NCERT Solutions for Class-XII Maths

Chapter-12.2

NCERT Chemistry Class 12

1. Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs Rs 60/kg and Food Q costs Rs 80/kg. Food P contains 3 units/kg of Vitamin A and 5 units / kg of Vitamin B while food Q contains 4 units/kg of Vitamin A and 2 units/kg of vitamin B. Determine the minimum cost of the mixture.

1. Let the mixture consists of x kg of food P and y kg of food Q.

$\therefore x \geq 0$ and $y \geq 0$

According to given situation in the question, the information can be tabulated as follows:

	Vitamin A (units / Kg)	**Vitamin B (units/kg)**	**Cost (Rs/kg)**
Food P	3	4	60
Food Q	5	2	80
Requirements (units/kg)	8	11	

The mixture must contain at least 8 units of vitamin A and 11 units of vitamin B.

$\therefore$ The constraints are:

$3x + 4y \geq 8$ and

$5x + 2y \geq 11$.

Where $x \geq 0$ and $y \geq 0$

Total cost of 'Z' of purchasing food is $Z = 6ox + 80y$

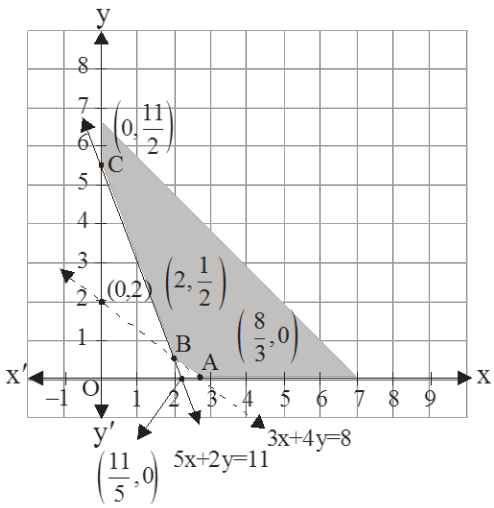
The mathematical formulation of the given problem is: minimize $Z = 6ox + 80y$…….1

Subject to constraints

$3x + 4y \geq 8$ and

$5x + 2y \geq 11$.

Where $x, y \geq 0$



The graphical representation shows the feasible region is unbounded.

The corner points of the feasible region are $A\left(\frac{8}{3},0\right)$, $B\left(2,\frac{1}{2}\right)$ *and* $C\left(0,\frac{11}{2}\right)$

The values of Z at these corner points are

Corner points	**Z= 60x + 80y**	
$A\left(\frac{8}{3},0\right)$	160	minimum
$B\left(2,\frac{1}{2}\right)$	160	minimum
$C\left(0,\frac{11}{2}\right)$	440	

As the feasible region is unbounded

∴ 160 may or may not be the minimum value of Z

But it can be seen that the feasible region has no common oint with 3x+4y <8

So the minimum cost of the mixture will be Rs160 at the line segment joining the points $A\left(\frac{8}{3},0\right)$ *and* $B\left(2,\frac{1}{2}\right)$.

2. One kind of cake requires 200g of flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes.

2. Let there be x cakes of first kind and y cakes of second kind. Therefore, x ≥ 0 and y ≥ 0

The given information can be complied in a table as follows.

	Flour (g)	Fat (g)
Cakes of first kind, x	200	25
Cakes of second kind, y	100	50
Availability	5000	1000

$\therefore 200x + 100y \leq 5000$

$\Rightarrow 2x + y \leq 50$

$25x + 50y \leq 1000$

$\Rightarrow x + 2y \leq 40$

Total numbers of cakes, z, that can be made are, $z = x + y$

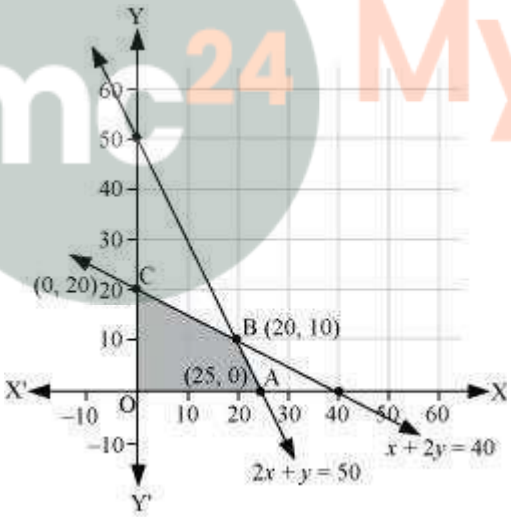
The mathematical formulation of the given problem is Maximize $z = x + y \ldots$ (1) subject to the constraints,

$2x + y \leq 50$

$x + 2y \leq 40$

$x, y \geq 0$

The feasible region determined by the system of constraints is as follows.



