

EXERCISE 22.9

Question. 1

Solution:

From the question it is given that, $x^2 dy + y(x + y) dx = 0$

The given differential equation can be written in standard form as,

$$dy/dx = - (y(x + y))/x^2$$

So, it is a homogeneous equation,

Let us assume, $y = vx$ and $(dy/dx) = v + x (dv/dx)$

Then, $v + x (dv/dx) = - (vx(x + vx))/x^2$

$$v + x (dv/dx) = -v - v^2$$

Transposing,

$$x (dv/dx) = -v - v - v^2$$

$$x (dv/dx) = -2v - v^2$$

Now, taking like variables on same side,

$$dv/(2v + v^2) = -dx/x$$

Integrating on both side we get,

$$\int (1/(2v + v^2))dv = - \int dx/x$$

$$\int (1/(v^2 + 2v + 1 - 1))dv = - \int dx/x$$

We know that, $(a + b)^2 = a^2 + 2ab + b^2$

$$\int (1/((v + 1)^2 - 1^2))dv = - \int dx/x$$

$$\frac{1}{2} \log [(v + 1 - 1)/(v + 1 + 1)] = - \log x + \log c$$

$$\log [v/(v + 2)]^{1/2} = - \log (c/x)$$

$$v/(v + 2) = c^2/x^2$$

$$((y/x)/((y/x) + 2)) = c^2/x^2$$

By simplification we get,

$$y/(y + 2x) = c^2/x^2$$

$$yx^2 = (y + 2x) c^2$$

Question. 2

Solution:

From the question it is given that, $(dy/dx) = (y - x)/(y + x)$

The given differential equation is a homogeneous equation,

Let us assume, $y = vx$ and $(dy/dx) = v + x (dv/dx)$

$$v + x (dv/dx) = (vx - x)/(vx + x)$$

$$\text{Then, } v + x (dv/dx) = (v - 1)/(v + 1)$$

$$x (dv/dx) = ((v - 1)/(v + 1)) - v$$

$$x \left(\frac{dv}{dx} \right) = (v - 1 - v^2 - v)/(v + 1)$$

On dividing we get,

$$x \left(\frac{dv}{dx} \right) = - (1 + v^2)/(v + 1)$$

Now, taking like variables on same side,

$$((v + 1)/(v^2 + 1)) = - dx/x$$

Integrating on both side we get,

$$\int ((v + 1)/(v^2 + 1)) dv = - \int dx/x$$

$$\int (v/(v^2 + 1)) dv + \int (1/(v^2 + 1)) dv = - \int dx/x$$

$$\frac{1}{2} \int (2v/(v^2 + 1)) dv + \int (1/(v^2 + 1)) dv = - \int dx/x$$

$$\frac{1}{2} \log [v^2 + 1] + \tan^{-1} v = - \log x + \log c$$

$$\text{Then, } \log [(y^2 + x^2)/x^2] + 2 \tan^{-1} (y/x) = 2 \log (c/x)$$

$$\log [x^2 + y^2] - 2 \log x + 2 \tan^{-1} (y/x) = 2 \log (c/x)$$

$$\log (x^2 + y^2) + 2 \tan^{-1} (y/x) = 2 \log c$$

$$\log (x^2 + y^2) + 2 \tan^{-1} (y/x) = k$$

Question. 3

Solution:

From the question it is given that, $(dy/dx) = (y^2 - x^2)/(2xy)$

The given differential equation is a homogeneous equation,

Let us assume, $y = vx$ and $(dy/dx) = v + x (dv/dx)$

$$v + x \left(\frac{dv}{dx} \right) = (v^2x^2 - x^2)/(2xvx)$$

$$\text{Then, } x \left(\frac{dv}{dx} \right) = ((v^2 - 1)/(2v)) - (v/1)$$

$$x \left(\frac{dv}{dx} \right) = (v^2 - 1 - 2v^2)/(2v)$$

$$x \left(\frac{dv}{dx} \right) = (-1 - v^2)/2v$$

Now, taking like variables on same side,

$$((2v)/(v^2 + 1)) dv = - dx/x$$

Integrating on both side we get,

$$\int ((2v)/(v^2 + 1)) dv = - \int dx/x$$

$$\log (1 + v^2) = - \log x + \log c$$

$$1 + v^2 = c/x$$

Now substitute the value of v ,

$$1 + y^2/x^2 = c/x$$

$$x^2 + y^2 = cx$$

Question. 4

Solution:

From the question it is given that, $x (dy/dx) = (x + y)/x$

The given differential equation can be written in standard form as,

$$dy/dx = (x + y)/x$$

So, it is a homogeneous equation,

Let us assume, $y = vx$ and $(dy/dx) = v + x (dv/dx)$

$$dy/dx = v + x (dv/dx)$$

$$\text{Then, } v + x (dv/dx) = (x + vx)/x$$

$$v + x (dv/dx) = (x/x) + (vx/x)$$

$$v + x (dv/dx) = 1 + v$$

Now, taking like variables on same side,

$$dv = dx/x$$

Integrating on both side we get,

$$\int dv = \int dx/x$$

$$v = \log x + c$$

Now substitute the value of v ,

$$y/x = \log x + c$$

$$y = x \log x + cx$$

Question. 5

Solution:

From the question it is given that, $(x^2 - y^2)dx - 2xy dy = 0$

The given differential equation can be written in standard form as,

$$dy/dx = (x^2 - y^2)/2xy$$

So, it is a homogeneous equation,

Let us assume, $y = vx$ and $(dy/dx) = v + x (dv/dx)$

$$v + x (dv/dx) = (x^2 - v^2x^2)/(2xvx)$$

$$\text{Then, } x (dv/dx) = ((1 - v^2)/(2v)) - (v/1)$$

$$x (dv/dx) = (1 - v^2 - 2v^2)/(2v)$$

$$x(dv/dx) = (1 - 3v^2)/2v$$

Now, taking like variables on same side,

$$((2v)/(1 - 3v^2)) dv = dx/x$$

Integrating on both side we get,

$$\int ((2v)/(1 - 3v^2)) dv = - \int dx/x$$

$$1/3 \int ((-6v)/(1 - 3v^2)) dv = - \int dx/x$$

$$\log (1 - 3v^2) = -3 \log x + \log c$$

$$1 - 3v^2 = c/x^3$$

Now substitute the value of v ,

$$x^3 (1 - (3y^2/x^2)) = c/$$

$$x^3 (x^2 - 3y^2)/x^2 = c$$

$$x(x^2 - 3y^2) = c$$



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