EXERCISE 1D

1. Simplify:

$$\frac{\sqrt{18}}{5\sqrt{18}+3\sqrt{72}+2\sqrt{162}}$$

Solution:

We have,

$$\frac{\sqrt{18}}{5\sqrt{18}+3\sqrt{72}-2\sqrt{162}}$$

It can be written as

$$=\frac{\sqrt{18}}{5\sqrt{18}+3\sqrt{4\times 18}-2\sqrt{9\times 18}}$$

$$=\frac{\sqrt{18}}{5\sqrt{18}+(3\times 2\sqrt{18})-(2\times 3\sqrt{18})}$$

So, we get

$$=\frac{\sqrt{18}}{5\sqrt{18}+6\sqrt{18}-6\sqrt{18}}$$

$$=\frac{\sqrt{18}}{5\sqrt{18}}=\frac{1}{5}$$

2. Simplify:

$$\frac{\sqrt{x^2+y^2}-y}{x-\sqrt{x^2-y^2}} \div \frac{\sqrt{x^2-y^2}+x}{\sqrt{x^2+y^2}+y}$$

Solution:

We have,

$$\frac{\sqrt{x^2+y^2}-y}{x-\sqrt{x^2-y^2}} \div \frac{\sqrt{x^2-y^2}+x}{\sqrt{x^2+y^2}+y}$$

It can be written as

$$=\frac{\sqrt{x^2+y^2}-y}{x-\sqrt{x^2-y^2}} \times \frac{\sqrt{x^2+y^2}+y}{\sqrt{x^2-y^2}+x}$$

$$=\frac{\sqrt{x^2+y^2}-y}{x-\sqrt{x^2-y^2}} \times \frac{\sqrt{x^2+y^2}+y}{x+\sqrt{x^2-y^2}}$$

Using the formula, $a^2 - b^2 = (a + b)(a - b)$

$$= \frac{(\sqrt{x^2+y^2})^2 - y^2}{x^2 - (\sqrt{x^2-y^2})^2}$$
$$= \frac{x^2+y^2-y^2}{x^2-x^2+y^2}$$
So, we get
$= x^2/y^2$

3. Evaluate, correct to one place of decimal. The expression 5/(√20 - √10), if √5 = 2.2 and √10 = 3.2.
Solution:

We have,
$$\frac{5}{\sqrt{20}-\sqrt{10}} = \frac{5}{\sqrt{4\times 5}-\sqrt{10}}$$
It can be written as
= 5/(2√5 - √10)
= 5/[(2 x 2.2) – 3.2)]
So, we get
= 5/(4.4 – 3.2)
= 5/1.2
= 4.2
[Note: In textual answer, the value of √20 has been directly taken, which is 4.5. Hence the answer 3.8!]

4. If x = √3 – √2. Find the value of:
(i) x + 1/x
(ii) $x^2 + 1/x^2$
(iii) $x^3 + 1/x^3$
(iv) $x^3 + 1/x^3 – 3(x^2 + 1/x^2) + x + 1/x$
Solution:

(i) We have,
x + 1/x
= (√3 - √2) + 1/(√3 - √2)
$$= \frac{(\sqrt{3}-\sqrt{2})^2+1}{(\sqrt{3}-\sqrt{2})}$$
$$= \frac{3-2\sqrt{3}\sqrt{2}+2+1}{(\sqrt{3}-\sqrt{2})}$$
$$= \frac{6-2\sqrt{3}\sqrt{2}}{(\sqrt{3}-\sqrt{2})}$$
$$= \frac{6-2\sqrt{6}}{(\sqrt{3}-\sqrt{2})} \times \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})}$$
$$= \frac{6\sqrt{3}-2\sqrt{6}\sqrt{3}+6\sqrt{2}-2\sqrt{6}\sqrt{2}}{1}$$