The corner points are A (25, 0), B (20, 10), O (0, 0), and C (0, 20).

The values of Z at these corner points are as follows.

Corner point	Z = x + y	
A (25, 0)	25	
B (20, 10)	30	→ Maximum
C (0, 20)	20	
O (0, 0)	0	

Thus, the maximum numbers of cakes that can be made are 30 (20 of one kind and 10 of the other kind).

3. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hour of machine time and 1 hour of craftman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time.

(i) What number of rackets and bats must be made if the factory is to work at full capacity?

(ii) If the profit on a racket and on a bat is Rs 20 and Rs 10 respectively, find the maximum profit of the factory when it works at full capacity.

3. Let the number of rackets and the number of bats to be made be x and y respectively.

The machine time available is not more than 42 hours.

$\therefore 1.5x + 3y \leq 42$

The craftsman`s time is not available for more than 24 hours

$\therefore 3x + y \leq 24$

The factory is to work at full capacity

$\therefore 1.5x + 3y = 42$

$3x + y = 24$

Where x and $y \geq 0$

Solving the two equations we get $x = 4$ and $y = 12$

Thus 4 rackets and 12 bats must be made

(i) The given information can be complied in the tables form as follows

	Tennis racket	**Cricket bat**	**Availability**
Machine time (h)	1.5	3	42
Craft`s man time(h)	3	1	24

The profit on a rackets is Rs 20 and on bat is Rs 10

Maximize , $Z = 20x + 10y$ ……1

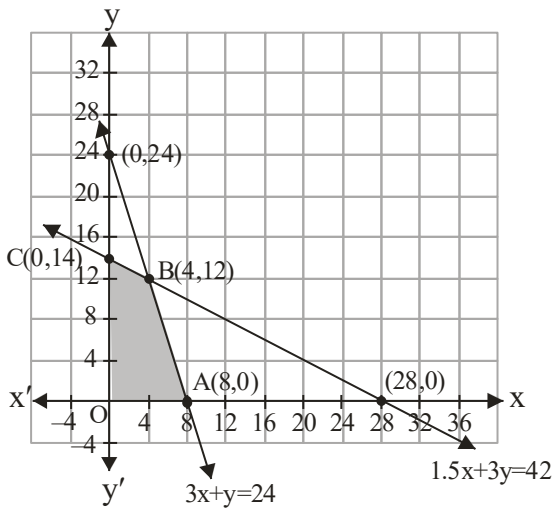
Subject to constraints

$1.5x + 3y = 42$

$3x + y = 24$

$x, y \geq 0$

the feasible region determined by the system of constraints is:



The corner points are A(8,0) , B(4, 12) , C(0, 14) and O(0,0)

The values of Z at these corner points are as follows:

Corner point	**Z= 20x+ 10y**	
A(8,0)	160	
B(4, 12)	200	maximum
C(0, 14)	140	
O(0,0)	0	

Thus, the maximum profit of the factory is when it works to its full capacity is Rs200.

4. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs17.50 per package on nuts and Rs 7.00 per package on bolts. How many packages of each should be produced each day so as to maximise his profit, if he operates his machines for at the most 12 hours a day?

4. Let the manufacturer produce x packages of nuts and y packages of bolts. Therefore, $x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows.

	Nuts	**Bolts**	**Availability**
Machine A (h)	1	3	12
Machine B (h)	3	1	12

The profit on a package of nuts is Rs 17.50 and on a package of bolts is Rs 7.

