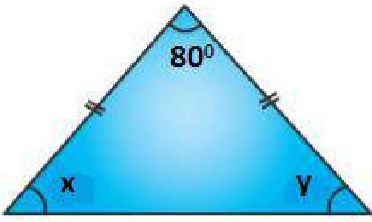


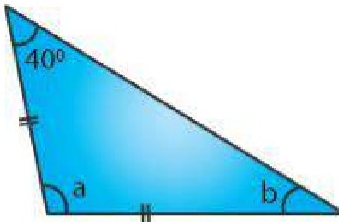
EXERCISE 15B

1. Find the unknown angles in the given figures:

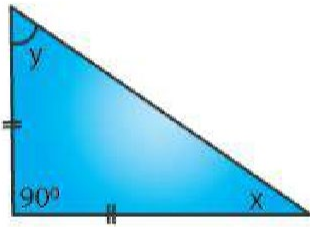
(i)



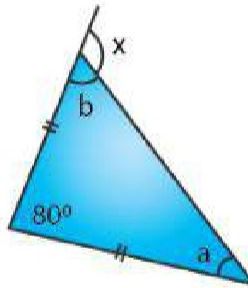
(ii)



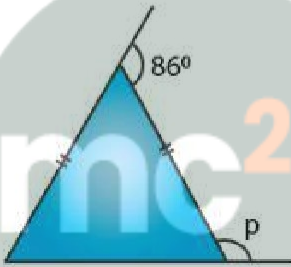
(iii)



(iv)



(v)



Myclass24
Your Class. Your Pace.

Solution:

(i) From the figure (i)

$x = y$ as the angles opposite to equal sides

In a triangle

$$x + y + 80^\circ = 180^\circ$$

Substituting the values

$$x + x + 80^\circ = 180^\circ$$

By further calculation

$$2x = 180^\circ - 80^\circ = 100^\circ$$

$$x = 100^\circ / 2 = 50^\circ$$

Therefore, $x = y = 50^\circ$.

(ii) From the figure (ii)

$b = 40^\circ$ as the angles opposite to equal sides

In a triangle

$$a + b + 40^\circ = 180^\circ$$

Substituting the values

$$a + 40^\circ + 40^\circ = 180^\circ$$

By further calculation

$$a = 180 - 80 = 100^\circ$$

Therefore, $a = 100^\circ$ and $b = 40^\circ$.

(iii) From the figure (iii)

$x = y$ as the angles opposite to equal sides

In a triangle

$$x + y + 90^\circ = 180^\circ$$

Substituting the values

$$x + x + 90^\circ = 180^\circ$$

By further calculation

$$2x = 180 - 90 = 90^\circ$$

$$x = 90/2 = 45^\circ$$

Therefore, $x = y = 45^\circ$.

(iv) From the figure (iv)

$a = b$ as the angles opposite to equal sides are equal

In a triangle

$$a + b + 80^\circ = 180^\circ$$

Substituting the values

$$a + a + 80^\circ = 180^\circ$$

By further calculation

$$2a = 180 - 80 = 100^\circ$$

$$a = 100/2 = 50^\circ$$

Here $a = b = 50^\circ$

We know that in a triangle the exterior angle is equal to sum of its opposite interior angles

$$x = a + 80^\circ$$

So we get

$$x = 50 + 80 = 130^\circ$$

Therefore, $a = 50^\circ$, $b = 50^\circ$ and $x = 130^\circ$.

(v) From the figure (v)

In an isosceles triangle consider each equal angle = x

$$x + x = 86^\circ$$

$$2x = 86^\circ$$

So we get

$$x = 86/2 = 43^\circ$$

For a linear pair

$$p + x = 180^\circ$$

Substituting the values

$$p + 43^\circ = 180^\circ$$

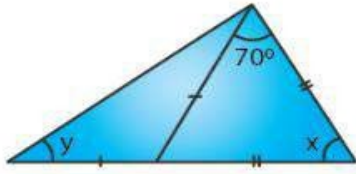
By further calculation

$$p = 180 - 43 = 137^\circ$$

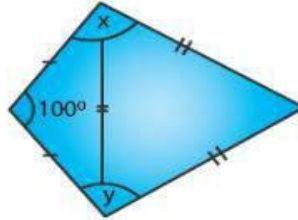
Therefore, $p = 137^\circ$.