$= 6\sqrt{3} - 2\sqrt{18} + 6\sqrt{2} - 2\sqrt{12}$
$= 6\sqrt{3} - 2\sqrt{(9 \times 2)} + 6\sqrt{2} - 2\sqrt{(4 \times 3)}$
$= 6\sqrt{3} - 2 \times 3\sqrt{2} + 6\sqrt{2} - 2 \times 2\sqrt{3}$
$= 6\sqrt{3} - 6\sqrt{2} + 6\sqrt{2} - 4\sqrt{3}$
$= 6\sqrt{3} - 4\sqrt{3}$
$= 2\sqrt{3}$

(ii) $x^2 + 1/x^2$
We have,
$= (\sqrt{3} - \sqrt{2})^2 + 1/(\sqrt{3} - \sqrt{2})^2$

$$= (3 - 2\sqrt{3}\sqrt{2} + 2) + \frac{1}{(3 - 2\sqrt{3}\sqrt{2} + 2)}$$

$$= (5 - 2\sqrt{6}) + \frac{1}{(5 - 2\sqrt{6})}$$

$$= \frac{25 - 10\sqrt{6} - 10\sqrt{6} + 4 \times 6 + 1}{(5 - 2\sqrt{6})}$$

$$= \frac{25 - 20\sqrt{6} + 25}{(5 - 2\sqrt{6})}$$

$$= \frac{50 - 20\sqrt{6}}{(5 - 2\sqrt{6})}$$

$$= \frac{10(5 - 2\sqrt{6})}{(5 - 2\sqrt{6})}$$

$$= 10$$

(iii) We have,
$x^3 + 1/x^3$
$= (\sqrt{3} - \sqrt{2})^3 + 1/(\sqrt{3} - \sqrt{2})^3$
We know that, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
$(\sqrt{3} - \sqrt{2})^3 = (\sqrt{3})^3 - (\sqrt{2})^3 - 3(\sqrt{3})(\sqrt{2})(\sqrt{3} - \sqrt{2})$
$= 3\sqrt{3} - 2\sqrt{2} - 3\sqrt{6}(\sqrt{3} - \sqrt{2})$
$= 3\sqrt{3} - 2\sqrt{2} - 3\sqrt{18} + 3\sqrt{12}$
$= 3\sqrt{3} - 2\sqrt{2} - 3\sqrt{(3^2 \times 2)} + 3\sqrt{(2^2 \times 3)}$
$= 3\sqrt{3} - 2\sqrt{2} - 3 \times 3\sqrt{2} + 3 \times 2\sqrt{3}$
$= 3\sqrt{3} - 2\sqrt{2} - 9\sqrt{2} + 6\sqrt{3}$
$= 9\sqrt{3} - 11\sqrt{2}$

$$\therefore \left(\sqrt{3} - \sqrt{2}\right)^3 + \frac{1}{(\sqrt{3} - \sqrt{2})^3} = \left(9\sqrt{3} - 11\sqrt{2}\right) + \frac{1}{(9\sqrt{3} - 11\sqrt{2})}$$

$$Considering\ \frac{1}{(9\sqrt{3}-11\sqrt{2})}$$
$$\frac{1}{(9\sqrt{3}-11\sqrt{2})}\times\frac{(9\sqrt{3}+11\sqrt{2})}{(9\sqrt{3}+11\sqrt{2})}$$
$$=\frac{(9\sqrt{3}+11\sqrt{2})}{(81\times 3)-(121\times 2)}$$
$$=\frac{(9\sqrt{3}+11\sqrt{2})}{(243)-(242)}$$
$$=(9\sqrt{3}+11\sqrt{2})$$

Now, $(9\sqrt{3} - 11\sqrt{2}) + 1/(9\sqrt{3} - 11\sqrt{2}) = (9\sqrt{3} - 11\sqrt{2}) + (9\sqrt{3} + 11\sqrt{2})$
$= 9\sqrt{3} - 11\sqrt{2} + 9\sqrt{3} + 11\sqrt{2}$
$= 9\sqrt{3} + 9\sqrt{3}$
$= 18\sqrt{3}$

