Exercise 7(C)

1. Evaluate:

(i) $9^{5/2} – 3 \times 8^0 – (1/81)^{-1/2}$

(ii) $(64)^{2/3} - \sqrt[3]{125} – 1/2^{-5} + (27)^{-2/3} \times (25/9)^{-1/2}$

(iii) $[(-2/3)^{-2}]^3 \times (1/3)^{-4} \times 3^{-1} \times 1/6$

Solution:

(i) We have, $9^{5/2} – 3 \times 8^0 – (1/81)^{-1/2}$

$= (3^2)^{5/2} – 3 \times 1 – (1/9^2)^{-1/2}$

$= 3^{2 \times 5/2} – 3 – (1/9)^{-2 \times ½}$

$= 3^5 – 3 – (1/9)^{-1}$

$= (3 \times 3 \times 3 \times 3 \times 3) – 3 – (9^{-1})^{-1}$

$= 243 – 3 – 9$

$= 231$

(ii) We have, $(64)^{2/3} - \sqrt[3]{125} – 1/2^{-5} + (27)^{-2/3} \times (25/9)^{-1/2}$

$= (4 \times 4 \times 4)^{2/3} - (5 \times 5 \times 5)^{1/3} – 1/2^{-5} + (3 \times 3 \times 3)^{-2/3} \times (5 \times 5/3 \times 3)^{-1/2}$

$= (4^3)^{2/3} – (5^3)^{1/3} – (2^{-1})^{-5} + (3^3)^{-2/3} \times (5^2/3^2)^{-1/2}$

$= (4)^{3 \times 2/3} – (5)^{3 \times 1/3} – (2)^{-1 \times -5} + (3)^{3 \times -2/3} \times (5/3)^{2 \times -1/2}$

$= (4)^2 – 5 – 2^5 + 3^{-2} \times (5/3)^{-1}$

$= 16 – 5 – 32 + 1/9 \times 3/5$

$= -21 + 1/15$

$= (-315 + 1)/15$

$= -314/15$

(iii) We have, $[(-2/3)^{-2}]^3 \times (1/3)^{-4} \times 3^{-1} \times 1/6$

$= (-2/3)^{-6} \times (3^{-1})^{-4} \times 3^{-1} \times ½ \times 1/3$

$= (-3/2)^6 \times 3^4 \times 3^{-1} \times 2^{-1} \times 3^{-1}$

$= (-1)^6 \times (3^6 \times 3^4 \times 3^{-1} \times 3^{-1}) \times (2^{-6} \times 2^{-1})$

$= 1 \times 3^{6+4-1-1} \times 2^{-6-1}$

$= 3^8 \times 2^{-7}$

$= 3^8/2^7$

2. Simplify: $\dfrac{3 \times 9^{n+1} - 9 \times 3^{2n}}{3 \times 3^{2n+3} - 9^{n+1}}$

Solution:

We have,

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$$\frac{3 \times 9^{n+1} - 9 \times 3^{2n}}{3 \times 3^{2n+3} - 9^{n+1}}$$

$$= \frac{3 \times (3^2)^{n+1} - 3^2 \times 3^{2n}}{3 \times 3^{2n+3} - (3^2)^{n+1}}$$

$$= \frac{3^{1+2n+2} - 3^{2+2n}}{3^{1+2n+3} - 3^{2n+2}}$$

$$= \frac{3^{3+2n} - 3^{2+2n}}{3^{4+2n} - 3^{2n+2}}$$

$$= \frac{3^{2n}(3^3 - 3^2)}{3^{2n}(3^4 - 3^2)}$$

= (27 – 9)/(81 - 9)
= 18/72
= 1/4

3. Solve: $3^{x-1} \times 5^{2y-3} = 225$.
Solution:

Given, $3^{x-1} \times 5^{2y-3} = 225$
$3^{x-1} \times 5^{2y-3} = 9 \times 25$
$3^{x-1} \times 5^{2y-3} = 3^2 \times 5^2$
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
x – 1 = 2 and 2y – 3 = 2
x = 2 + 1 and 2y = 2 + 3
x = 3 and 2y = 5
x = 3 and y = 5/2 = 2.5

