 1.67×10^{-27} kg

v = speed of Hydrogen

p = momentum of photon as well as Hydrogen atom

v = p/m

$v = 1.05 \times 10^{-27}/1.67 \times 10^{-27} = 0.63$ m/sec.

Required speed of Hydrogen atom is 0.63 m/sec.

5. The energy flux of sunlight reaching the surface of the earth is 1.388×10^3 W/m^2. How many photons (nearly) per square metre are incident on the Earth per second? Assume that the photons in the sunlight have an average wavelength of 550 nm.

5. Energy flux of sunlight reaching the surface of earth, $\Phi = 1.388 \times 10^3$ W/m^2

Hence, power of sunlight per square metre, $P = 1.388 \times 10^3$ W

Speed of light, $c = 3 \times 10^8$ m/s

Planck's constant, $h = 6.626 \times 10^{-34}$ Js

Average wavelength of photons present in sunlight, $\lambda = 550$ nm $= 550 \times 10^{-9}$ m

Number of photons per square metre incident on earth per second = n Hence, the equation for power can be written as:

$P = nE$

$$\therefore n = \frac{P}{E} = \frac{P\lambda}{hc}$$

$$= \frac{1.388 \times 10^3 \times 550 \times 10^{-9}}{6.626 \times 10^{-34} \times 3 \times 10^8} = 3.84 \times 10^{21} \text{ photons}/\text{m}^2/\text{s}$$

Therefore, every second 3.84×10^{21} photons are incident per square metre on earth.

6. In an experiment on photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be 4.12×10^{-15} V s.

Calculate the value of Planck's constant.

6. Photoelectric equation

$eV_0 = hv - Ǿ_0$ eq.01

Where e = Charge on an electron $= 1.6 \times 10^{-19}$ C

V_0 = Stopping Potential

h = plank's constant

v = frequency of photon

$Ǿ_0$ = Work function of the metal

Rewriting equation no. 1

$V_0 = hv/e - Ǿ_0/e$ ………………………………equ.02

h/e is slop of the line represented by above equation 02.

Therefore h/e = slop of the line $= 4.12 \times 10^{-15}$ Vs

$h = 1.6 \times 10^{-19} \times 4.12 \times 10^{-15} = 6.594 \times 10^{-34}$ Js

Value of plank's constant 'h' from above experiment is equal to 6.594×10^{-34} Js.

7. A 100W sodium lamp radiates energy uniformly in all directions. The lamp is located at the centre of a large sphere that absorbs all the sodium light which is incident on it. The wavelength of the sodium light is 589 nm. (a) What is the energy per photon associated with the sodium light? (b) At what rate are the photons delivered to the sphere?

7. Power of the sodium lamp, P = 100 W

Wavelength of the emitted sodium light, λ = 589 nm = 589 nm = 589×10^{-9} m

Planck's constant, h = 6.626×10^{-34} Js

Speed of light, c = 3×10^{8} m/s

(a) The energy per photon associated with the sodium light is given as:

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{589 \times 10^{-9}} = 3.37 \times 10^{-19}\,J$$

$$= \frac{3.37 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.11\,eV$$

(b) Number of photons delivered to the sphere = n

The equation for power can be written as: P = nE

$$\Rightarrow n = \frac{P}{E} = \frac{100}{3.37 \times 10^{-19}} = 2.96 \times 10^{20} \text{ photons / s}$$

Therefore, every second, 2.96×10^{20} photons are delivered to the sphere.

8. The threshold frequency for a certain metal is 3.3×10^{14} Hz. If the light of frequency 8.2×10^{14} Hz is incident on the metal, predict the cutoff voltage for the photoelectric emission.

8. Photoelectric equation

$eV_0 = hv - \acute{\O}_0$ .... equation (1)

Where e = Charge on an electron = 1.6×10^{-19} C

V_0 = Stopping Potential

h = plank's constant

v = frequency of photon

$\acute{\O}_0$ = Work function of the metal

Threshold frequency is the minimum frequency at which photoemission starts taking place from a metal surface. It is represented by v_0.

At threshold frequency, stopping potential is zero and energy of the emitted electron is also zero. Energy provided by the incident photon is just sufficient to eject out electrons from the metal surface.