2. Apply the properties of isosceles and equilateral triangles to find the unknown angles in the given figures:

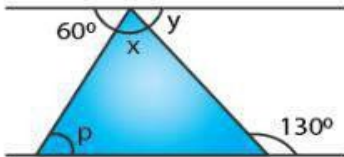
(i)



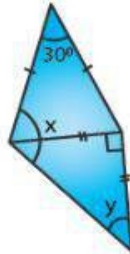
(ii)



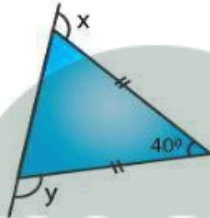
(iii)



(iv)



(v)



Solution:

(i) $a = 70^\circ$ as the angles opposite to equal sides are equal

In a triangle

$$a + 70^\circ + x = 180^\circ$$

Substituting the values

$$70^\circ + 70^\circ + x = 180^\circ$$

By further calculation

$$x = 180 - 140 = 40^\circ$$

$y = b$ as the angles opposite to equal sides are equal

Here $a = y + b$ as the exterior angle is equal to sum of interior opposite angles

$$70^\circ = y + y$$

So we get

$$2y = 70^\circ$$

$$y = 70^\circ / 2 = 35^\circ$$

Therefore, $x = 40^\circ$ and $y = 35^\circ$.

(ii) From the figure (ii)

Each angle is 60° in an equilateral triangle

In a isosceles triangle

Consider each base angle = a

$$a + a + 100^\circ = 180^\circ$$

By further calculation

$$2a = 180 - 100 = 80^\circ$$

So we get

$$a = 80^\circ / 2 = 40^\circ$$

$$x = 60^\circ + 40^\circ = 100^\circ$$

$$y = 60^\circ + 40^\circ = 100^\circ$$

(iii) From the figure (iii)

$130^\circ = x + p$ as the exterior angle is equal to the sum of interior opposite angles

It is given that the lines are parallel

Here $p = 60^\circ$ is the alternate angles and $y = a$

In a linear pair

$$a + 130^\circ = 180^\circ$$

By further calculation

$$a = 180 - 130 = 50^\circ$$

Here $x + p = 130^\circ$

Substituting the values

$$x + 60^\circ = 130^\circ$$

By further calculation

$$x = 130 - 60 = 70^\circ$$

Therefore, $x = 70^\circ$, $y = 50^\circ$ and $p = 60^\circ$.

(iv) From the figure (iv)

$$x = a + b$$

Here $b = y$ and $a = c$ as the angles opposite to equal sides are equal

$$a + c + 30^\circ = 180^\circ$$

Substituting the values

$$a + a + 30^\circ = 180^\circ$$

By further calculation

$$2a = 180 - 30 = 150^\circ$$

$$a = 150 / 2 = 75^\circ$$

We know that

$$b + y = 90^\circ$$

Substituting the values

$$y + y = 90^\circ$$

$$2y = 90^\circ$$

$$y = 90 / 2 = 45^\circ$$

$$\text{where } b = 45^\circ$$

Therefore, $x = a + b = 75 + 45 = 120^\circ$ and $y = 45^\circ$.

(v) From the figure (v)

$$a + b + 40^\circ = 180^\circ$$

So we get

$$a + b = 180 - 40 = 140^\circ$$

The angles opposite to equal sides are equal

$$a = b = 140/2 = 70^\circ$$

$$x = b + 40^\circ = 70^\circ + 40^\circ = 110^\circ$$

Here the exterior angle of a triangle is equal to the sum of its interior opposite angles