Therefore, the constraints are

$x + 3y \leq 12$

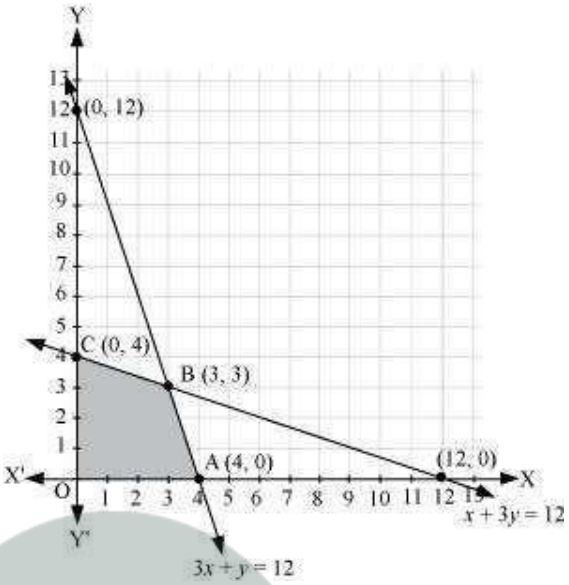
$3x + y \leq 12$

Total profit, $z = 17.5x + 7y$

The mathematical formulation of the given problem

is Maximise $Z = 17.5x + 7y \ldots (1)$ subject to the constraints, $x + 3y \le 12 \ldots (2)$ $3x + y \le 12 \ldots (3)$ x, $y \ge 0 \ldots (4)$

The feasible region determined by the system of constraints is as follows.



The corner points are A (4, 0), B (3, 3), and C (0, 4).

The values of Z at these corner points are as follows.

Corner point	Z = 17.5x + 7y	
O (0, 0)	0	
A (4, 0)	70	
B (3, 3)	73.5	→ Maximum
C (0, 4)	28	

The maximum value of Z is Rs 73.50 at (3, 3).

The maximum value of Z is Rs 73.50 at (3, 3).

Thus, 3 packages of nuts and 3 packages of bolts should be produced each day to get the maximum profit of Rs 73.50.

5. A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of Rs 7 and screws B at a profit of Rs 10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximise his profit? Determine the maximum profit.

5. Let the factory manufacture x screws of type A and y screws of type B on each day.

$\therefore$ x and y ≥ 0

The tabular form of the given data in the question is

	Screw A	**Screw B**	**Availability**
Automatic Machine(min)	4	6	4 × 60= 240
Hand operated machine(min)	6	3	4 × 60 = 240

The profit on a package of screws A is Rs 7 and on package of screws B is Rs 10.

$\therefore$ the constraints are

$4x+ 6y \leq 240$

And $6x + 3y \leq 240$ and $x, y \geq 0$

And total profit is $Z = 7x+ 10y$

The mathematical formulation of the data is

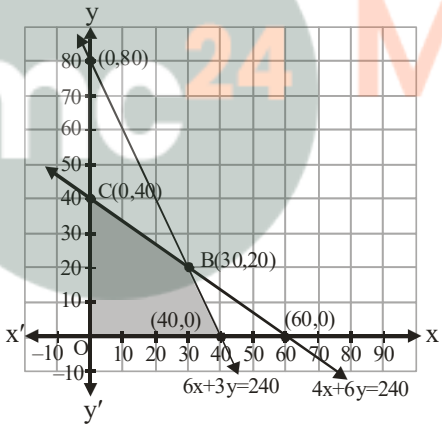
Maximizing $Z = 7x+ 10y$

Subject to constraints

$4x+ 6y \leq 240$

And $6x + 3y \leq 240$ and $x, y \geq 0$

The feasible reason is determined by the system of constraints is:



The corner points are A(40, 0) , B(30 , 20) , C (0 , 40)

The value of Z at these points is

Corner point	**Z = 7z + 10y**	
A (40, 0)	280	
B (30, 20)	410	Maximum
C (0 ,40)	400	

The maximum value of Z is at point B(30, 20)

Therefore factory should produce 30 packages of screw A and 20 packages of screw B to get the maximum profit of Rs 410.

6. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is Rs 5 and that from a shade is Rs 3. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximise his profit?

6. Let the cottage industry manufacture x pedestal lamps and y wooden shades.

Therefore, $x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows.