$\acute{\O}_0 = hv_0$ equ.02

V_0 = Threshold frequency

Putting value of $\acute{\O}_0$ from equ.02 to equ.01

$eV_0 = hv - hv_0$

$\therefore V_0 = h(v - v_0)/e = 6.63 \times 10^{-34} \times (8.2 - 3.3) \times 10^{14}/1.6 \times 10^{-19}$

$= 2.03$ V

For photo emission cut of potential is equal to 2.03 V.

9. The work function for a certain metal is 4.2 eV. Will this metal give photoelectric emission for incident radiation of wavelength 330 nm?

9. No

Work function of the metal, $\Phi_0 = 4.2$ eV

Charge on an electron, $e = 1.6 \times 10^{-19}$ C

Planck's constant, $h = 6.626 \times 10^{-34}$ Js

Wavelength of the incident radiation, $\lambda = 330$ nm $= 330 \times 10^{-9}$ m

Speed of light, $c = 3 \times 10^8$ m/s

The energy of the incident photon is given as:

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{330 \times 10^{-9}} = 6.0 \times 10^{-19}\,\text{J}$$

$$= \frac{6.0 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.76\,\text{eV}$$

It can be observed that the energy of the incident radiation is less than the work function of the metal. Hence, no photoelectric emission will take place.

10. Light of frequency 7.21×10^{14} Hz is incident on a metal surface. Electrons with a maximum speed of 6.0×10^5 m/s are ejected from the surface. What is the threshold frequency for photoemission of electrons?

10. Photoelectric equation

$K.\,E_{max} = hv - hv_0$ ....equ. 01

Where $K.E_{max}$ = maximum kinetic energy of the photoelectron

h = plank's constant

v = frequency of photon

v_0 = threshold frequency

K. $E_{max} = ½ mv^2_{max}$ ....equ. 02

Where 'm' = mass of the electron = 9.1×10^{-31} kg

V_{max} = maximum speed of photoelectrons

From equ.01 and equ.02

$½ mv^2_{max} = hv - hv_0$

$\therefore v_0 = v - ½ mv^2_{max} / h$

$$\Rightarrow v_0 = 7.21\times10^{14}\,\text{Hz} - 9.1\times10^{-31}\,\text{Kg}\times\frac{\left(6.1\times10^{5}\right)^2}{6.63\,10^{-34}\,\text{J}-\text{sec}\times2}$$

$\Rightarrow v_0 = 7.21 \times 10^{14} - 2.47 \times 10^{14}$

$\Rightarrow v_0 = 4.74 \times 10^{14}$ Hz

Threshold frequency for photoemission is equal to 4.74×10^{14} Hz.

11. Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the emitter, the stopping (cut-off) potential of photoelectrons is 0.38 V. Find the work function of the material from which the emitter is made.

11. Wavelength of light produced by the argon laser,

$\lambda = 488 \text{ nm} = 488 \times 10^{-9}$ m

Stopping potential of the photoelectrons,

$V_0 = 0.38$ V

We know that 1eV = 1.6×10^{-19} J, therefore

$$V_0 = \frac{0.38}{1.6\times10^{-19}}\,\text{eV}$$

Planck's constant, h = 6.6×10^{-34} Js

Charge on an electron, e = 1.6×10^{-19} C

Speed of light, c = 3 × 10 m/s

From Einstein's photoelectric effect, we have the relation involving the work function Φ_0 of the material of the emitter as:

$$eV_0 = \frac{hc}{\lambda} - \phi_0$$

$$\phi_0 = \frac{hc}{\lambda} - eV_0$$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}\times488\times10^{-9}} - \frac{1.6\times10^{-19}\times0.38}{1.6\times10^{-19}}$$

$= 2.54 - 0.38 = 2.16$ eV

Therefore, the material with which the emitter is made has the work function of 2.16 eV.

12. Calculate the

(a) momentum, and

(b) de Broglie wavelength of the electrons accelerated through a potential difference of 56 V.

