$\therefore a = (-2) + 3 = 1,$

$b = -(-2) - 4 = -2$

$c = -6(-2) - 5 = 7$

Therefore, co-ordinates of point of intersection of given line and plane are

$P \equiv (1, -2, 7)$

**21. Question**

Find the distance of the point (2, 3, 4) from the plane 3x + 2y + 2z +5 =0, measured parallel to the line $\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$.

**Answer**

Given :

Equation of plane : 3x + 2y + 2z + 5 = 0

Equation of line :

$$\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$$

Point : P = (2, 3, 4)

To Find : Distance of point P from the given plane parallel to the given line.

Formula :

1) Equation of line :

Equation of line passing through A = $(x_1, y_1, z_1)$ & having direction ratios (a, b, c) is

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

2) Distance formula :

The distance between two points A = $(a_1, a_2, a_3)$ & B = $(b_1, b_2, b_3)$ is

$$d = \sqrt{(a_1-b_1)^2 + (a_2-b_2)^2 + (a_3-b_3)^2}$$

For the given line,

$$\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$$

Direction ratios are (a, b, c) = (3, 6, 2)

Let Q be the point on the plane such that $\overline{PQ}$ is parallel to the given line.

Therefore direction ratios of given line and line PQ will be same.

Therefore equation of line PQ with point P = (2, 3, 4) and with direction ratios (3, 6, 2) is

$$\frac{x-2}{3} = \frac{y-3}{6} = \frac{z-4}{2}$$

Let co-ordinates of Q be (u, v, w)

As point Q lies on the line PQ, we can write,

$$\frac{u-2}{3} = \frac{v-3}{6} = \frac{w-4}{2} = k(say)$$

⇒u = 3k+2, v = 6k+3, w = 2k+4 .........(1)

Also point Q lies on the plane

3u + 2v + 2w = -5

⇒3(3k+2) + 2(6k+3) + 2(2k+4) = -5 .....from (1)

⇒9k + 6 + 12k + 6 + 4k + 8 = -5

⇒25k = -25

⇒k = -1

$\therefore u = 3(-1) + 2 = -1,$

$v = 6(-1) + 3 = -3$

$w = 2(-1) + 4 = 2$

Therefore, co-ordinates of point Q are

$Q = (-1, -3, 2)$

Now distance between points P and Q by distance formula is

$d = \sqrt{(2+1)^2 + (3+3)^2 + (4-2)^2}$

$= \sqrt{(3)^2 + (6)^2 + (2)^2}$

$= \sqrt{9 + 36 + 4}$

$= \sqrt{49}$

= 7

Therefore distance of point P from the given plane measured parallel to the given line is

d = 7 units

**22. Question**

Find the distance of the point (0, -3, 2) from the plane x + 2y -z = 1, measured parallel to the line $\frac{x+1}{3} = \frac{y+1}{2} = \frac{z}{3}$.

**Answer**

Given :

Equation of plane : x + 2y - z = 1

Equation of line :

$$\frac{x+1}{3} = \frac{y+1}{2} = \frac{z}{3}$$

Point : P = (0, -3, 2)

To Find : Distance of point P from the given plane parallel to the given line.

Formula :

1) Equation of line :

Equation of line passing through A = ($x_1$, $y_1$, $z_1$) & having direction ratios (a, b, c) is

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

2) Distance formula :

The distance between two points A = ($a_1$, $a_2$, $a_3$) & B = ($b_1$, $b_2$, $b_3$) is

$d = \sqrt{(a_1-b_1)^2 + (a_2-b_2)^2 + (a_3-b_3)^2}$

For the given line,

$$\frac{x+1}{3} = \frac{y+1}{2} = \frac{z}{3}$$

Direction ratios are (a, b, c) = (3, 2, 3)

Let Q be the point on the plane such that $\overline{PQ}$ is parallel to the given line.

Therefore direction ratios of given line and line PQ will be same.

Therefore equation of line PQ with point P = (0, -3, 2) and with direction ratios (3, 2, 3) is

$$\frac{x-0}{3} = \frac{y+3}{2} = \frac{z-2}{3}$$

Let co-ordinates of Q be (u, v, w)

As point Q lies on the line PQ, we can write,

$$\frac{u}{3} = \frac{v+3}{2} = \frac{w-2}{3} = k(say)$$

⇒u = 3k, v = 2k-3, w = 3k+2 .........(1)

Also point Q lies on the plane

u + 2v - w = 1

⇒(3k) + 2(2k-3) - (3k+2) = 1 .....from (1)

⇒3k + 4k - 6 - 3k - 2 = 1

⇒4k = 9

$\Rightarrow k = \frac{9}{4}$

$\therefore u = 3\left(\frac{9}{4}\right) = \frac{27}{4},$

$v = 2\left(\frac{9}{4}\right) - 3 = \frac{6}{4}$

$w = 3\left(\frac{9}{4}\right) + 2 = \frac{35}{4}$

Therefore, co-ordinates of point Q are

$Q \equiv \left(\frac{27}{4}, \frac{6}{4}, \frac{35}{4}\right)$

Now distance between points P and Q by distance formula is

$$d = \sqrt{\left(0 - \frac{27}{4}\right)^2 + \left(-3 - \frac{6}{4}\right)^2 + \left(2 - \frac{35}{4}\right)^2}$$

$$= \sqrt{\left(\frac{-27}{4}\right)^2 + \left(\frac{-18}{4}\right)^2 + \left(\frac{-27}{4}\right)^2}$$

$= \sqrt{45.5625 + 20.25 + 45.5625}$

$= \sqrt{111.375}$

$= 10.55$

Therefore distance of point P from the given plane measured parallel to the given line is

d = 10.55 units

**23. Question**

Find the equation of the line passing through the point P(4, 6, 2) and the point of intersection of the line $\frac{x-1}{3} = \frac{y}{2} = \frac{z+1}{7}$ and the plane x + y -Z = 8.

