NCERT Solutions for Class 10 Maths Chapter 12 Area Related to Circles Ex. 12.3

Unless stated otherwise, use $\pi = \frac{22}{7}$.

1. Find the area of the shaded region in Fig. 12.19, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle

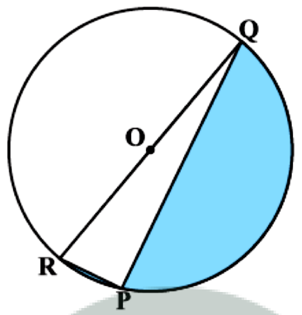

Fig. 12.19

Solution: Here, OR is the hypotenuse of ΔPQR, as lines from two ends of diameter always make a right angle when they meet at the circumference of that circle.

Hence,

$OR^2 = PQ^2 + PR^2$

$OR^2 = 24^2 + 7^2$

$OR^2 = 576 + 49$

OR = 25 cm

Or, radius = 12.5cm

Area of triangle = ½ * base *height

= ½ * 24 *7

Area of semicircle = $\frac{1}{2} \times \pi r^2$

$= \frac{1}{2} \times \pi \times 12.5^2$

= 245.3125 sq cm

Hence,

Area of shaded region = 245.3125 – 84

= 161.3125 sq cm

2. Find the area of the shaded region in Fig. 12.20, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and ∠ AOC = 40°

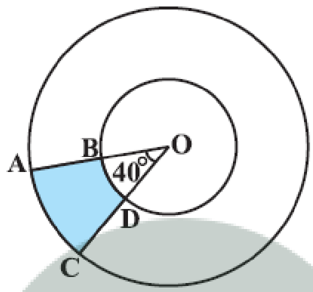

Fig. 12.20

Solution: Area of the shaded region = Area of Bigger sector –area of Smaller Sector

If R and r are the two radii of Bigger and Smaller circles respectively, then:

Area of shaded region = $\frac{40°}{360°}\pi(R^2 - r^2)$

$= \frac{1}{9} \times \frac{22}{7}(14^2 - 7^2)$

= 51.33 sq cm

3. Find the area of the shaded region in Fig. 12.21, if ABCD is a square of side 14 cm and APD and BPC are semicircles

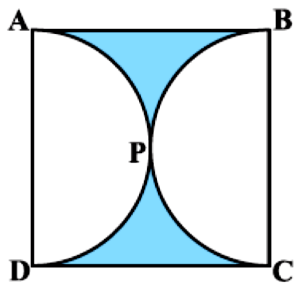

Fig. 12.21

Solution: It can be observed from the figure that the radius of each semi-circle is 7 cm

Area of each semi-circle $=\frac{1}{2}\pi r^2$

$= \frac{1}{2} * \frac{22}{7} * (7)^2$

= 77 cm^2

Area of square ABCD = $(\text{Side})^2$

$= (14)^2$

= 196 cm^2

Area of the shaded region= Area of square ABCD - Area of semi-circle APD - Area of semi-circle BPC

= 196 - 77 - 77

= 196 - 154

= 42 cm^2

4. Find the area of the shaded region in Fig. 12.22, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre

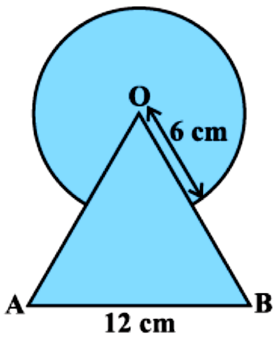

Fig. 12.22

Solution: We know that each interior angle of an equilateral triangle is of measure 60°

Area of sector OCDE = $\frac{60}{360}\pi r^2$

= $\frac{1}{6} * \frac{22}{7} * 6 * 6$

= $\frac{132}{7}$ cm^2

Area of triangle OAB = $\frac{\sqrt{3}}{4} * (12)^2$

= $\frac{\sqrt{3}*12*12}{4}$

= $36\sqrt{3}$ cm^2

Area of circle = πr^2

= $\frac{22}{7} * 6 * 6$

= $\frac{792}{7}$ cm^2

Area of shaded region = Area of ΔOAB + Area of circle - Area of sector OCDE

= $36\sqrt{3} + \frac{792}{7} - \frac{132}{7}$

= $(36\sqrt{3} + \frac{660}{7})$ cm^2

5. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut andalso a circle of diameter 2 cm is cut as shown in Fig. 12.23. Find the area of the remainingportion of the square