12. (a) When an charged particle having charge q is accelerated by a potential difference V volts it gain kinetic energy given by

K = Vq, which imparts it with velocity

We know kinetic energy of a Particle with mass m kg and velocity v is given by

$$K = \frac{1}{2}mv^2$$

Equating both equations

$$V_q = \frac{1}{2}mv^2$$

We get velocity of the particle as

$$v = \sqrt{\frac{2Vq}{m}}$$

But we know momentum is given by relation

P = mv

Where P is the momentum of particle having mass m and moving with velocity v

So we get

$$p = mv = m \times \sqrt{\frac{2Vq}{m}} = \sqrt{2mVq}$$

Here Potential difference

V = 56 volts

$q = 1.6 \times 10^{-19}$ C

$m = 9.1 \times 10^{-31}$ Kg

so momentum of the electron,

$$p = \sqrt{2 \times 1.6 \times 10^{-19} C \times 9.1 \times 10^{-31} Kg \times 56V}$$

$$\therefore p = 4.04 \times 10^{-24} Kgms^{-1}$$

So momentum of electron is 4.04×10^{-24} $Kgms^{-1}$

(b) Now we know de Broglie wavelength of a Particle is given by relation

$\lambda = h/mv$

Where λ is de Broglie wavelength of the particle having mass m and moving with velocity v

But we know momentum is given by relation

P = mv

Where P is the momentum of particle having mass m and moving with velocity v

So substituting we get de Broglie wavelength of Particle as

λ = h/P

$h = 6.6 \times 10^{-34}$ Js

$p = 4.04 \times 10^{-24}$ $Kgms^{-1}$

So putting the values in equation we get

$$\lambda = \frac{6.6\times10^{-34}\,Js}{4.04\times10^{-24}\,Kgms^{-1}} = 1.64\times10^{-10}\,m = 0.164\text{ nm}$$

So the de Broglie wavelength of electron is 0.164 nm.

Note: we could have also used the direct relation when an electron is accelerated through a potential difference of V volts its de Broglie wavelength is given by $\lambda = \frac{12.27}{\sqrt{V}} A^\circ$

$1A^o = 10^{-10}$ m

13. What is the

(a) momentum,

(b) speed, and

(c) de Broglie wavelength of an electron with kinetic energy of 120 eV.

13. Kinetic energy of the electron, $E_k = 120$ eV

Planck's constant, $h = 6.6 \times 10^{-34}$ Js

Mass of an electron, $m = 9.1 \times 10^{-31}$ kg

Charge on an electron, $e = 1.6 \times 10^{-19}$ C

(a) For the electron, we can write the relation for kinetic energy as: $E_k = \frac{1}{2}mv^2$

Where, v = Speed of the electron

$$\therefore v^2 = \sqrt{\frac{2eE_1}{m}}$$

$$= \sqrt{\frac{2\times1.6\times10^{-19}\times120}{9.1\times10^{-31}}}$$

$$= \sqrt{41.198\times10^{12}} = 6.496\times10^6\ m/s$$

Momentum of the electron, $p = mv = 9.1 \times 10^{-31} \times 6.496 \times 10^6$

$= 5.91 \times 10^{-24}$ kg ms^{-1}

Therefore, the momentum of the electron is 5.91×10^{-24} kg m s^{-1}.

(b) Speed of the electron, $v = 6.496 \times 10^{6}$ m/s

(c) De Broglie wavelength of an electron having a momentum p, is given as:

$$\lambda = \frac{h}{p}$$

$$= \frac{6.6\times10^{-34}}{5.91\times10^{-24}} = 1.116\times10^{-10}\,\text{m}$$

= 0.112 nm

= 0.112 nm

Therefore, the de Broglie wavelength of the electron is 0.112 nm.

14. The wavelength of light from the spectral emission line of sodium is 589 nm. Find the kinetic energy at which

(a) an electron, and

(b) a neutron, would have the same de Broglie wavelength.

14. Now we know de Broglie wavelength of a Particle is given by relation

$\lambda = h/mv$

Where λ is de Broglie wavelength of the particle having mass m and moving with velocity v.

But we know momentum is given by relation

$p = mv$

So, substituting we get de Broglie wavelength of Particle as

$\lambda = h/p$

Or we can say momentum of particle is

$p = h/\lambda$

We know kinetic energy of a particle is given by the relation

$$K = \frac{1}{2}mv^2$$

Where K is the kinetic energy of a particle of mass m moving with velocity v

Now multiplying L.H.S. and R.H.S. of the equation by m we get

$$mK = \frac{1}{2}m^2v^2$$

$$\text{or } mK = \frac{1}{2}(mv)^2$$

But we know momentum is given by relation

P = mv

So, substituting we get

$$mK = \frac{1}{2}p^2$$

$$\text{or } K = \frac{p^2}{2m}$$

we get kinetic energy of particle K in terms of momentum P and mass of particle m

so putting P = h/λ

we get kinetic energy of particle as

$$K = \frac{h^2}{2m\lambda^2}$$

Where h is the Planck's constant, m is mass of particle having de Broglie wavelength λ.