**Answer**

Given :

Equation of plane : x + y - z = 8

Equation of line :

$\frac{x-1}{3} = \frac{y}{2} = \frac{z+1}{7}$

Point : P = (4, 6, 2)

To Find : Equation of line.

Formula :

Equation of line passing through A = ($x_1$, $y_1$, $z_1$) &

B = ($x_2$, $y_2$, $z_2$) is

$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} = \frac{z - z_1}{z_1 - z_2}$

let Q (a, b, c) be point of intersection of plane and line.

As point Q lies on the line, we can write,

$\frac{a-1}{3} = \frac{b}{2} = \frac{c+1}{7} = k(say)$

⇒a =3k+1, b= 2k, c= 7k-1

Also point Q lies on the plane,

a + b - c = 8

⇒(3k+1) + (2k) - (7k-1) = 8

⇒3k + 1 + 2k - 7k + 1 = 8

⇒-2k = 6

⇒k = -3

$\therefore a = 3(-3) + 1 = -8,$

$b = -2(-3) = -6$

$c = 7(-3) - 1 = -22$

Therefore, co-ordinates of point of intersection of given line and plane are Q = (-8, -6, -22)

Now, equation of line passing through P(4,6,2) and

Q(-8, -6, -22) is

$$\frac{x-4}{4+8}=\frac{y-6}{6+6}=\frac{z-2}{2+22}$$

$$\therefore \frac{x-4}{12}=\frac{y-6}{12}=\frac{z-2}{24}$$

$$\therefore \frac{x-4}{1}=\frac{y-6}{1}=\frac{z-2}{2}$$

This is the equation of required line

**24. Question**

Show that the distance of the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane x -y + z = 5 from the point (-1, -5 -10) is 13 units.

**Answer**

Given :

Equation of plane : x - y + z = 5

Equation of line :

$$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$$

Point : P = (-1, -5, -10)

To Prove : Distance of point P from the given plane parallel to the given line is 13 units.

Formula :

1) Equation of line :

Equation of line passing through A = ($x_1$, $y_1$, $z_1$) & having direction ratios (a, b, c) is

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

2) Distance formula :

The distance between two points A = ($a_1$, $a_2$, $a_3$) & B = ($b_1$, $b_2$, $b_3$) is

$$d=\sqrt{(a_1-b_1)^2+(a_2-b_2)^2+(a_3-b_3)^2}$$



For the given line,

$$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$$

Direction ratios are (a, b, c) = (3, 4, 12)

Let Q be the point on the plane such that $\overline{PQ}$ is parallel to the given line.

Therefore direction ratios of given line and line PQ will be same.

Therefore equation of line PQ with point P = (-1, -5, -10) and with direction ratios (3, 4, 12) is

$$\frac{x+1}{3}=\frac{y+5}{4}=\frac{z+10}{12}$$

Let co-ordinates of Q be (u, v, w)

As point Q lies on the line PQ, we can write,

$$\frac{u+1}{3}=\frac{v+5}{4}=\frac{w+10}{12}=k(say)$$

⇒u = 3k-1, v = 4k-5, w = 12k-10 .........(1)

Also point Q lies on the plane

u - v + w = 5

⇒(3k-1) - (4k-5) + (12k-10) = 5 .....from (1)

⇒3k - 1 - 4k + 5 + 12k - 10 = 5

⇒11k = 11

⇒$k = 1$

$\therefore u = 3(1) - 1 = 2,$

$v = 4(1) - 5 = -1$

$w = 12(1) - 10 = 2$

Therefore, co-ordinates of point Q are

Q ≡ $(2, -1, 2)$

Now distance between points P and Q by distance formula is

$d = \sqrt{(-1-2)^2 + (-5+1)^2 + (-10-2)^2}$

$= \sqrt{(-3)^2 + (-4)^2 + (-12)^2}$

$= \sqrt{9 + 16 + 144}$

$= \sqrt{169}$

= 13

Therefore distance of point P from the given plane measured parallel to the given line is

d = 13 units

Hence proved.

**25. Question**

Find the distance of the point A(-1, -5, -10) from the point of intersection of the line $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$.

HINT: Convert the equations of the line and the plane to Cartesian form.

**Answer**

Given :

Equation of plane : $\bar{r}.(\hat{\imath} - \hat{\jmath} + \hat{k}) = 5$

Equation of line :

$\bar{r} = (2\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \lambda(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k})$

Point : P = (-1, -5, -10)

To Find : Distance of point P from the given plane parallel to the given line.