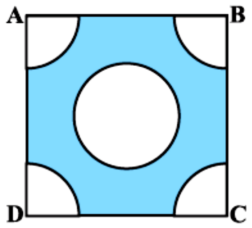

Fig. 12.23

Solution:Each quadrant is a sector of 90° in a circle of 1 cm radius

Area of each quadrant = $\frac{90}{360} * \pi r^2$

= $\frac{1}{4} * \frac{22}{7} * (1)^2$

= $\frac{22}{28}$ cm^2

Area of square = (Side)^2

= $(4)^2$

= 16 cm^2

Area of circle = $\pi r^2 = \pi\ (1)^2$

= $\frac{22}{7}$ cm^2

Area of the shaded region = Area of square - Area of circle - 4 × Area of quadrant

= $16 - \frac{22}{7} - 4 * \frac{22}{28}$

= $16 - \frac{22}{7} - \frac{22}{7}$

= $16 - \frac{44}{7}$

= $\frac{112-44}{7}$

$= \frac{68}{7}$ cm^2

6. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in Fig. 12.24. Find the area of the design (shaded region)

Fig. 12.24

Solution: Radius (r) of circle = 32 cm

AD is the median of triangle ABC

AO = $\frac{2}{3}$ AD = 32

AD = 48 cm

In triangle ABD,

$AB^2 = AD^2 + BD^2$

$AB^2 = (48)^2 + (\frac{AB}{2})^2$

$\frac{3AB*AB}{4} = (48)^2$

AB = $\frac{48*2}{\sqrt{3}}$

$= \frac{96}{\sqrt{3}}$

$= 32\sqrt{3}$ cm

Area of equilateral triangle, ABC = $\frac{\sqrt{3}}{4} * (32\sqrt{3})^2$

$= \frac{\sqrt{3}}{4} * 32 * 32 * 3$

$= 96 * 8 * \sqrt{3}$

$= 768\sqrt{3}$ cm^2

Area of design = Area of circle - Area of ΔABC

$= (\frac{22528}{7} - 768\sqrt{3})$ cm^2

7. In Fig. 12.25, ABCD is a square of side 14 cm. With centers A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region

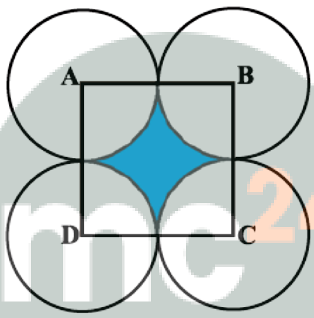

Fig. 12.25

Solution: Area of each of the 4 sectors is equal to each other and is a sector of 90° in a circle of 7 cm radius

Area of each sector $= \frac{90}{360} * \pi (7)^2$

$= \frac{1}{4} * \frac{22}{7} * 7 * 7$

$= \frac{77}{2}$ cm^2

Area of square ABCD = (Side)^2

$= (14)^2$

$= 196$ cm^2

Area of shaded portion = Area of square ABCD - 4 × Area of each sector

$= 196 - 4 * \frac{77}{2}$

$= 196 - 154$

$= 42\ cm^2$

Therefore, the area of shaded portion is $42\ cm^2$

8. Fig. 12.26 depicts a racing track whose left and right ends are semicircular.

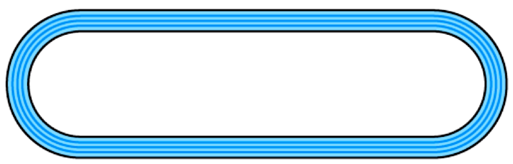

Fig. 12.26

The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:

(i) The distance around the track along its inner edge

(ii) The area of the track

Solution: (i) Distance around the track along its inner edge = AB + arc BEC + CD + arc DFA

$= 106 + \frac{1}{2} * 2\pi r + 106 + \frac{1}{2} * 2\pi r$

$= 212 + \frac{1}{2} * 2 * \frac{22}{7} * 30 + \frac{1}{2} * 2 * \frac{22}{7} * 30$

$= 212 + 2 * \frac{22}{7} * 30$

$= \frac{1484+1320}{7}$

$= \frac{2804}{7}$ m

(ii) Area of the track = (Area of GHIJ - Area of ABCD) + (Area of semi-circle HKI - Area of semi-circle BEC) + (Area of semi-circle GLJ - Area of semi-circle)

$= 106 * 80 - 106 * 60 + \frac{1}{2} * \frac{22}{7} * (40)^2 - \frac{1}{2} * \frac{22}{7} * (30)^2 + \frac{1}{2} * \frac{22}{7} * (40)^2 - \frac{1}{2} * \frac{22}{7} * (30)^2$