	Lamps	**Shades**	**Availability**
Grinding/Cutting Machine (h)	2	1	12
Sprayer (h)	3	2	20

The profit on a lamp is Rs 5 and on the shades is Rs 3 . Therefore, the constraints are

$2x + y \leq 12$

$3x + 2y \leq 20$

Total profit, $z = 5x + 3y$

The mathematical formulation of the given problem is Maximize

$z = 5x + 3y \ldots (1)$

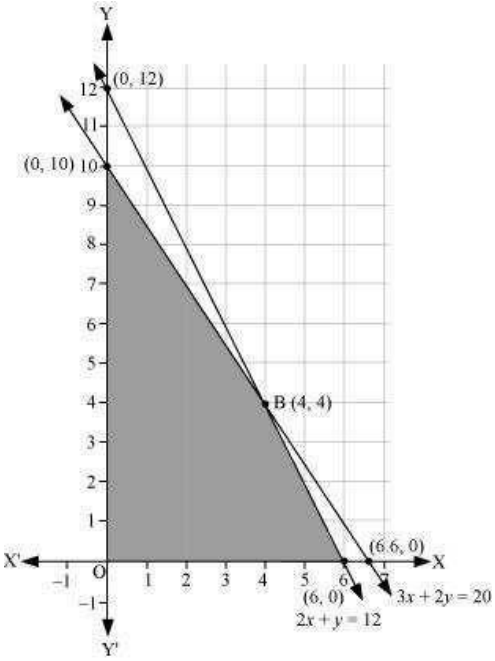
subject to the constraints,

$2x + y \leq 12 \ldots (2)$

$3x + 2y \leq 20 \ldots.(3)$

$x, y \geq 0 \quad \ldots(4)$

The feasible region determined by the system of constraints is as follows.



The corner points are A (6, 0), B (4, 4), and C (0, 10).

The values of Z at these corner points are as follows

Corner point	**Z = 5x + 3y**	
A (6, 0)	30	
B (4, 4)	32	→ Maximum
C (0, 10)	30	

The maximum value of Z is 32 at (4, 4).

Thus, the manufacturer should produce 4 pedestal lamps and 4 wooden shades to maximize his profits.

7. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit is Rs 5 each for type A and Rs 6 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximise the profit?

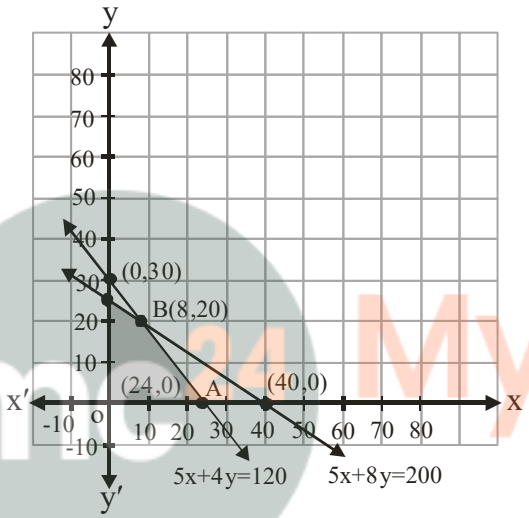
7. Let the company manufacturer x souvenirs of type A and y souvenirs of type B

$\therefore$ x and y ≥ 0

The tabular representation of the given data is:

	Type A	**Type B**	**Availability**
Cutting (min)	5	8	3 × 60 +20= 200
Manufacturing (min)	10	8	4× 60 =240

The profit on type A souvenirs is Rs 5 and that on type B souvenirs is Rs 6.
The constraints here are of the form:
$5x+ 8y \leq 200$
And $10x+ 8y \leq 240$ or $5x + 4y \leq 120$
The function is to maximize the profit $Z = 5x+ 6y$
The mathematical formulation of the data
Maximise $Z = 5x+ 6y$
Subject to constraints
$5x+ 8y \leq 200$
$5x + 4y \leq 120$
x and $y \geq 0$
The feasible region is determined by the system of constraints is as follows:



The corner points here are A(24, 0) , B(8, 20) and C(0 , 25)
The value of Z at these corner points are :

Corner points	Z= 5x + 6y	
A (24,0)	120	
B (8,20)	160	Maximum
C ((0,25)	150	

The maximum value of Z is at the point (8, 20)
Thus, the firm should produce 8 number of souvenirs of type A and 20 type B souvenirs each day in order to maximize its profit of Rs160.

8. A merchant plans to sell two types of personal computers – a desktop model and a portable model that will cost Rs 25000 and Rs 40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit

if he does not want to invest more than Rs 70 lakhs and if his profit on the desktop model is Rs 4500 and on portable model is Rs 5000.

8. Let the merchant stock x desktop models and y portable models. Therefore, $x \geq 0$ and $y \geq 0$

The cost of a desktop model is Rs 25000 and of a portable model is Rs 4000. However, the merchant can invest a maximum of Rs 70 lakhs.