Formula :

1) Equation of line :

Equation of line passing through A = ($x_1$, $y_1$, $z_1$) & having direction ratios (a, b, c) is

$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$

2) Distance formula :

The distance between two points A = ($a_1$, $a_2$, $a_3$) & B = ($b_1$, $b_2$, $b_3$) is

$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$

for the given plane,

$\bar{r}.(\hat{\imath} - \hat{\jmath} + \hat{k}) = 5$

Here, $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$(x\hat{\imath} + y\hat{\jmath} + z\hat{k}).(\hat{\imath} - \hat{\jmath} + \hat{k}) = 5$

⇒x - y + z = 5 .........eq(1)

For the given line,

$\bar{r} = (2\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \lambda(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k})$

Here, $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\therefore (3\hat{\imath} + 4\hat{\jmath} + 2\hat{k})\lambda = (x\hat{\imath} + y\hat{\jmath} + z\hat{k}) - (2\hat{\imath} - \hat{\jmath} + 2\hat{k})$

$\therefore 3\lambda\hat{\imath} + 4\lambda\hat{\jmath} + 2\lambda\hat{k} = (x-2)\hat{\imath} + (y+1)\hat{\jmath} + (z-2)\hat{k}$

Comparing coefficients of $\hat{\imath}, \hat{\jmath}$ & $\hat{k}$

$\Rightarrow 3\lambda = (x-2), 4\lambda = (y+1)$ & $2\lambda = (z-2)$

$\Rightarrow \lambda = \frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2}$ .........eq(2)

Direction ratios for above line are (a, b, c) = (3, 4, 2)

Let Q be the point on the plane such that $\overline{PQ}$ is parallel to the given line.

Therefore direction ratios of given line and line PQ will be same.

Therefore equation of line PQ with point P = (-1, -5, -10) and with direction ratios (3, 4, 2) is

$$\frac{x+1}{3} = \frac{y+5}{4} = \frac{z+10}{2}$$

Let co-ordinates of Q be (u, v, w)

As point Q lies on the line PQ, we can write,

$$\frac{u+1}{3} = \frac{v+5}{4} = \frac{w+10}{2} = k(say)$$

⇒u = 3k-1, v = 4k-5, w = 2k-10 .........(3)

Also point Q lies on the given plane

Therefore from eq(1), we can write,

u - v + w = 5

⇒(3k-1) - (4k-5) + (2k-10) = 5 .....from (3)

⇒3k - 1 - 4k + 5 + 2k - 10 = 5

⇒k = 11

⇒$k = 11$

$\therefore u = 3(11) - 1 = 32,$

$v = 4(11) - 5 = 39$

$w = 2(11) - 10 = 12$

Therefore, co-ordinates of point Q are

Q ≡ $(32, 39, 12)$

Now the distance between points P and Q by distance formula is

$d = \sqrt{(-1-32)^2 + (-5-39)^2 + (-10-12)^2}$

$= \sqrt{(-33)^2 + (-44)^2 + (-22)^2}$

$= \sqrt{1089 + 1936 + 484}$

$= \sqrt{3509}$

= 59.24

Therefore distance of point P from the given plane measured parallel to the given line is

d = 59.24 units



**26. Question**

Prove that the normals to the planes 4x + 11y + 2z + 3 = 0 and 3x - 2y + 5z = 8 are perpendicular to each other.

**Answer**

Given :

Equations of plane are :

4x + 11y + 2z + 3 = 0

3x – 2y + 5z = 8

To Prove : $\overline{n_1}$ & $\overline{n_2}$ are perpendicular.

Formula :

**1)** Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

Note :

Direction ratios of the plane given by

ax + by + cz = d

are (a, b, c).

For plane

4x + 11y + 2z + 3 = 0

direction ratios of normal vector are (4, 11, 2)

therefore, equation of normal vector is

$\overline{n_1} = 4\hat{\imath} + 11\hat{\jmath} + 2\hat{k}$

And for plane

3x - 2y + 5z = 8

direction ratios of the normal vector are (3, -2, 5)

therefore, the equation of normal vector is

$\overline{n_2} = 3\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$

Now,

$\overline{n_1}.\overline{n_2} = (4\hat{\imath} + 11\hat{\jmath} + 2\hat{k}).(3\hat{\imath} - 2\hat{\jmath} + 5\hat{k})$

$= (4 \times 3) + (11 \times (-2)) + (2 \times 5)$

= 12 - 22 + 10

= 0

$\therefore \overline{n_1}.\overline{n_2} = 0$

Therefore, normals to the given planes are perpendicular.

**27. Question**

Show that the line $\bar{r} = (2\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ is parallel to the plane $\bar{r} \cdot (\hat{i} + 5\hat{j} + \hat{k}) = 7$.

**Answer**

Given :

Equation of plane : : $\bar{r}.(\hat{\imath} + 5\hat{\jmath} + \hat{k}) = 7$

Equation of a line :

$\bar{r} = (2\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) + \lambda(\hat{\imath} - \hat{\jmath} + 4\hat{k})$

To Prove : Given line is parallel to the given plane.

Comparing given plane i.e.

$\bar{r}.(\hat{\imath} + 5\hat{\jmath} + \hat{k}) = 7$

with $\bar{r}.\bar{n} = \bar{a}.\bar{n}$ , we get,

$\bar{n} = \hat{\imath} + 5\hat{\jmath} + \hat{k}$

This is the vector perpendicular to the given plane.

Now, comparing the given the equation of line i.e.

$\bar{r} = (2\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) + \lambda(\hat{\imath} - \hat{\jmath} + 4\hat{k})$

with $\bar{r} = \bar{a} + \lambda\bar{b}$ , we get,

$\bar{b} = \hat{\imath} - \hat{\jmath} + 4\hat{k}$

Now,

$\bar{n}.\bar{b} = (\hat{\imath} + 5\hat{\jmath} + \hat{k}).(\hat{\imath} - \hat{\jmath} + 4\hat{k})$

$= (1 \times 1) + (5 \times (-1)) + (1 \times 4)$

= 1 - 5 + 4

= 0

$\therefore \bar{n}.\bar{b} = 0$

Therefore, a vector normal to the plane is perpendicular to the vector parallel to the line.

Hence, the given line is parallel to the given plane.

**28. Question**

Find the equation of a plane which is at a distance of 3√3 units from the origin and the normal to which is equally inclined to the coordinate axes.