$= 106\ (80 - 60) + \frac{22}{7} * (40)^2 - \frac{22}{7} * (30)^2$

$= 106\ (20) + \frac{22}{7}\ [(40)^2 - (30)^2]$

$= 2120 + \frac{22}{7}\ (40 - 30)\ (40 + 30)$

$= 2120 + (\frac{22}{7})\ (10)\ (70)$

$= 2120 + 2200$

$= 4320\ m^2$

9. In Fig. 12.27, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region

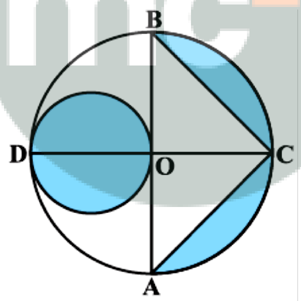

Fig. 12.27

Solution: Radius ($r1$) of larger circle = 7 cm

Radius ($r2$) of smaller circle = $\frac{7}{2}$ cm

Area of smaller circle = $\pi r2^2$

$= \frac{22}{7} * \frac{7}{2} * \frac{7}{2}$

$= \frac{77}{2}\ cm^2$

Area of semi-circle AECFB of larger circle = $\frac{1}{2} \pi r1^2$

$= \frac{1}{2} * \frac{22}{7} * (7)^2$

$= 77\ cm^2$

Area of triangle ABC $= \frac{1}{2} * AB * OC$

$= \frac{1}{2} * 14 * 7$

$= 49\ cm^2$

Area of the shaded region

= Area of smaller circle + Area of semi-circle AECFB - Area of ΔABC

$= \frac{77}{2} + 77 - 49$

$= 28 + \frac{77}{2}$

$= 28 + 38.5$

$= 66.5\ cm^2$

10. The area of an equilateral triangle ABC is 17320.5cm2. With each vertex of the triangle as centre, acircle is drawn with radius equal to half the length of the side of the triangle (see Fig. 12.28). Find the area of the shaded region. (Use π = 3.14 and $\sqrt{3}$ = 1.73205)

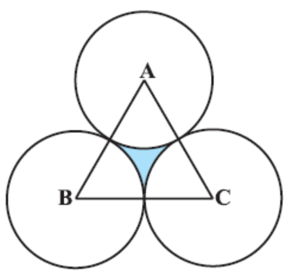

Fig. 12.28

Solution:Let the side of the equilateral triangle be *a*

Area of equilateral triangle $= 17320.5\ cm^2$

$\frac{\sqrt{3}}{4}$ $(a)^2$ = 17320.5

$\frac{1.73205}{4}$ $(a)^2$ = 17320.5

a^2 = 4 * 10000

a = 200 cm

Each sector is of measure 60°

Area of sector ADEF = $\frac{60}{360} * \pi * r^2$

= $\frac{1}{6} * \pi * (100)^2$

= $\frac{3.14*10000}{6}$

= $\frac{15700}{3}$ cm^2

Area of shaded region = Area of equilateral triangle - 3 × Area of each sector

= 17320.5 – 3 * $\frac{15700}{3}$

= 17320.5 – 15700

= 1620.5 cm^2

11. On a square handkerchief, nine circular designs each of radius 7 cm are made (see Fig. 12.29). Find the area of the remaining portion of the handkerchief

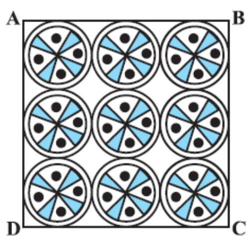

Fig. 12.29

Solution: From the figure, it can be observed that the side of the square is 42 cm