(iv) $x^3 + 1/x^3 – 3(x^2 + 1/x^2) + x + 1/x$
According to the results obtained in (i), (ii) and (iii), we get
$x^3 + 1/x^3 – 3(x^2 + 1/x^2) + x + 1/x = 18\sqrt{3} – 3(10) + 2\sqrt{3}$
$= 20\sqrt{3} – 30$
$= 10(2\sqrt{3} - 3)$

5. Show that:
(i) Negative of an irrational number is irrational.
Solution:

Let the irrational number be $\sqrt{2}$
Considering the negative of $\sqrt{2}$, we get $-\sqrt{2}$
We know that $-\sqrt{2}$ is an irrational number
Hence, negative of an irrational number is irrational

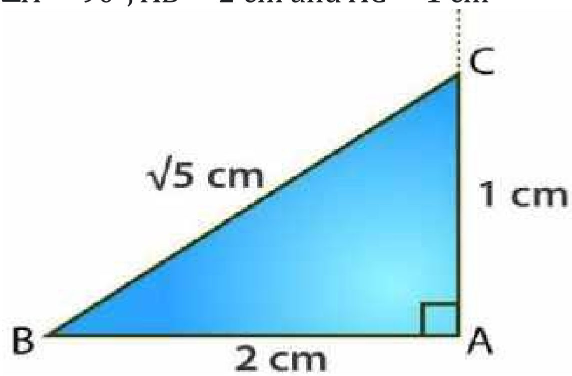
(ii) The product of a non-zero rational number and an irrational number is an irrational number.
Solution:

Let the non-zero rational number be 3
Let the irrational number be $\sqrt{5}$
Then, according to the question
$3 \times \sqrt{5} = 3\sqrt{5} = 3 \times 2.2 = 6.6$, which is irrational

6. Draw a line segment of length $\sqrt{5}$ cm.
Solution:

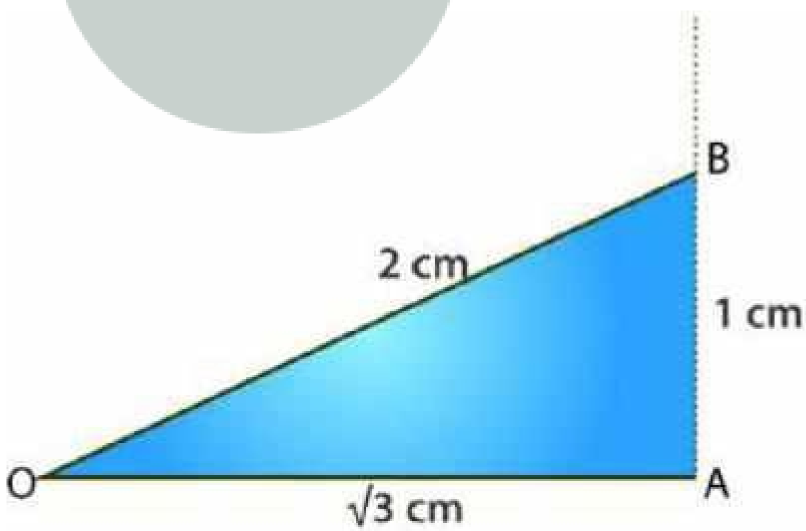
We know that, $\sqrt{5} = \sqrt{(2^2 + 1^2)}$
Which relates to: Hypotenuse $= \sqrt{[(\text{side } 1)^2 + (\text{side } 2)^2]}$ … [Pythagoras theorem]

Hence, considering
Side 1 = 2 and Side 2 = 1,
We get a right-angled triangle such that:
$\angle A = 90°$, AB = 2 cm and AC = 1 cm