**Answer**

Given :

$d = 3\sqrt{3}$

$\alpha = \beta = \gamma$

To Find : Equation of plane

Formulae :

**1)** Distance of plane from the origin :

If $\bar{n} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ is the vector normal to the plane, then distance of the plane from the origin is

$$d = \frac{p}{|\bar{n}|}$$

Where, $|\bar{n}| = \sqrt{a^2 + b^2 + c^2}$

**2)** $l^2 + m^2 + n^2 = 1$

Where $l = \cos\alpha, m = \cos\beta, n = \cos\gamma$

**3)** Equation of plane :

If $\bar{n} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ is the vector normal to the plane, then equation of the plane is

$$\bar{r}.\bar{n} = p$$

As $\alpha = \beta = \gamma$

$\therefore \cos\alpha = \cos\beta = \cos\gamma$

⇒ l = m = n

$l^2 + m^2 + n^2 = 1$

$\therefore 3l^2 = 1$

$\therefore l = \frac{1}{\sqrt{3}}$

Therefore equation of normal vector of the plane having direction cosines l, m, n is

$\bar{n} = l\hat{\imath} + m\hat{\jmath} + n\hat{k}$

$\therefore \bar{n} = \frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{1}{\sqrt{3}}\hat{k}$

$\therefore |\bar{n}| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2}$

$= \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}}$

$= \sqrt{1}$

$= 1$

Now,

distance of the plane from the origin is

$$d = \frac{p}{|\bar{n}|}$$

$\therefore 3\sqrt{3} = \frac{p}{1}$

$\therefore p = 3\sqrt{3}$

Therefore equation of required plane is

$\bar{r}.\bar{n} = p$

$\therefore (x\hat{\imath} + y\hat{\jmath} + z\hat{k}).\left(\frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{1}{\sqrt{3}}\hat{k}\right) = 3\sqrt{3}$

$\therefore \frac{x}{\sqrt{3}} + \frac{y}{\sqrt{3}} + \frac{z}{\sqrt{3}} = 3\sqrt{3}$

$\therefore x + y + z = 3\sqrt{3}.\sqrt{3}$

$\therefore x + y + z = 9$

This is the required equation of the plane.

**29. Question**

A vector $\bar{n}$ of magnitude 8 units is inclined to the x-axis at $45^0$, y-axis at $60^0$ and an acute angle with the z-axis, if a plane passes through a point (√2, -1, 1) and is normal to find its equation in vector form.

**Answer**

Given :

$|\bar{n}| = 8$

$\alpha = 45°$

$\beta = 60°$

P = (√2, -1, 1)

To Find : Equation of plane

Formulae :

**1)** $l^2 + m^2 + n^2 = 1$

Where $l = \cos\alpha\,, m = \cos\beta\,, n = \cos\gamma$

**2)** Equation of plane :

If $\bar{n} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ is the vector normal to the plane, then equation of the plane is

$\bar{r}.\bar{n} = p$

As $\alpha = 45°\ \&\ \beta = 60°$

$\therefore l = \cos\alpha = \cos 45° = \frac{1}{\sqrt{2}}$ and

$m = \cos\beta = \cos 60° = \frac{1}{2}$

But, $l^2 + m^2 + n^2 = 1$

$\therefore \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + n^2 = 1$

$\therefore \frac{1}{2} + \frac{1}{4} + n^2 = 1$

$\therefore n^2 = 1 - \frac{3}{4}$

$\therefore n^2 = \frac{1}{4}$

$\therefore n = \frac{1}{2}$

Therefore direction cosines of the normal vector of the plane are (l, m, n)

Hence direction ratios are (kl, km, kn)

Therefore the equation of normal vector is

$\bar{n} = kl\hat{\imath} + km\hat{\jmath} + kn\hat{k}$

$\therefore |\bar{n}| = \sqrt{(kl)^2 + (km)^2 + (kn)^2}$

$\therefore |\bar{n}| = \sqrt{\left(\frac{k}{\sqrt{2}}\right)^2 + \left(\frac{k}{2}\right)^2 + \left(\frac{k}{2}\right)^2}$

$\therefore 8 = \sqrt{\frac{k^2}{2} + \frac{k^2}{4} + \frac{k^2}{4}}$

$\therefore 8 = \sqrt{k^2}$

$\therefore k = 8$

$\bar{n} = \left(\frac{8}{\sqrt{2}}\right)\hat{\imath} + \left(\frac{8}{2}\right)\hat{\jmath} + \left(\frac{8}{2}\right)\hat{k}$

$\therefore \bar{n} = 4\sqrt{2}\hat{\imath} + 4\hat{\jmath} + 4\hat{k}$

Now, equation of the plane is

$\bar{r}.\bar{n} = p$

$\therefore \bar{r}.\left(4\sqrt{2}\hat{\imath} + 4\hat{\jmath} + 4\hat{k}\right) = p$ .........eq(1)

But $\bar{r} = (x\hat{\imath} + y\hat{\jmath} + z\hat{k})$

$\therefore (x\hat{\imath} + y\hat{\jmath} + z\hat{k}).(4\sqrt{2}\hat{\imath} + 4\hat{\jmath} + 4\hat{k}) = p$

⇒4√2x + 4y + 4z = p

As point P (√2, -1, 1) lies on the plane by substituting it in above equation,

4√2(√2) + 4(-1) + 4(1) = p

⇒8 - 4 + 4 = p

⇒P = 8

From eq(1)

$\therefore \bar{r}.(4\sqrt{2}\hat{\imath} + 4\hat{\jmath} + 4\hat{k}) = 8$

Dividing throughout by 4

$\therefore \bar{r}.(\sqrt{2}\hat{\imath} + \hat{\jmath} + \hat{k}) = 2$

This is the equation of required plane.