4. If $[(a^{-1}b^2)/(a^2b^{-4})]^7 \div [(a^3b^{-5})/(a^{-2}b^3)] = a^x.b^y$, find x + y.
Solution:

We have,

$$\left(\frac{a^{-1}b^{2}}{a^{2}b^{-4}}\right)^{7} \div \left(\frac{a^{3}b^{-5}}{a^{-2}b^{3}}\right)^{-5} = a^{x} \cdot b^{y}$$

$$\Rightarrow \left(\frac{b^{6}}{a^{3}}\right)^{7} \div \left(\frac{a^{5}}{b^{8}}\right)^{-5} = a^{x} \cdot b^{y}$$

$$\Rightarrow \left(\frac{b^{6}}{a^{3}}\right)^{7} \div \left(\frac{b^{8}}{a^{5}}\right)^{5} = a^{x} \cdot b^{y}$$

$(b^{42}/a^{21}) \div (b^{40}/a^{25}) = a^{x}.\ b^{y}$
$(b^{42}/a^{21}) \times (a^{25}/b^{40}) = a^{x}.\ b^{y}$
$b^{42-40} \times a^{25-21} = a^{x}.\ b^{y}$
$a^{4} \times b^{2} = a^{x}.\ b^{y}$
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
$x = 4$ and $y = 2$
$x + y = 4 + 2 = 6$

5. If $3^{x+1} = 9^{x-3}$, find the value of 2^{1+x}.
Solution:

We have,
$3^{x+1} = 9^{x-3}$
$3^{x} \times 3^{1} = (3^{2})^{x-3}$
$3^{x} \times 3^{1} = (3)^{2x-6}$
$3^{x} = (3)^{2x-6}/3$
$3^{x} = (3)^{2x-6} \times 3^{-1}$
$3^{x} = (3)^{2x-6-1}$
$3^{x} = 3^{2x-7}$
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
$x = 2x - 7$
$x = 7$
Now,
$2^{1+x} = 2^{1+7} = 2^{8} = 256$

6. If $2^{x} = 4^{y} = 8^{z}$ and $1/2x + 1/4y + 1/8z = 4$, find the value of x.
Solution:

Given, $2^{x} = 4^{y} = 8^{z}$
$2^{x} = (2^{2})^{y} = (2^{3})^{z}$
$2^{x} = 2^{2y} = 2^{3z}$
On comparing the powers, we get
$x = 2y = 3z$
$y = x/2$ and $z = x/3$
Now,

1/2x + 1/4y + 1/8z = 4
1/2x + 1/4(x/2) + 1/8(x/3) = 4
1/2x + 2/4x + 3/8x = 4
1/2x + 1/2x + 3/8x = 4
(4 + 4 + 3)/8x = 4
11/8x = 4
4 × 8x = 11
32x = 11
x = 11/32

7. If $\frac{9^n \cdot 3^2 \cdot 3^n - (27)^n}{(3^m \cdot 2)^3} = 3^{-3}$.
Show that: m – n = 1
Solution:

We have,

$$\frac{9^n . 3^2 . 3^n - (27)^n}{(3^m . 2)^3} = 3^{-3}$$

$$\frac{3^{2n} . 3^2 . 3^n - (3)^{3n}}{3^{3m} . (2)^3} = \frac{1}{3^3}$$

$$\frac{3^{3n} . 3^2 - 3^{3n}}{3^{3m} . 2^3} = \frac{1}{3^3}$$

$$\frac{3^{3n}(3^2 - 1)}{3^{3m} \times 8} = \frac{1}{3^3}$$

$$\frac{3^{3n} \times 8}{3^{3m} \times 8} = \frac{1}{3^3}$$

$$\frac{1}{3^{3(m-n)}} = \frac{1}{3^{3\times1}}$$

On comparing the powers, we get
m – n = 1

8. Solve for x: $(13)^{\sqrt{x}} = 4^4 - 3^4 - 6$.
Solution:

We have,
$(13)^{\sqrt{x}} = 4^4 - 3^4 - 6$

$= 256 - 81 - 6$
$= 169$
$= 13^2$
$13^{\sqrt{x}} = 13^2$
On comparing the powers, we get
$\sqrt{x} = 2$
Squaring on both sides,
$x = 2^2$
$x = 4$