(a) Now we are given an electron whose de Broglie Wavelength should be

$\lambda = 589 \text{ nm} = 589 \times 10^{-9}$ m

We know mass of electron is, $m = 9.1 \times 10^{-31}$ Kg

and value of Planck's constant, $h = 6.6 \times 10^{-34}$Js

putting values in the relation $K = \frac{h^2}{2m\lambda^2}$ to find the kinetic energy of electron K we get

$$K = \frac{\left(6.6\times10^{-34}\,\text{Js}\right)^2}{2\times9.1\times10^{-31}\,\text{Kg}\times\left(589\times10^{-9}\,\text{m}\right)^2} = \frac{43.56}{6313962.2}\times10^{-19}\,\text{J}$$

$= 6.9 \times 10^{-25}$ J

So kinetic energy of the electron is 6.9×10^{-25} J

Converting it to eV

$1 \text{ eV} = 1.6 \times 10^{-19}$J

i.e.

$1\text{J} = (1/1.6 \times 10^{-19})$ eV

So we get kinetic energy of electron as

$$K = \frac{6.9\times10^{-25}}{1.6\times10^{-19}}\text{eV} = 4.31\times10^{-6}\text{eV}$$

Or we get Kinetic energy of electron as 4.31 μeV.

(b) Now we are given an neutron whose de Broglie Wavelength should be

$\lambda = 589 \text{ nm} = 589 \times 10^{-9}$ m

We know mass of electron is

$m = 1.66 \times 10^{-27}$ Kg

and value of Planck's constant

$h = 6.6 \times 10^{-34} Js$

putting values in the relation $K = \frac{h^2}{2m\lambda^2}$ to find the kinetic energy of neutron K we get

$$K = \frac{\left(6.6 \times 10^{-34} Js\right)^2}{2 \times 1.66 \times 10^{-27} Kg \times \left(589 \times 10^{-9} m\right)^2} = \frac{43.56}{1151777.72} \times 10^{-23} J$$

So kinetic energy of the electron is $3.78 \times 10^{-28} J$

Converting it to eV

We know

$1 eV = 1.6 \times 10^{-19} J$

i.e.

$1J = (1/1.6 \times 10^{-19}) eV$

So we get kinetic energy of electron as

$$K = \frac{3.78 \times 10^{-28}}{1.6 \times 10^{-19}} eV = 2.36 \times 10^{-9} eV$$

Or we get Kinetic energy of electron as 2.36 neV.

15. What is the de Broglie wavelength of
(a) a bullet of mass 0.040 kg travelling at the speed of 1.0 km/s,
(b) a ball of mass 0.060 kg moving at a speed of 1.0 m/s, and
(c) a dust particle of mass 1.0×10^{-9} kg drifting with a speed of 2.2 m/s?

15. (a) Mass of the bullet, m = 0.040 kg

Speed of the bullet, v = 1.0 km/s = 1000 m/s

Planck's constant, $h = 6.6 \times 10^{-34}$ Js

De Broglie wavelength of the bullet is given by the relation:

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.6 \times 10^{-34}}{0.040 \times 1000} = 1.65 \times 10^{-35} m$$

(b) Mass of the ball, m = 0.060 kg

Speed of the ball, v = 1.0 m/s

De Broglie wavelength of the ball is given by the relation:

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.6 \times 10^{-34}}{0.060 \times 1} = 1.1 \times 10^{-32} m$$

(c) Mass of the dust particle, $m = 1 \times 10^{-9}$ kg

Speed of the dust particle, $v = 2.2$ m/s

De Broglie wavelength of the dust particle is given by the relation:

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.6\times10^{-34}}{2.2\times1\times10^{-9}} = 3.0\times10^{-25}\,\text{m}$$

16. An electron and a photon each have a wavelength of 1.00 nm. Find

(a) their momenta,

(b) the energy of the photon, and

(c) the kinetic energy of electron.