**30. Question**

Find the vector equation of a line passing through the point $(2\hat{i} - 3\hat{j} - 5\hat{k})$ and perpendicular to the plane $\vec{r}\cdot(6\hat{i} - 3\hat{j} + 5\hat{k}) + 2 = 0.$

Also, find the point of intersection of this line and the plane.

**Answer**

Given :

$\bar{a} = 2\hat{\imath} - 3\hat{\jmath} - 5\hat{k}$

Equation of plane : $\bar{r}.(6\hat{\imath} - 3\hat{\jmath} + 5\hat{k}) = -2$

To Find :

Equation of line

Point of intersection

Formula :

Equation of line passing through point A with position vector $\bar{a}$ and parallel to vector $\bar{b}$ is

$\bar{r} = \bar{a} + \lambda\bar{b}$

Where, $\bar{r} = (x\hat{\imath} + y\hat{\jmath} + z\hat{k})$

From the given equation of the plane

$\bar{r}.(6\hat{\imath} - 3\hat{\jmath} + 5\hat{k}) = -2$ .........eq(1)

The normal vector of the plane is

$\bar{n} = 6\hat{\imath} - 3\hat{\jmath} + 5\hat{k}$

As the given line is perpendicular to the plane therefore $\bar{n}$ will be parallel to the line.

$\therefore \bar{n} = \bar{b}$

Now, the equation of the line passing through $\bar{a} = (2\hat{\imath} - 3\hat{\jmath} - 5\hat{k})$ and parallel to $\bar{b} = (6\hat{\imath} - 3\hat{\jmath} + 5\hat{k})$ is

$\bar{r} = \bar{a} + \lambda\bar{b}$

$\therefore \bar{r} = (2\hat{\imath} - 3\hat{\jmath} - 5\hat{k}) + \lambda(6\hat{\imath} - 3\hat{\jmath} + 5\hat{k})$

.........eq(2)

This is the required equation line.

Substituting $\bar{r} = (x\hat{\imath} + y\hat{\jmath} + z\hat{k})$ in eq(1)

$(x\hat{\imath} + y\hat{\jmath} + z\hat{k}).(6\hat{\imath} - 3\hat{\jmath} + 5\hat{k}) = -2$

⇒6x - 3y + 5z = -2 .........eq(3)

Also substituting $\bar{r} = (x\hat{\imath} + y\hat{\jmath} + z\hat{k})$ in eq(2)

$(x\hat{\imath} + y\hat{\jmath} + z\hat{k}) = (2\hat{\imath} - 3\hat{\jmath} - 5\hat{k}) + \lambda(6\hat{\imath} - 3\hat{\jmath} + 5\hat{k})$

$\therefore (6\hat{\imath} - 3\hat{\jmath} + 5\hat{k})\lambda = (x\hat{\imath} + y\hat{\jmath} + z\hat{k}) - (2\hat{\imath} - 3\hat{\jmath} - 5\hat{k})$

$\therefore 6\lambda\hat{\imath} - 3\lambda\hat{\jmath} + 5\lambda\hat{k} = (x-2)\hat{\imath} + (y+3)\hat{\jmath} + (z+5)\hat{k}$

Comparing coefficients of $\hat{\imath}, \hat{\jmath}$ & $\hat{k}$

$\Rightarrow 6\lambda = (x-2), -3\lambda = (y+3)$ & $5\lambda = (z+5)$

$\lambda = \frac{x-2}{6} = \frac{y+3}{-3} = \frac{z+5}{5}$ .........eq(4)

Let Q(a, b, c) be the point of intersection of given line and plane

As point Q lies on the given line.

Therefore from eq(4)

$$\frac{a-2}{6} = \frac{b+3}{-3} = \frac{c+5}{5} = k(say)$$

⇒a = 6k+2, b = -3k-3, c = 5k-5

Also point Q lies on the plane.

Therefore from eq(3)

6a - 3b + 5c = -2

⇒6(6k+2) - 3(-3k-3) + 5(5k-5) = -2

⇒36k + 12 + 9k + 9 + 25k - 25 = -2

⇒70k = 2

$\Rightarrow k = \frac{1}{35}$

$$\therefore a = 6\left(\frac{1}{35}\right) + 2 = \frac{76}{35}$$

$$b = -3\left(\frac{1}{35}\right) - 3 = \frac{-108}{35}$$

$$c = 5\left(\frac{1}{35}\right) - 5 = \frac{-170}{35} = \frac{-34}{7}$$

Therefore co-ordinates of the point of intersection of line and plane are

Q ≡ $\left(\frac{76}{35}, \frac{-108}{35}, \frac{-34}{7}\right)$



## Exercise 28C

**1. Question**

Find the distance of the point $\left(2\hat{i} - \hat{j} - 4\hat{k}\right)$ from the plane $\vec{r} \cdot \left(3\hat{i} - 4\hat{j} + 12\hat{k}\right) = 9$.

**Answer**

Formula : $Distance\ = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$

where $(x_1, y_1, z_1)$ is point from which distance is to be calculated

Therefore ,

Plane $r.(3i - 4j\ +\ 12k)\ =\ 9$ can be written in cartesian form as

3x - 4y + 12z = 9

3x - 4y + 12z - 9 = 0

Point = (2i - j - 4k)

Which can be also written as

Point = ( 2 , - 1 , - 4 )

Distance = $\frac{|(2\times 3) + (-1\times -4) + (-4\times 12) + (-9)|}{\sqrt{(3)^2 + (-4)^2 + 12^2}}$

$$= \frac{|6\ +\ 4 - 48 - 9|}{\sqrt{9\ +\ 16\ +\ 144}}$$

$$= \frac{|-47|}{\sqrt{169}}$$

$$= \frac{47}{13}\ units$$

**2. Question**

Find the distance of the point $\left(\hat{i} + 2\hat{j} + 5\hat{k}\right)$ from the plane $\vec{r} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) + 17 = 0$.