$\therefore 25000x + 40000y \leq 7000000$

$5x + 8y \leq 1400$

The monthly demand of computers will not exceed 250 units.

$\therefore x + y \leq 1400$

The profit on a desktop model is Rs 4500 and the profit on a portable model is Rs 5000.

Total profit, Z = 4500x + 5000y

Thus, the mathematical formulation of the given problem is

Maximum Z = 4500x + 5000 y (1)

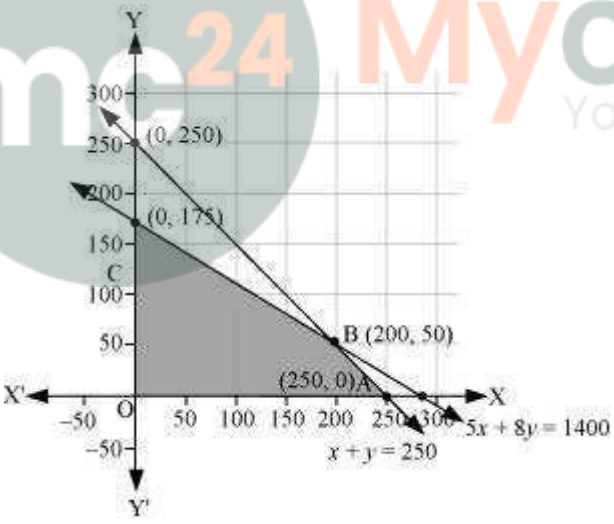
subject to the constraints,

5x + 5y ≤ 1400 (2)

x + y ≤ 250 (3)

x, y ≤ 0 (4)

The feasible region determined by the system of constraints is as follows.



The corner points are A (250, 0), B (200, 50), and C (0, 175).

The values of Z at these corner points are as follows.

Corner point	Z = 4500x + 5000y	
A (250, 0)	1125000	
B (200, 50)	1150000	→ Maximum
C (0, 175)	875000	

The maximum value of Z is 1150000 at (200, 50).

Thus, the merchant should stock 200 desktop models and 50 portable models to get the maximum profit of Rs 1150000.

9. A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs Rs 4 per unit food and F_2 costs Rs 6 per unit. One unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

9. let the diet contain x units of food F_1 and y units of food F_2.

$\therefore$ x and y ≥ 0

The tabular representation of the data is

	Vitamin A (units)	**Minerals**	**Cost per unit(Rs)**
food F_1	3	4	4
food F_2	6	3	6
Requirement	80	100	

The cost of food F_1 is Rs 4/unit and the cost of food F_2 is Rs 6 per unit.

The constraints here are

$3x + 6y \geq 80$

$4x + 3y \geq 100$

x and y ≥ 0

Total cost of the diet $Z = 4x + 6y$

The mathematical formulation of the given data is

maximize $Z = 4x + 6y$

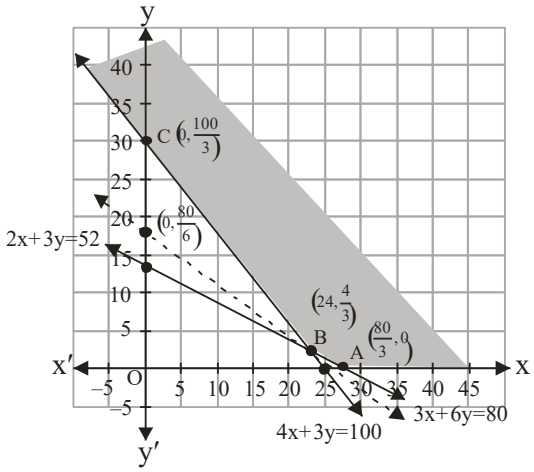
Subject to constraints

$3x + 6y \geq 80$

$4x + 3y \geq 100$

x and y ≥ 0

the feasible region by the system of constraints is as follows:



It can be seen the feasible region is unbounded with

The corner points of the feasible region are $A(\frac{80}{3},0), B\left(24,\frac{4}{3}\right)$ *and* $C\left(0,\frac{100}{3}\right)$

The values of Z at the corner points are

Corner points	**Z = 4x + 6y**	
$A(\frac{80}{3},0)$,	106.67	
$B\left(24,\frac{4}{3}\right)$	104	minimum
$C\left(0,\frac{100}{3}\right)$	200	

As the feasible region is unbounded therefore 104 may or may not be the minimum value of Z.