9. If $3^{4x} = (81)^{-1}$ and $(10)^{1/y} = 0.0001$, find the value of $2^{-x} \times 16^y$.
Solution:

We have, $3^{4x} = (81)^{-1}$ and $(10)^{1/y} = 0.0001$
$3^{4x} = (3^4)^{-1}$ and $(10)^{1/y} = 1/10000$
$3^{4x} = 3^{-4}$ and $10^{1/y} = 1/10^{-4}$
$3^{4x} = 3^{-4}$ and $10^{1/y} = 1/10^4$
$3^{4x} = 3^{-4}$ and $10^{1/y} = 10^{-4}$
On comparing the powers, we get
$4x = -4$ and $1/y = -4$
$x = -1$ and $y = -1/4$
Now, value of
$2^{-x} \times 16^y = 2^{-(-1)} \times 16^{-1/4}$
$= 2^1 \times (2^4)^{-1/4}$
$= 2 \times 2^{-1}$
$= 2/2$
$= 1$

10. Solve: $3(2^x + 1) - 2^{x+2} + 5 = 0$.
Solution:

We have, $3(2^x + 1) - 2^{x+2} + 5 = 0$
$(3 \times 2^x + 3) - (2^x \times 2^2) + 5 = 0$
$2^x(3 - 2^2) + 3 + 5 = 0$
$2^x(3 - 4) + 8 = 0$
$2^x(-1) + 8 = 0$
$-2^x + 8 = 0$
$2^x = 8$
$2^x = 2^3$
On comparing the powers, we get
$x = 3$

11. If $(a^m)^n = a^m .a^n$, find the value of: $m(n - 1) - (n - 1)$
Solution:

We have, $(a^m)^n = a^m .a^n$

$a^{mn} = a^{m+n}$
On comparing the powers, we get
mn = m + n … (i)
Now,
m(n - 1) – (n - 1) = mn – m – n + 1
= (m + n) – m – n + 1 … [Form (i)]
= 1

12. If m = $\sqrt[3]{15}$ and n = $\sqrt[3]{14}$, find the value of m – n – 1/(m² + mn + n²)
Solution:

We have,
m = $\sqrt[3]{15}$ and n = $\sqrt[3]{14}$
⇒ m = 15^3 and n = 14^3

$$\therefore\ m - n - \frac{1}{m^2 + mn + n^2} = \frac{(m^3 + m^2n + mn^2) - (m^2n + mn^2 + n^3) - 1}{m^2 + mn + n^2}$$
$$= \frac{m^3 + m^2n + mn^2 - m^2n - mn^2 - n^3 - 1}{m^2 + mn + n^2}$$
$$= \frac{m^3 - n^3 - 1}{m^2 + mn + n^2}$$
$$= \frac{15 - 14 - 1}{m^2 + mn + n^2}$$
$$= \frac{1 - 1}{m^2 + mn + n^2}$$
$$= 0$$

13. Evaluate: $\dfrac{2^n \times 6^{m+1} \times 10^{m-n} \times 15^{m+n-2}}{4^m \times 3^{2m+n} \times 25^{m-1}}$
Solution:

We have,

$$\frac{2^n \times 6^{m+1} \times 10^{m-n} \times 15^{m+n-2}}{4^m \times 3^{2m+n} \times 25^{m-1}}$$

$$= \frac{2^n \times 6^m \times 6 \times 10^m \times 10^{-n} \times 15^m \times 15^n \times 15^{-2}}{4^m \times (3^2)^m \times 3^n \times 25^m \times 25^{-1}}$$

$$= \frac{\left(2 \times \frac{1}{10} \times 15\right)^n \times (6 \times 10 \times 15)^m \times 6 \times \frac{1}{15^2}}{3^n \times \left(4 \times 3^2 \times 25\right)^m \times \frac{1}{25}}$$

$$= \frac{3^n \times 900^m \times \frac{6}{225}}{3^n \times 900^m \times \frac{1}{25}}$$