7. Draw a line segment of length √**3** cm.
Solution:

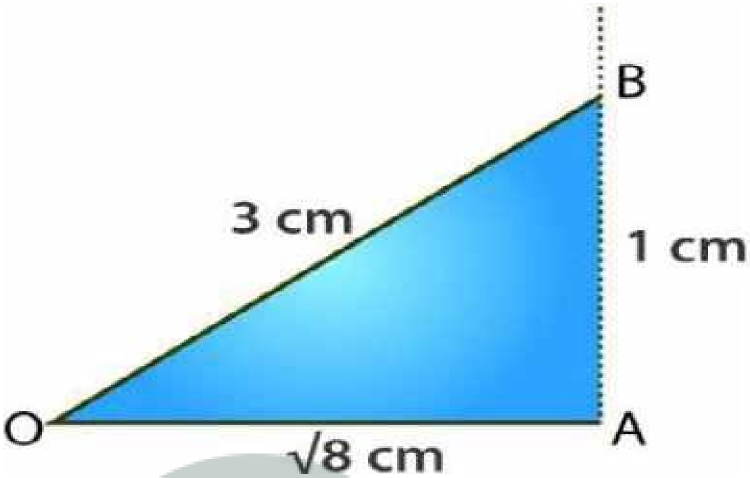
We know that, $\sqrt{3} = \sqrt{(2^2 - 1^2)}$
Which relates to: Hypotenuse= $\sqrt{[(\text{side } 1)^2 + (\text{side } 2)^2]}$ … [Pythagoras theorem]
Hypotenuse^2 – Side 1^2 = Side 2^2
Hence, considering Hypotenuse = 2 cm and Side 1 = 1 cm,
We get a right-angled triangle OAB such that:
$\angle O = 90°$, OB = 2 cm and AB = 1 cm



8. Draw a line segment of length √8 cm.
Solution:

We know that, $\sqrt{8} = \sqrt{(3^2 - 1^2)}$
Which relates to: Hypotenuse = $\sqrt{[(\text{side } 1)^2 + (\text{side } 2)^2]}$ … (Pythagoras theorem)

$Hypotenuse^2 - (Side\ 1)^2 = (Side\ 2)^2$
Hence, considering Hypotenuse = 3
cm and Side 1 =1 cm,
We get a right-angled triangle OAB such that:
$\angle A = 90°$, OB = 3 cm and AB=1 cm



9. Show that:

$$\frac{4-\sqrt{5}}{4+\sqrt{5}}+\frac{2}{5+\sqrt{3}}+\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{2}{5-\sqrt{3}}=\frac{52}{11}$$

Solution:

We have,

$$\frac{4-\sqrt{5}}{4+\sqrt{5}}+\frac{2}{5+\sqrt{3}}+\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{2}{5-\sqrt{3}}=\frac{52}{11}$$

Here,

Considering $\frac{4-\sqrt{5}}{4+\sqrt{5}}$

$$\Rightarrow \frac{4-\sqrt{5}}{4+\sqrt{5}}\times\frac{4-\sqrt{5}}{4-\sqrt{5}}=\frac{(4-\sqrt{5})^2}{16-5}=\frac{(4-\sqrt{5})^2}{11}$$

Now, Considering $\frac{2}{5+\sqrt{3}}$

$$\Rightarrow \frac{2}{5+\sqrt{3}}\times\frac{5-\sqrt{3}}{5-\sqrt{3}}=\frac{10-2\sqrt{3}}{25-3}=\frac{10-2\sqrt{3}}{22}$$

Now, Considering $\frac{4+\sqrt{5}}{4-\sqrt{5}}$

$$\Rightarrow \frac{4+\sqrt{5}}{4-\sqrt{5}}\times\frac{4+\sqrt{5}}{4+\sqrt{5}}=\frac{(4+\sqrt{5})^2}{16-5}=\frac{(4+\sqrt{5})^2}{11}$$