In the same way

$$y = a + 40^\circ$$

Substituting the values

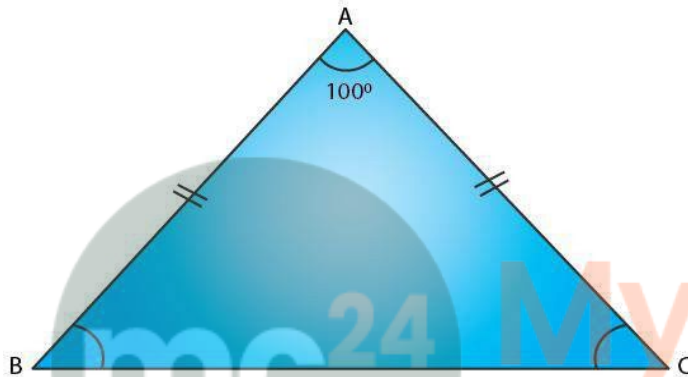
$$y = 70^\circ + 40^\circ = 110^\circ$$

Therefore, $x = y = 110^\circ$.

3. The angle of vertex of an isosceles triangle is 100° . Find its base angles.

Solution:

Consider $\triangle ABC$



Here $AB = AC$ and $\angle B = \angle C$

We know that

$$\angle A = 100^\circ$$

In a triangle

$$\angle A + \angle B + \angle C = 180^\circ$$

Substituting the values

$$100^\circ + \angle B + \angle B = 180^\circ$$

By further calculation

$$2\angle B = 180^\circ - 100^\circ = 80^\circ$$

$$\angle B = 80/2 = 40^\circ$$

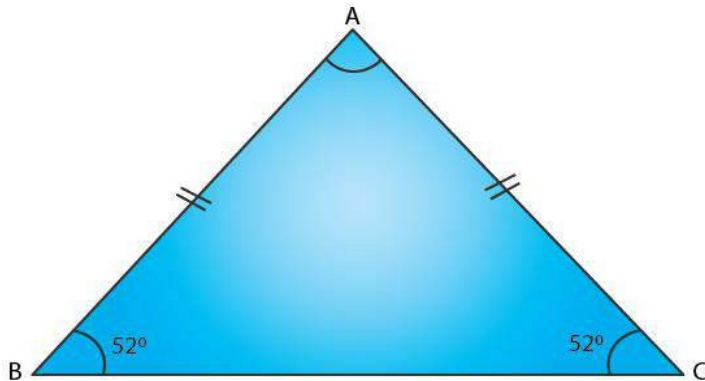
Therefore, $\angle B = \angle C = 40^\circ$.

4. One of the base angles of an isosceles triangle is 52° . Find its angle of vertex.

Solution:

It is given that the base angles of isosceles triangle $\triangle ABC = 52^\circ$

$$\text{Here } \angle B = \angle C = 52^\circ$$



In a triangle

$$\angle A + \angle B + \angle C = 180^\circ$$

Substituting the values

$$\angle A + 52^\circ + 52^\circ = 180^\circ$$

By further calculation

$$\angle A = 180 - 104 = 76^\circ$$

Therefore, $\angle A = 76^\circ$.

5. In an isosceles triangle, each base angle is four times of its vertical angle. Find all the angles of the triangle.

Solution:

Consider the vertical angle of an isosceles triangle = x

So the base angle = $4x$

In a triangle

$$x + 4x + 4x = 180^\circ$$

By further calculation

$$9x = 180^\circ$$

$$x = 180/9 = 20^\circ$$

So the vertical angle = 20°

Each base angle = $4x = 4 \times 20^\circ = 80^\circ$

6. The vertical angle of an isosceles triangle is 15° more than each of its base angles. Find each angle of the triangle.

Solution:

Consider the angle of the base of isosceles triangle = x°

So the vertical angle = $x + 15^\circ$

In a triangle

$$x + x + x + 15^\circ = 180^\circ$$

By further calculation

$$3x = 180 - 15 = 165^\circ$$

$$x = 165/3 = 55^\circ$$

Therefore, the base angle = 55°

Vertical angle = $55 + 15 = 70^\circ$.