= 6/225 x 25/1

= 6/9

= 2/3

14. Evaluate:

$(x^q/x^r)^{1/qr} \times (x^r/x^p)^{1/rp} \times (x^p/x^q)^{1/pq}$

Solution:

We have,

$$\left(\frac{x^q}{x^r}\right)^{\frac{1}{qr}} \times \left(\frac{x^r}{x^p}\right)^{\frac{1}{rp}} \times \left(\frac{x^p}{x^q}\right)^{\frac{1}{pq}}$$

$$= \frac{x^{q \times \frac{1}{qr}}}{x^{r \times \frac{1}{qr}}} \times \frac{x^{r \times \frac{1}{rp}}}{x^{p \times \frac{1}{rp}}} \times \frac{x^{p \times \frac{1}{pq}}}{x^{q \times \frac{1}{pq}}}$$

$$= \frac{x^{\frac{1}{r}}}{x^{\frac{1}{q}}} \times \frac{x^{\frac{1}{p}}}{x^{\frac{1}{r}}} \times \frac{x^{\frac{1}{q}}}{x^{\frac{1}{p}}}$$

$$= 1$$

15. (i) Prove that: $a^{-1}/(a^{-1} + b^{-1}) + a^{-1}/(a^{-1} - b^{-1}) = 2b^2/(b^2 - a^2)$

Solution:

We have,

$$\frac{a^{-1}}{a^{-1}+b^{-1}}+\frac{a^{-1}}{a^{-1}-b^{-1}}=\frac{2}{b^2-a^2}$$

L.H.S. = $\frac{a^{-1}}{a^{-1}+b^{-1}}+\frac{a^{-1}}{a^{-1}-b^{-1}}$

= $\frac{\frac{1}{a}}{\frac{1}{a}+\frac{1}{b}}+\frac{\frac{1}{a}}{\frac{1}{a}-\frac{1}{b}}$

= $\frac{\frac{1}{a}}{\frac{b+a}{ab}}+\frac{\frac{1}{a}}{\frac{b-a}{ab}}$

= 1/a x ab/(b + a) + 1/a x ab/(b - a)
= b/(b + a) + b/(b - a)
= $(b^2 - ab + b^2 + ab)/(b^2 - a^2)$
= $2b^2/(b^2 - a^2)$
= R.H.S

(ii) Prove that: $(a + b + c)/(a^{-1}b^{-1} + b^{-1}c^{-1} + c^{-1}a^{-1}) = abc$
Solution:

Taking L.H.S., we have

L.H.S. = $\frac{a+b+c}{a^{-1}b^{-1}+b^{-1}c^{-1}+c^{-1}a^{-1}}$

= $\frac{a+b+c}{\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}}$

= $\frac{a+b+c}{\frac{c+a+b}{abc}}$

= $\frac{(a+b+c)(abc)}{a+b+c}$

= abc
= R.H.S

16. Evaluate: $4/(216)^{-2/3} + 1/(256)^{-3/4} + 2/(243)^{-1/5}$
Solution:

We have,

$$\frac{4}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}}$$

$$= \frac{4}{\left(6^3\right)^{-\frac{2}{3}}} + \frac{1}{\left(4^4\right)^{-\frac{3}{4}}} + \frac{2}{\left(3^5\right)^{-\frac{1}{5}}}$$

$= 4/6^{-2} + 1/4^{-3} + 2/3^{-1}$

$= 4 \times 6^2 + 4^3 + 2 \times 3^1$

$= 4 \times 36 + 64 + 2 \times 3$

$= 144 + 64 + 6$

$= 214$