16. (a) In order to find momentum, we know de Broglie wavelength for any particle is given by

$\lambda = h/P$

Or $P = h/\lambda$

Where h is Planck's constant

$h = 6.63 \times 10^{-34}$ Js

and P is the momentum of particle

here we are given the de Broglie wavelength

$\lambda = 1.00\text{ nm} = 1.00 \times 10^{-9}$ m

Note: We can see that momentum only depends on de Broglie wavelength and Planck's Constant which is same for both electron or Photon(Light Particle) so the momentum for both will also be same.

Putting values we get momentum

$$P = \frac{6.63\times10^{-34}\,\text{Js}}{1.00\times10^{-9}\,\text{m}} = 6.63\times10^{-25}\,\text{Kgms}^{-1}$$

So momentum of electron and photon is 6.63×10^{-25} Kgms^{-1}

(b) we know energy of an photon is given by the relation

$E = hc/\lambda$

Where E is the energy of Photon having wavelength λ and speed c, h is Planck's constant.

$h = 6.63 \times 10^{-34}$ Js

Speed of light or photon in free space

$c = 3 \times 10^{8}$ m/s

And we are given wavelength

$\lambda = 1.00 \text{ nm} = 1.00 \times 10^{-9}$ m

Putting these values we get

$$E = \frac{6.63\times10^{-34}\,\text{Js}\times3\times10^{8}\,\text{ms}^{-1}}{1.00\times10^{-9}\,\text{m}} = 19.89\times10^{-17}\,\text{J}$$

Converting it to eV

We know

$1 \text{ eV} = 1.6 \times 10^{-19}$J

i.e.

$1\text{J} = (1/1.6 \times 10^{-19})$ eV

So, we get energy of Photon as

$$E = \frac{19.89\times10^{-17}}{1.6\times10^{-19}}\,\text{eV} = 1.24\times10^{3}\,\text{eV} = 1.24\,\text{KeV}$$

so we get energy of Photon as 1.24KeV.

(c) We know kinetic energy of a particle is given by the relation $K = \frac{1}{2}mv^2$

Where K is the kinetic energy of a particle of mass m moving with velocity v

Now multiplying L.H.S. and R.H.S. of the equation by m we get

$$mK = \frac{1}{2}m^2v^2$$

$$\text{or } mK = \frac{1}{2}(mv)^2$$

But we know momentum is given by relation

P = mv

Where m is mass of Particle moving with velocity v

So, substituting we get

$$mK = \frac{1}{2}p^2$$

or we get kinetic energy of particle K in terms of momentum P and mass of particle m as

$$K = \frac{p^2}{2m}$$

The particle is electron and mass of electron is

$m = 9.1 \times 10^{-31}$ Kg

Here the momentum of electron

$P = 6.63 \times 10^{-25}$ Kgms^{-1}

Putting these values in above equation we have

$$K = \frac{\left(6.63\times10^{-25}\,\text{Kgms}^{-1}\right)^2}{2\times9.1\times10^{-31}\,\text{Kg}} = 2.415\times10^{-19}\,\text{J}$$

Converting it to eV

We know

1 eV = 1.6×10^{-19} J

i.e.

1J = $(1/1.6 \times 10^{-19})$eV

So, we get Kinetic energy of electron as

$$E = \frac{2.415\times10^{-19}}{1.6\times10^{-19}}\,\text{eV} = 1.51\,\text{eV}$$

so we get Kinetic energy of electron as 1.51eV.

17. (a) For what kinetic energy of a neutron will the associated de Broglie wavelength be 1.40×10^{-10} m?

(b) Also find the de Broglie wavelength of a neutron, in thermal equilibrium with matter, having an average kinetic energy of (3/2) kT at 300 K.