Now, Considering $\frac{2}{5-\sqrt{3}}$

$$\Rightarrow \frac{2}{5-\sqrt{3}}\times\frac{5+\sqrt{3}}{5+\sqrt{3}}=\frac{10+2\sqrt{3}}{25-3}=\frac{10+2\sqrt{3}}{22}$$

$$\therefore \frac{4-\sqrt{5}}{4+\sqrt{5}}+\frac{2}{5+\sqrt{3}}+\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{2}{5-\sqrt{3}}$$

$$=\frac{(4-\sqrt{5})^2}{11}+\frac{10-2\sqrt{3}}{22}+\frac{(4+\sqrt{5})^2}{11}+\frac{10+2\sqrt{3}}{22}$$

$$=\frac{(4-\sqrt{5})^2}{11}+\frac{5-\sqrt{3}}{11}+\frac{(4+\sqrt{5})^2}{11}+\frac{5+\sqrt{3}}{11}$$

$$=\frac{16-8\sqrt{5}+5+5-\sqrt{3}+16+8\sqrt{5}+5+5+\sqrt{3}}{11}$$

$$=\frac{52}{11}$$

Hence proved

10. Show that:

(i) $x^3 + 1/x^3 = 52$, if $x = 2 + \sqrt{3}$

(ii) $x^2 + 1/x^2 = 34$, if $x = 3 + 2\sqrt{2}$

(iii) $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{2\sqrt{3}}{\sqrt{3}-\sqrt{2}}=11$

Solution:

(i) We know that, $(a + b)^3 = a^3 + b^3$
$+ 3ab(a + b)$
$x^3 + 1/x^3 = (2 + \sqrt{3})^3 + 1/(2 + \sqrt{3})^3$
Here, taking
$(2 + \sqrt{3})^3 = 2^3 + (\sqrt{3})^3 + 3(2)(\sqrt{3})(2 + \sqrt{3})$
$= 8 + 3\sqrt{3} + 6\sqrt{3}(2 + \sqrt{3})$
$= 8 + 3\sqrt{3} + 12\sqrt{3} + 6(\sqrt{3})^2$
$= 8 + 3\sqrt{3} + 12\sqrt{3} + (6 \times 3)$
$= 8 + 15\sqrt{3} + 18$
$= 26 + 15\sqrt{3}$

$$Now,\ (2+\sqrt{3})^3 + \frac{1}{(2+\sqrt{3})^3} = 26 + 15\sqrt{3} + \frac{1}{26+15\sqrt{3}}$$

$$Taking\ \frac{1}{26+15\sqrt{3}},$$

$$\Rightarrow \frac{1}{26+15\sqrt{3}} \times \frac{26-15\sqrt{3}}{26-15\sqrt{3}} = \frac{26-15\sqrt{3}}{676-675} = 26 - 15\sqrt{3}$$

$$= 26 + 15\sqrt{3} + 26 - 15\sqrt{3} = 52$$

- Hence, proved.

(ii) We know that, $(a + b)^2 = a^2 + b^2 + 2ab$
$x^2 + 1/x^2 = (3 + 2\sqrt{2})^2 + 1/(3 + 2\sqrt{2})^2$
$= (9 + 8 + 2 \times 3 \times 2\sqrt{2}) + 1/(9 + 8 + 2 \times 3 \times 2\sqrt{2})$
$= (17 + 12\sqrt{2}) + 1/(17 + 12\sqrt{2})$

$$Taking\ \frac{1}{(17+12\sqrt{2})}\ we\ get:$$

$$\frac{1}{(17+12\sqrt{2})} \times \frac{(17-12\sqrt{2})}{(17-12\sqrt{2})} = \frac{(17-12\sqrt{2})}{289-288} = 17 - 12\sqrt{2}$$

$$\therefore (17+12\sqrt{2}) + \frac{1}{(17+12\sqrt{2})} = 17 + 12\sqrt{2} + 17 - 12\sqrt{2} = 34$$

- Hence, proved.