Area of square = $(\text{Side})^2$

= $(42)^2$

= 1764 cm^2

Area of each circle = πr^2

= $\frac{22}{7} * (7)^2$

= 154 cm^2

Area of 9 circles = 9 × 154

= 1386 cm^2

Area of the remaining portion of the handkerchief = 1764 - 1386

= 378 cm^2

12. In Fig. 12.30, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the

(i) Quadrant OACB

(ii) Shaded region

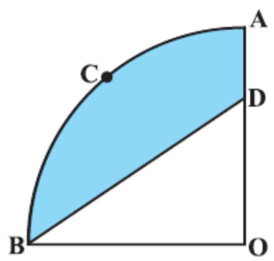

Fig. 12.30

Solution: (i) Since OACB is a quadrant, it will subtend 90° angle at O

Area of quadrant OACB = $\frac{90}{360} * \pi r^2$

= $\frac{1}{4} * \frac{22}{7} * (3.5)^2$

$= \frac{1}{4} * \frac{22}{7} * (\frac{7}{2})^2$

$= \frac{11*7*7}{2*7*2*2}$

$= \frac{77}{8}$ cm^2

(ii) Area of ΔOBD = $\frac{1}{2}$ * OB * OD

$= \frac{1}{2}$ * 3.5 * 2

$= \frac{1}{2} * \frac{7}{2} * 2$

$= \frac{7}{2}$ cm^2

Area of the shaded region = Area of quadrant OACB - Area of ΔOBD

$= \frac{77}{8} - \frac{7}{2}$

$= \frac{77-28}{8}$

$= \frac{49}{8}$ cm^2

13. In Fig. 12.31, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. (Use π = 3.14)

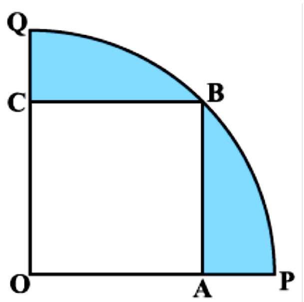

Fig. 12.31

Solution: In ΔOAB,

OB^2 = OA^2 + AB^2

$= (20)^2 + (20)^2$

$= 20\sqrt{2}$

Radius (r) of circle = $20\sqrt{2}$ cm

Area of quadrant OPBQ = $\frac{90}{360} * 3.14 * (20\sqrt{2})^2$

$= \frac{1}{4} * 3.14 * 800$

= 628 cm^2

Area of OABC = $(Side)^2$

$= (20)^2$

= 400 cm^2

Area of shaded region = Area of quadrant OPBQ - Area of OABC

= (628 - 400)

= 228 cm^2

14. AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see Fig. 12.32). If ∠ AOB = 30°, find the area of the shaded region

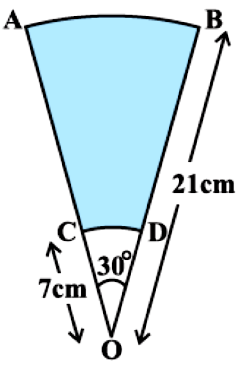

Fig. 12.32

Solution: Area of the shaded region = Area of sector OAEB - Area of sector OCFD

$= \frac{30}{360} * \pi * (21)^2 - \frac{30}{360} * \pi * (7)^2$

$= \frac{1}{12} * \pi [(21)^2 – (7)^2]$

$= \frac{1}{12} * \frac{22}{7} [(21 – 7) (21 + 7)]$

$= \frac{22*14*28}{12*7}$

$= \frac{308}{3}$ cm^2

15. In Fig. 12.33, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.

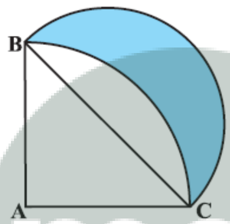

Fig. 12.33

Solution: Area of quadrant = $\frac{1}{4} \times \pi r^2$

$= \frac{1}{4} \times \pi \times 14^2$

=154 sq cm

Area of triangle= ½ *b *h

= ½ *14 *14

= 98 sq cm

Area of segment made by hypotenuse of triangle = 154 – 98

= 56 sq cm

Since other two sides of triangle are 14cm

Therefore,

Diameter of external semicircle = $14\sqrt{2}\ cm$

So, area of the semicircle = ½ πr^2

$$= \frac{1}{4} \times \pi \times \left(14\sqrt{2}\right)^2$$

= 154 sq cm

Area of the shaded region = 154 – 56

= 98 sq cm

16. Calculate the area of the designed region in Fig. 12.34 common between the two quadrants of circles of radius 8 cm each

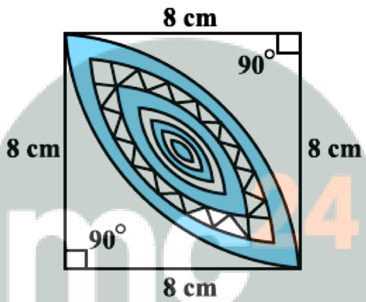

Fig. 12.34

Solution: Area of square = Side^2

= 8^2

= 64 sq cm

Area of the shaded region = Double the area of segment formed by diagonal of the square

Area of two quadrants = ½ πr^2

$$= \frac{1}{4} \times \pi \times (8)^2$$

= 100.48 sq cm

Now,

Area of square = area of two triangles formed by radii and the cord

Hence,

Area of the shaded region = 100.48 -48

= 36.48 sq cm