17. (a) De Broglie wavelength of the neutron, $\lambda = 1.40 \times 10^{-10}$ m

Mass of a neutron, $m_n = 1.66 \times 10^{-27}$ kg

Planck's constant, $h = 6.6 \times 10^{-34}$ Js

Kinetic energy (K) and velocity (v) are related as:

$$K = \frac{1}{2} m_n v^2 \qquad \ldots(1)$$

De Broglie wavelength (λ) and velocity (v) are related as:

$$\lambda = \frac{h}{m_n v} \qquad \ldots(2)$$

Using equation (2) in equation (1), we get:

$$K = \frac{1}{2}\frac{m_n h^2}{\lambda^2 m_n^2} = \frac{h^2}{2\lambda^2 m_n}$$

$$= \frac{\left(6.63\times10^{-34}\right)^2}{2\times\left(1.40\times10^{-10}\right)^2\times1.66\times10^{-27}} = 6.75\times10^{-21}\,\text{J}$$

$$= \frac{6.75\times10^{-21}}{1.6\times10^{-19}} = 4.219\times10^{-2}\,\text{eV}$$

Hence, the kinetic energy of the neutron is 6.75×10^{-21} J or 4.219×10^{-2} eV.

(b) Temperature of the neutron, T = 300 K

Boltzmann constant, $k = 1.38 \times 10^{-23}$ kg $m^2s^{-2}K^{-1}$ Average kinetic energy of the neutron:

$$K' = \frac{3}{2}kT$$

$$= \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 = 6.21 \times 10^{-21} J$$

The relation for the de Broglie wavelength is given as: $\lambda' = \frac{h}{\sqrt{2K'm_n}}$

Where, $m_n = 1.66 \times 10^{-27}$ kg, $h = 6.6 \times 10^{-34}$ Js and $K' = 6.75 \times 10^{-21}$ J. Therefore,

$$\lambda' = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 6.21 \times 10^{-21} \times 1.66 \times 10^{-27}}}$$

$= 1.46 \times 10^{-10}$ m = 0.146 nm

Therefore, the de Broglie wavelength of the neutron is 0.146 nm.

18. Show that the wavelength of electromagnetic radiation is equal to the de Broglie wavelength of its quantum (photon).

18. We know the relation between energy and momentum P of a electromagnetic radiation having velocity C and energy E is

P = E/C

and we know energy of electromagnetic radiation is given by relation

E = hV

Where E is the energy of Photon, v is the frequency of electromagnetic radiation and h is Planck's constant

So putting value of E in equation P = E/c

we get

P = hv/c

We know relation between frequency wavelength and velocity of electromagnetic radiation as

c = vλ

Where v is the frequency and λ is wavelength of electromagnetic radiation having velocity c

So re arranging we get

V = c/λ

Putting the value of v in equation P = hv/c

We get

P = hc/λc = h/λ

Or we can say the wavelength of electromagnetic radiation is given as

λ = h/P

Where h is Planck's constant and P is momentum of electromagnetic radiation

Now we know de Broglie wavelength of quantum (photon) can be given by relation

λ = h/mv

Where λ is de Broglie wavelength of a photon having mass m and moving with velocity v (here the velocity of photon v = C)

But we know momentum is given by relation

P = mv.

So, substituting we get de Broglie wavelength of quantum (photon) as

λ = h/P

This is same as wavelength of electromagnetic radiation.

19. What is the de Broglie wavelength of a nitrogen molecule in air at 300 K? Assume that the molecule is moving with the root mean square speed of molecules at this temperature. (Atomic mass of nitrogen = 14.0076 u)

19. Temperature of the nitrogen molecule, T = 300 K

Atomic mass of nitrogen = 14.0076 u

Hence, mass of the nitrogen molecule, m = 2 × 14.0076 = 28.0152 u

But 1 u = 1.66×10^{-27} kg

But 1 u = 1.66×10^{-27} kg

∴ m = $28.0152 \times 1.66 \times 10^{-27}$ kg

Planck's constant, k = 1.38×10^{-23} J K^{-1}

We have the expression that relates mean kinetic energy $\left(\frac{3}{2}kT\right)$ of the nitrogen molecule with the root mean square speed (v_{rms}) as:

$$\frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$$

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

Hence, the de Broglie wavelength of the nitrogen molecule is given as:

$$\lambda = \frac{h}{mv_{rms}} = \frac{h}{\sqrt{3mkT}}$$

$$= \frac{6.63\times10^{-34}}{\sqrt{3\times280152\times1.66\times10^{-27}\times1.38\times10^{-23}\times300}}$$

$= 0.028 \times 10^{-9}$ m

$= 0.028$ nm

Therefore, the de Broglie wavelength of the nitrogen molecule is 0.028 nm.