7. The base angle of an isosceles triangle is 15° more than its vertical angle. Find its each angle.

Solution:

Consider the vertical angle of the isosceles triangle = x°
Here each base angle = $x + 15^\circ$

In a triangle

$$x + 15^\circ + x + 15^\circ + x = 180^\circ$$

By further calculation

$$3x + 30^\circ = 180^\circ$$

$$3x = 180 - 30 = 150^\circ$$

$$x = 150/3 = 50^\circ$$

Therefore, vertical angle = 50° and each base angle = $50 + 15 = 65^\circ$.

8. The vertical angle of an isosceles triangle is three times the sum of its base angles. Find each angle.

Solution:

Consider each base of an isosceles triangle = x
Vertical angle = $3(x + x) = 3(2x) = 6x$

In a triangle

$$6x + x + x = 180^\circ$$

By further calculation

$$8x = 180^\circ$$

$$x = 180/8 = 22.5^\circ$$

Therefore, each base angle = 22.5° and vertical angle = $3(22.5 + 22.5) = 3 \times 45 = 135^\circ$.

9. The ratio between a base angle and the vertical angle of an isosceles triangle is 1 : 4. Find each angle of the triangle.

Solution:

It is given that the ratio between a base angle and the vertical angle of an isosceles triangle = 1 : 4

Consider base angle = x

Vertical angle = $4x$

In a triangle

$$x + x + 4x = 180^\circ$$

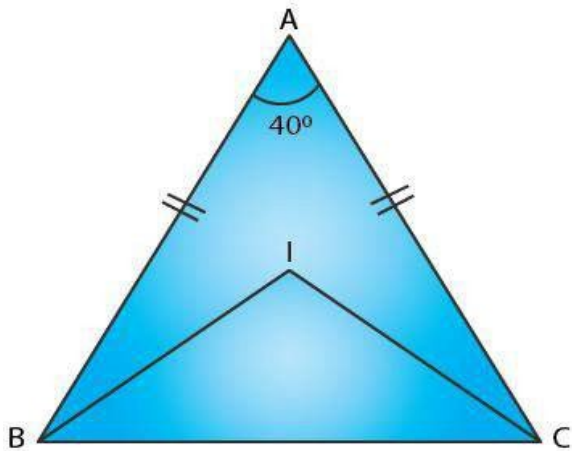
By further calculation

$$6x = 180^\circ$$

$$x = 180/6 = 30^\circ$$

Therefore, each base angle = $x = 30^\circ$ and vertical angle = $4x = 4 \times 30^\circ = 120^\circ$.

10. In the given figure, BI is the bisector of $\angle ABC$ and CI is the bisector of $\angle ACB$. Find $\angle BIC$.



Solution:

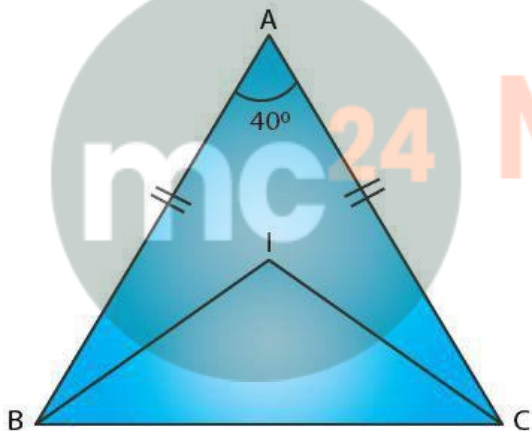
In $\triangle ABC$

BI is the bisector of $\angle ABC$ and CI is the bisector of $\angle ACB$

Here $AB = AC$

$\angle B = \angle C$ as the angles opposite to equal sides are equal

We know that $\angle A = 40^\circ$



In a triangle

$$\angle A + \angle B + \angle C = 180^\circ$$

Substituting the values

$$40^\circ + \angle B + \angle B = 180^\circ$$

By further calculation

$$40^\circ + 2\angle B = 180^\circ$$

$$2\angle B = 180 - 40 = 140^\circ$$

$$\angle B = 140/2 = 70^\circ$$

Here BI and CI are the bisectors of $\angle ABC$ and $\angle ACB$

$$\angle IBC = \frac{1}{2} \angle ABC = \frac{1}{2} \times 70^\circ = 35^\circ$$

$$\angle ICB = \frac{1}{2} \angle ACB = \frac{1}{2} \times 70^\circ = 35^\circ$$

In $\triangle IBC$

$$\angle BIC + \angle IBC + \angle ICB = 180^\circ$$

Substituting the values

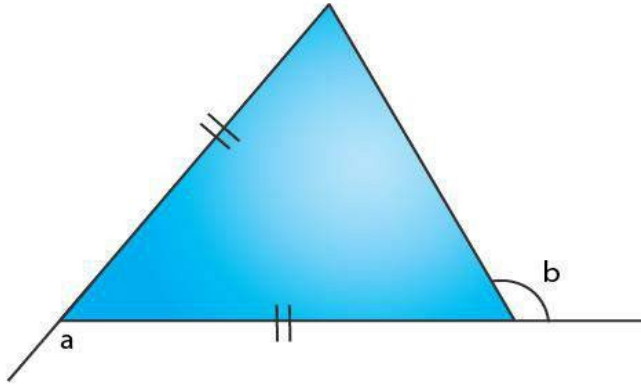
$$\angle BIC + 35^\circ + 35^\circ = 180^\circ$$

By further calculation

$$\angle BIC = 180 - 70 = 110^\circ$$

Therefore, $\angle BIC = 110^\circ$.

11. In the given figure, express a in terms of b .



Solution:

From the ΔABC

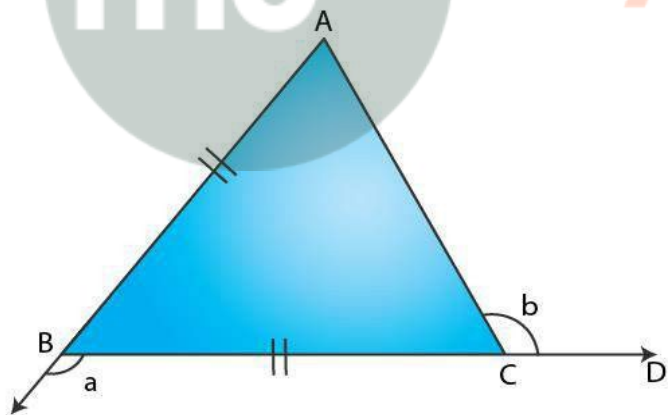
$$BC = BA$$

$$\angle BCA = \angle BAC$$

Here the exterior $\angle CBE = \angle BCA + \angle BAC$

$$a = \angle BCA + \angle BCA$$

$$a = 2\angle BCA \dots\dots (1)$$



$$\text{Here } \angle ACB = 180^\circ - b$$

Where $\angle ACD$ and $\angle ACB$ are linear pair

$$\angle BCA = 180^\circ - b \dots\dots (2)$$

We get

$$a = 2 \angle BCA$$

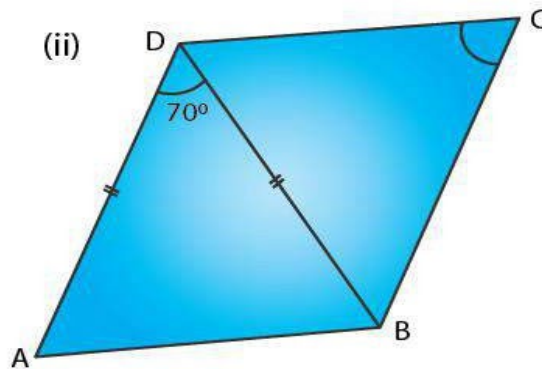
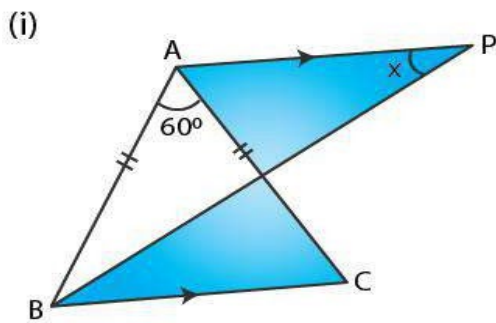
Substituting the values

$$a = 2 (180^\circ - b)$$

$$a = 360^\circ - 2b$$

12. (a) In Figure (i) BP bisects $\angle ABC$ and $AB = AC$. Find x .