For this we will draw a graph of inequality 4x+ 6y < 104

It can be seen that the feasible region has no common points with 4x+ 6y < 104

∴ the maximum cost of the mixture will be Rs104.

10. There are two types of fertilisers F_1 and F_2. F_1 consists of 10% nitrogen and 6% phosphoric acid and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If F_1 costs Rs 6/kg and F_2 costs Rs 5/kg, determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?

10. Let the farmer buy x kg of fertilizer F1 and y kg of fertilizer F2. Therefore, $x \geq 0$ and $y \geq 0$

The given information can be complied in a table as follows.

	Nitrogen (%)	**Phosphoric Acid (%)**	**Cost (Rs/kg)**

F_1 (x)	10	6	6
F_2 (y)	5	10	5
Requirement (kg)	14	14	

F_1 consists of 10% nitrogen and F_2 consists of 5% nitrogen. However, the farmer requires at least 14 kg of nitrogen.

$\therefore$ 10% of x + 5% of y $\geq$ 14

$$\frac{x}{10}+\frac{y}{20}\geq 14$$

$$2x+y\geq 280$$

F_1 consists of 6% phosphoric acid and F_2 consists of 10% phosphoric acid. However, the farmer requires at least 14 kg of phosphoric acid.

$\therefore 6\%$ of $x+10\%$ of $y\geq 14$

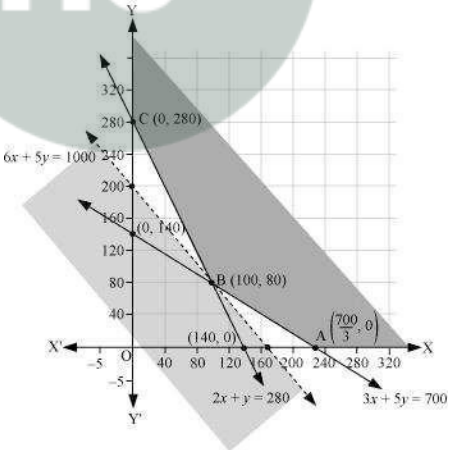
$$\frac{6x}{100}+\frac{10y}{100}\geq 14$$

$$3x+56y\geq 700$$

Total cost of fertilizers, $z=6x+5y$

The mathematical formulation of the given problem is Minimize $Z=6x+5y\ldots$ (1) subject to the constraints, $2x+y\geq 280\ldots$ (2) $3x+5y\geq 700\ldots$ (3) $x,y\geq 0\ldots$ (4)

The feasible region determined by the system of constraints is as follows.



It can be seen that the feasible region is unbounded.

The corner points are $A\left(\frac{700}{3},0\right), B(100,80)$, and $C(0.280)$.

The values of Z at these points are as follows.

Corner point	Z = 6x + 5y	

A $\left(\frac{700}{3},0\right)$	1400	
B(100, 80)	1000	→ Minimum
C(0, 280)	1400	

As the feasible region is unbounded, therefore, 1000 may or may not be the minimum value of Z.

For this, we draw a graph of the inequality, $6x + 5y < 1000$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $6x + 5y < 1000$

Therefore, 100 kg of fertiliser F_1 and 80 kg of fertilizer F_2 should be used to minimize the cost. The minimum cost is Rs 1000.

11. The corner points of the feasible region determined by the following system of linear inequalities:

$2x + y \leq 10$, $x + 3y \leq 15$, $x, y \geq 0$ are (0, 0), (5, 0), (3, 4) and (0, 5).

Let $Z = px + qy$, where $p, q > 0$.

Condition on p and q so that the maximum of Z occurs at both (3, 4) and (0, 5) is

A. $p = q$ B. $p = 2q$

C. $p = 3q$ D. $q = 3p$

11. In the given question the constraints are

$2x + y \leq 10$,

$x + 3y \leq 15$,

and $x, y \geq 0$

The function is to maximize $Z = px + qy$

The corner points are (0, 0), (5, 0), (3, 4) and (0, 5).

The value of Z at these points are

Corner points	Value of Z
(0,0)	0
(5,0)	5p
(3,4)	30p+ 4q
(0,5)	5q

Since the maximum value of z occurs on (3, 4) and (0, 5)

Hence the value on (3, 4) = Value on (0, 5)

$3p + 4q = 5q$

$3p = q$

∴ Value of Z will be maximum if $q = 3p$

Hence D is the correct answer.