**Answer**

Formula : $Distance = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$

where $(x_1, y_1, z_1)$ is point from which distance is to be calculated

Therefore ,

Plane $r.(i + j + k) + 17 = 0$ can be written in cartesian form as

x + y + z + 17 = 0

Point = (i + 2j + 5k)

Which can be also written as

Point = ( 1 , 2 , 5 )

Distance = $\frac{|(1\times1) + (2\times1) + (5\times1) + (17)|}{\sqrt{(1)^2 + (1)^2 + 1^2}}$

$= \frac{|1 + 2 + 5 + 17|}{\sqrt{1 + 1 + 1}}$

$= \frac{|25|}{\sqrt{3}}$

$= \frac{25\sqrt{3}}{3}\ units$

**3. Question**

Find the distance of the point (3, 4, 5) from the plane $\vec{r} \cdot (2\hat{i} - 5\hat{j} + 3\hat{k}) = 13$.

**Answer**

Formula : $Distance = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$

where $(x_1, y_1, z_1)$ is point from which distance is to be calculated

Therefore ,

Plane $r.(2i - 5j + 3k) = 13$ can be written in cartesian form as

2x - 5y + 3z = 13

2x - 5y + 3z - 13 = 0

Point = ( 3 , 4 , 5 )

Distance = $\frac{|(3\times2) + (4\times-5) + (5\times3)-(13)|}{\sqrt{(2)^2 + (-5)^2 + 3^2}}$

$= \frac{|6 - 20 + 15 - 13|}{\sqrt{4 + 25 + 9}}$

$= \frac{|-12|}{\sqrt{38}}$

$= \frac{12\sqrt{38}}{38} = \frac{6\sqrt{38}}{19}\ units$

**4. Question**

Find the distance of the point (1, 1, 2) from the plane $\vec{r} \cdot (2\hat{i} - 2\hat{j} + 4\hat{k}) + 5 = 0$.

**Answer**

Formula : $Distance = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$

where $(x_1, y_1, z_1)$ is point from which distance is to be calculated

Therefore ,

Plane $r.(2i - 2j + 4k) + 5 = 0$ can be written in cartesian form as

2x - 2y + 4z + 5 = 0

Point = ( 1 , 1 , 2 )

Distance = $\frac{|(1\times2) + (1\times-2) + (2\times4) + (5)|}{\sqrt{(2)^2 + (-2)^2 + (4)^2}}$

$= \frac{|2 - 2 + 8 + 5|}{\sqrt{4 + 4 + 16}}$

$= \frac{|13|}{\sqrt{24}}$

$= \frac{13}{2\sqrt{6}} = \frac{13\sqrt{6}}{12}\ units$

**5. Question**

Find the distance of the point (2, 1, 0) from the plane 2x + y + 2z + 5 = 0.

**Answer**

Formula : $Distance\ = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$

where $(x_1, y_1, z_1)$ is point from which distance is to be calculated

Therefore ,

2x + y + 2z + 5 = 0

Point = ( 2 , 1 , 0 )

Distance = $\frac{|(2\times2) + (1\times1) + (0\times2) + (5)|}{\sqrt{(2)^2 + (1)^2 + (2)^2}}$

$= \frac{|4 + 1 + 0 + 5|}{\sqrt{4 + 1 + 4}}$

$= \frac{|10|}{\sqrt{9}}$

$= \frac{10}{3}\ units$

**6. Question**

Find the distance of the point (2, 1, - 1) from the plane x – 2y + 4z = 9.

**Answer**

Formula : $Distance\ = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$

where $(x_1, y_1, z_1)$ is point from which distance is to be calculated

Therefore ,

x - 2y + 4z = 9

x - 2y + 4z – 9 = 0

Point = ( 2 , 1 , - 1 )

Distance = $\frac{|(2\times1) + (1\times-2) + (-1\times4)-(9)|}{\sqrt{(1)^2 + (-2)^2 + (4)^2}}$

$= \frac{|2 - 2 - 4 - 9|}{\sqrt{1 + 4 + 16}}$

$= \frac{|-13|}{\sqrt{21}}$

$= \frac{13}{\sqrt{21}} = \frac{13\sqrt{21}}{21}\ units$

**7. Question**

Show that the point (1, 2, 1) is equidistant from the planes $\vec{r}\cdot\left(\hat{i}+2\hat{j}-2\hat{k}\right)=5$ and $\vec{r}\cdot\left(2\hat{i}-2\hat{j}+\hat{k}\right)+3=0$.

**Answer**

Formula : $Distance\ = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$

where $(x_1, y_1, z_1)$ is point from which distance is to be calculated

Therefore ,

First Plane $r.(i\ +\ 2j - 2k)\ =\ 5$ can be written in cartesian form as

x + 2y - 2z = 5

x + 2y - 2z – 5 = 0

Point = ( 1 , 2 , 1 )

Distance for first plane = $\frac{|(1\times1)+(2\times2)+(1\times-2)-(5)|}{\sqrt{(1)^2+(2)^2+(-2)^2}}$

$= \frac{|1+4-2-5|}{\sqrt{1+4+4}}$

$= \frac{|-2|}{\sqrt{9}}$

$= \frac{2}{3}\ units$

Second Plane $r.(2i-2j+k)+3=0$ can be written in cartesian form as

2x - 2y + z + 3 = 0

Point = ( 1 , 2 , 1 )

Distance for second plane = $\frac{|(1\times2)+(2\times-2)+(1\times1)+(3)|}{\sqrt{(2)^2+(-2)^2+(1)^2}}$

$= \frac{|2-4+1+3|}{\sqrt{4+4+1}}$

$= \frac{|2|}{\sqrt{9}}$

$= \frac{2}{3}\ units$

Hence proved.