(b) Find x in Figure (ii) Given: $DA = DB = DC$, BD bisects $\angle ABC$ and $\angle ADB = 70^\circ$.



Solution:

(a) From the figure (i)

$AB = AC$ and BP bisects $\angle ABC$

AP is drawn parallel to BC

Here PB is the bisector of $\angle ABC$

$\angle PBC = \angle PBA$

$\angle APB = \angle PBC$ are alternate angles

$x = \angle PBC \dots (1)$

In $\triangle ABC$

$\angle A = 60^\circ$

Since $AB = AC$ we get $\angle B = \angle C$

In a triangle

$\angle A + \angle B + \angle C = 180^\circ$

Substituting the values

$60^\circ + \angle B + \angle C = 180^\circ$

We get

$60^\circ + \angle B + \angle B = 180^\circ$

By further calculation

$2\angle B = 180 - 60 = 120^\circ$

$\angle B = 120/2 = 60^\circ$

$\frac{1}{2}\angle B = 60/2 = 30^\circ$

$\angle PBC = 30^\circ$

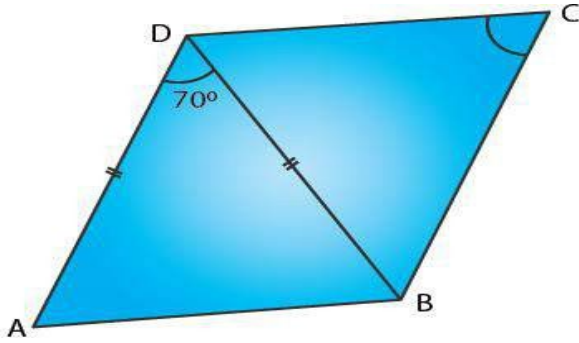
So from figure (i) $x = 30^\circ$

(b) From the figure (ii)

$DA = DB = DC$

Here BD bisects $\angle ABC$ and $\angle ADB = 70^\circ$

Myclass24
Your Class. Your Pace.



In a triangle

$$\angle ADB + \angle DAB + \angle DBA = 180^\circ$$

Substituting the values

$$70^\circ + \angle DBA + \angle DBA = 180^\circ$$

By further calculation

$$70^\circ + 2\angle DBA = 180^\circ$$

$$2\angle DBA = 180 - 70 = 110^\circ$$

$$\angle DBA = 110/2 = 55^\circ$$

Here BD is the bisector of $\angle ABC$

$$\text{So } \angle DBA = \angle DBC = 55^\circ$$

In $\triangle DBC$

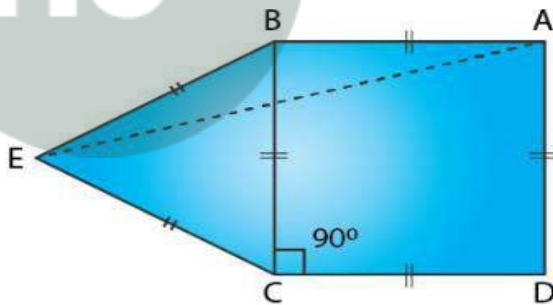
$$DB = DC$$

$$\angle DCB = \angle DBC$$

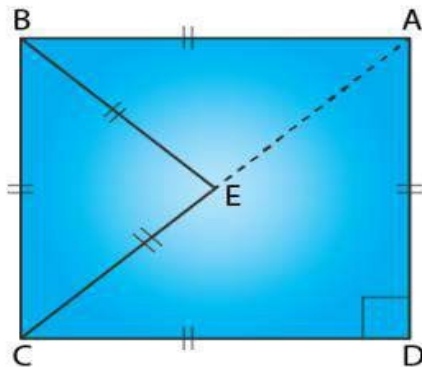
$$\text{Hence, } x = 55^\circ.$$

13. In each figure, given below, ABCD is a square and $\triangle BEC$ is an equilateral triangle.

(i)