(iii) We have,

$$\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{2\sqrt{3}}{\sqrt{3}-\sqrt{2}}$$

$$First, taking\ \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}},$$

$$\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}\times\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}=\frac{(3\sqrt{2}-2\sqrt{3})^2}{18-12}=\frac{18-12\sqrt{6}+12}{6}$$

$$=\frac{6(3-2\sqrt{6}+2)}{6}=5-2\sqrt{6}$$

$$Now, taking\ \frac{2\sqrt{3}}{\sqrt{3}-\sqrt{2}},$$

$$\frac{2\sqrt{3}}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}=\frac{6+2\sqrt{6}}{3-2}=6+2\sqrt{6}$$

$$\therefore\ \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{2\sqrt{3}}{\sqrt{3}-\sqrt{2}}=5-2\sqrt{6}+6+2\sqrt{6}=11$$

- Hence, proved.

11. Show that x is rational if:
(i) $x^2 = 6$
(ii) $x^2 = 0.009$
(iii) $x^2 = 27$

Solution:

(i) $x^2 = 6$
$x = \sqrt{6} = 2.449$ … which is irrational.

(ii) $x^2 = 0.009$
$x = \sqrt{0.009} = 0.0948$ … which is irrational.

(iii) $x^2 = 27$
$x = \sqrt{27} = 5.1961$ … which is irrational.

12. Show that x is rational if:
(i) $x^2 = 16$
(ii) $x^2 = 0.0004$
(iii) $x^2 = 1\frac{7}{9}$

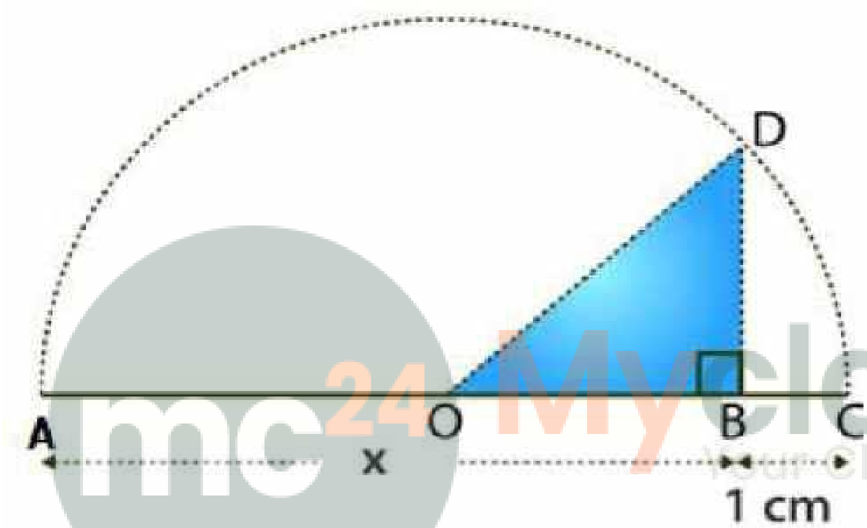
Solution:

(i) $x^2 = 16$
$x = \sqrt{16} = 4$, which is rational.

(ii) $x^2 = 0.0004$
$x = \sqrt{0.0004} = 0.02$, which is rational.

(iii)
$x^2 = 1\frac{7}{9}$
$x = \sqrt{1\frac{7}{9}} = \sqrt{\frac{16}{9}} = \frac{4}{3}$, which is rational.

13. Using the following figure, show that $BD = \sqrt{x}$.



Solution:

Let's assume AB = x, BC = 1 and AC = x + 1
Here, AC is diameter and O is the centre
OA = OC = OD = radius = (x + 1)/2
And,
OB = OC – BC
= (x + 1)/2 – 1
= (x + 1 - 2)/2
= (x - 1)/2
Now, using Pythagoras theorem, we have
$OD^2 = OB^2 + BD^2$

$$\left(\frac{x+1}{2}\right)^2 = \left(\frac{x-1}{2}\right)^2 + BD^2$$

$$\Rightarrow BD^2 = \left(\frac{x+1}{2}\right)^2 - \left(\frac{x-1}{2}\right)^2$$

$$= \frac{x^2 + 2x + 1 - x^2 + 2x - 1}{4}$$

= 4x/x

$= x$

$\therefore BD = \sqrt{x}$

- Hence, proved.