**8. Question**

Show that the points ( - 3, 0, 1) and (1, 1, 1) are equidistant from the plane 3x + 4y – 12z + 13 = 0.

**Answer**

Formula : $Distance = \frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}$

where $(x_1, y_1, z_1)$ is point from which distance is to be calculated

Therefore ,

Plane = 3x + 4y - 12z + 13 = 0

First Point = ( - 3 , 0 , 1 )

Distance for first point = $\frac{|(-3\times3)+(0\times4)+(1\times-12)+(13)|}{\sqrt{(3)^2+(4)^2+(-12)^2}}$

$= \frac{|-9+0-12+13|}{\sqrt{9+16+144}}$

$= \frac{|-8|}{\sqrt{169}}$

$= \frac{8}{13}\ units$

Plane = 3x + 4y - 12z + 13 = 0

Second Point = ( 1 , 1 , 1 )

Distance for first point = $\frac{|(1\times3)+(1\times4)+(1\times-12)+(13)|}{\sqrt{(3)^2+(4)^2+(-12)^2}}$

$= \frac{|3+4-12+13|}{\sqrt{9+16+144}}$

$= \frac{|8|}{\sqrt{169}}$

$= \frac{8}{13}\ units$

Hence proved.

**9. Question**

Find the distance between the parallel planes 2x + 3y + 4 = 4 and 4x + 6y + 8z = 12.

**Answer**

Formula : The distance between two parallel planes, say

Plane 1:ax + by + cz + d1 = 0 &

Plane 2:ax + by + cz + d2 = 0 is given by the formula

$$Distance = \frac{|d_2 - d_1|}{\sqrt{a^2 + b^2 + c^2}}$$

where $(d_1, d_2)$ are constants of the planes

Therefore ,

First Plane 2x + 3y + 4 = 4

2x + 3y + 4 – 4 = 0 …… (1)

Second plane 4x + 6y + 8z = 12

4x + 6y + 8z - 12 = 0

2(2x + 3y + 4z – 6) = 0

2x + 3y + 4z – 6 = 0 …… (2)

Using equation (1) and (2)

Distance between both planes = $\frac{|-6-(-4)|}{\sqrt{(2)^2+(3)^2+(4)^2}}$

$$= \frac{|-6 + 4|}{\sqrt{4 + 9 + 16}}$$

$$= \frac{|-2|}{\sqrt{29}}$$

$$= \frac{2}{\sqrt{29}} = \frac{2\sqrt{29}}{29}\ units$$

**10. Question**

Find the distance between the parallel planes x + 2y - 2z + 4 = 0 and x + 2y - 2z - 8 = 0.



**Answer**

Formula : The distance between two parallel planes, say

Plane 1:ax + by + cz + d1 = 0 &

Palne 2:ax + by + cz + d2 = 0 is given by the formula

$$Distance = \frac{|d_2 - d_1|}{\sqrt{a^2 + b^2 + c^2}}$$

where $(d_1, d_2)$ are costants of the planes

Therefore ,

First Plane x + 2y - 2z + 4 = 0 …… (1)

Second plane x + 2y – 2z – 8 = 0 …… (2)

Using equation (1) and (2)

Distance between both planes = $\frac{|-8-(4)|}{\sqrt{(1)^2+(2)^2+(2)^2}}$

$$= \frac{|-12|}{\sqrt{1 + 4 + 4}}$$

$$= \frac{12}{\sqrt{9}}$$

$$= \frac{12}{3} = 4\ units$$

**11. Question**

Find the equation of the planes parallel to the plane x – 2y + 2z – 3 = 0, each one of which is at a unit distance from the point (1, 1, 1).

**Answer**

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same

Therefore ,

Parallel Plane x – 2y + 2z – 3 = 0

Normal vector = (i - 2j + 2k)

∴ Normal vector of required plane = (i - 2j + 2k)

Equation of required planes r . (i - 2j + 2k) = d

In cartesian form x – 2y + 2y = d

It should be at unit distance from point (1,1,1)

Distance = $\frac{|(1\times1)+(1\times-2)+(1\times2)-(d)|}{\sqrt{(1)^2+(-2)^2+(2)^2}}$

$= \frac{|1-2\ +\ 2-d|}{\sqrt{1\ +\ 4\ +\ 4}}$

$= \frac{|1-d|}{\sqrt{9}}$

$1\ = \frac{\pm(1-d)}{3}$

$3\ =\ \pm(1-d)$

For + sign = > 3 = 1 - d = > d = - 2

For - sign = > 3 = - 1 + d = > d = 4

Therefore equations of planes are : -

For d = - 2 For d = 4

x – 2y + 2y = d x – 2y + 2y = d

x – 2y + 2y = - 2 x – 2y + 2y = 4

x – 2y + 2y + 2 = 0 x – 2y + 2y - 4 = 0

Required planes = x – 2y + 2y + 2 = 0

x – 2y + 2y – 4 = 0

**12. Question**

Find the equation of the plane parallel to the plane 2x – 3y + 5z + 7 = 0 and passing through the point (3, 4, - 1). Also, find the distance between the two planes.