(ii)



Find, in each case: (i) $\angle ABE$ (ii) $\angle BAE$

Solution:

The sides of a square are equal and each angle is 90°
 In an equilateral triangle three sides and angles are 60°
 In figure (i) ABCD is a square and $\triangle BEC$ is an equilateral triangle

(i) $\angle ABE = \angle ABC + \angle CBE$

Substituting the values
 $\angle ABE = 90^\circ + 60^\circ = 150^\circ$

(ii) In $\triangle ABE$

$\angle ABE + \angle BEA + \angle BAE = 180^\circ$

Substituting the values

$150^\circ + \angle BAE + \angle BAE = 180^\circ$

By further calculation

$2\angle BAE = 180 - 150 = 30^\circ$

$\angle BAE = 30/2 = 15^\circ$

In figure (ii) ABCD is a square and $\triangle BEC$ is an equilateral triangle

(i) $\angle ABE = \angle ABC - \angle CBE$

Substituting the values
 $\angle ABE = 90^\circ - 60^\circ = 30^\circ$

(ii) In $\triangle ABE$

$\angle ABE + \angle BEA + \angle BAE = 180^\circ$

Substituting the values

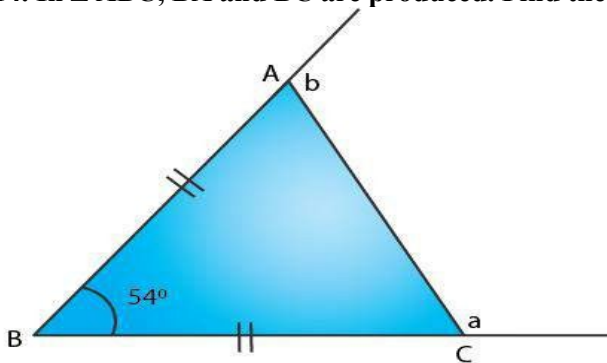
$30^\circ + \angle BAE + \angle BAE = 180^\circ$

By further calculation

$2\angle BAE = 180 - 30 = 150^\circ$

$\angle BAE = 150/2 = 75^\circ$

14. In $\triangle ABC$, BA and BC are produced. Find the angles a and h, if $AB = BC$.



Solution:

In $\triangle ABC$, BA and BC are produced
 $\angle ABC = 54^\circ$ and $AB = BC$

In $\triangle ABC$

$\angle BAC + \angle BCA + \angle ABC = 180^\circ$

Substituting the values

$$\angle BAC + \angle BAC + 54^\circ = 180^\circ$$

$$2\angle BAC = 180 - 54 = 126^\circ$$

$$\angle BAC = 126/2 = 63^\circ$$

$$\angle BCA = 63^\circ$$

In a linear pair

$$\angle BAC + b = 180^\circ$$

Substituting the value

$$63^\circ + b = 180^\circ$$

So we get

$$b = 180 - 63 = 117^\circ$$

In a linear pair

$$\angle BCA + a = 180^\circ$$

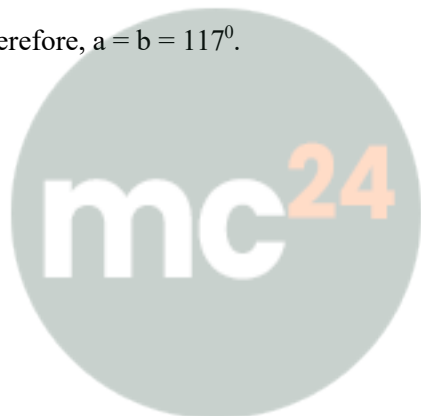
Substituting the value

$$63^\circ + a = 180^\circ$$

So we get

$$a = 180 - 63 = 117^\circ$$

Therefore, $a = b = 117^\circ$.



Myclass24
Your Class. Your Pace.