**Answer**

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

The distance between two parallel planes, say

Plane 1:ax + by + cz + d1 = 0 &

Palne 2:ax + by + cz + d2 = 0 is given by the formula

$$Distance\ = \frac{|d_2 - d_1|}{\sqrt{a^2\ +\ b^2\ +\ c^2}}$$

If two planes are parallel , then their normal vectors are same

Therefore ,

Parallel Plane 2x – 3y + 5z + 7 = 0

Normal vector = (2i - 3j + 5k)

∴ Normal vector of required plane = (2i - 3j + 5k)

Equation of required plane r . (2i - 3j + 5k) = d

In cartesian form 2x – 3y + 5y = d

Plane passes through point (3,4, - 1) therefore it will satisfy it.

2(3) – 3(4) + 5( - 1) = d

6 – 12 - 5 = d

d = - 11

Equation of required plane 2x – 3y + 5z = - 11

2x – 3y + 5z + 11 = 0

Therefore ,

First Plane 2x – 3y + 5z + 7 = 0 ...... (1)

Second plane 2x – 3y + 5z + 11 = 0 ...... (2)

Using equation (1) and (2)

Distance between both planes = $\frac{|11-(7)|}{\sqrt{(2)^2+(-3)^2+(5)^2}}$

$= \frac{|4|}{\sqrt{4 + 9 + 25}}$

$= \frac{4}{\sqrt{38}}$

$= \frac{4\sqrt{38}}{38} = \frac{2\sqrt{38}}{19}\ units$

**13. Question**

Find the equation of the plane mid - parallel to the planes 2x – 3y + 6z + 21 = 0 and 2x – 3y + 6z – 14 = 0

**Answer**

Formula : The equation of mid parallel plane is , say

Plane 1:ax + by + cz + d1 = 0 &

Plane 2:ax + by + cz + d2 = 0 is given by the formula

$Mid\ parallel\ plane\ = ax\ +\ by\ +\ cy\ +\ \frac{(d_1\ +\ d_2)}{2}\ =\ 0$

where $(d_1, d_2)$ are constants of the planes

Therefore ,

First Plane 2x – 3y + 6z + 21 = 0 ...... (1)

Second plane 2x – 3y + 6z – 14 = 0 ...... (2)

Using equation (1) and (2)

Mid parallel plane = $2x - 3y\ +\ 6z\ +\ \frac{21-14}{2}\ =\ 0$

4x – 6y + 12z + 7 = 0

## Exercise 28D

**1. Question**

Show that the planes 2x – y + 6z = 5 and 5x – 2.5y + 15z = 12 are parallel.

**Answer**

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are either same or proportional to each other

Therefore ,

Plane 1 : - 2x – y + 6z = 5

Normal vector (Plane 1) = (2i – j + 6k) .....(1)

Plane 2 : - 5x – 2.5y + 15z = 12

Normal vector (Plane 2) = (5i – 2.5j + 15k) .....(2)

Multiply equation(1) by 5 and equation(2) by 2

Normal vector (Plane 1) = 5(2i – j + 6k)

= 10i – 5j + 30k

Normal vector (Plane 2) = 2(5i – 2.5j + 15k)

= 10i – 5j + 30k

Since, both normal vectors are same .Therefore both planes are parallel

**2. Question**

Find the vector equation of the plane through the point $\left(3\hat{i}+4\hat{j}-\hat{k}\right)$ and parallel to the plane $\vec{r}\cdot\left(2\hat{i}-3\hat{j}+5\hat{k}\right)+5=0.$

**Answer**

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same.

Therefore ,

Parallel Plane r . (2i – 3j + 5k) + 5 = 0

Normal vector = (2i - 3j + 5k)

∴ Normal vector of required plane = (2i - 3j + 5k)

Equation of required plane r . (2i - 3j + 5k) = d

In cartesian form 2x – 3y + 5z = d

Plane passes through point (3,4, - 1) therefore it will satisfy it.

2(3) – 3(4) + 5( - 1) = d

6 – 12 - 5 = d

d = - 11

Equation of required plane r . (2i - 3j + 5k) = - 11

r . (2i - 3j + 5k) + 11 = 0



**3. Question**

Find the vector equation of the plane passing through the point (a, b, b) and parallel to the plane $\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=2.$

There is a error in question ...... the point should be (a,b,c) instead of (a,b,b) to get the required answer.

**Answer**

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same.

Therefore ,

Parallel Plane r . (i + j + k) = 2

Normal vector = (i + j + k)

∴ Normal vector of required plane = (i + j + k)

Equation of required plane r . (i + j + k) = d

In cartesian form x + y + z = d

Plane passes through point (a,b,c) therefore it will satisfy it.

(a) + (b) + (c) = d

d = a + b + c

Equation of required plane r . (i + j + k) = a + b + c

**4. Question**

Find the vector equation of the plane passing through the point (1, 1, 1) and parallel to the plane $\vec{r}\cdot\left(2\hat{i}-\hat{j}+2\hat{k}\right)=5.$

**Answer**

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same.

Therefore ,

Parallel Plane r . (2i - j + 2k) = 5

Normal vector = (2i - j + 2k)

∴ Normal vector of required plane = (2i - j + 2k)

Equation of required plane r . (2i - j + 2k) = d

In cartesian form 2x - y + 2z = d

Plane passes through point (1,1,1) therefore it will satisfy it.

2(1) - (1) + 2(1) = d

d = 2 – 1 + 2 = 3

Equation of required plane r . (2i - j + 2k) = 3

**5. Question**

Find the equation of the plane passing through the point (1, 4, - 2) and parallel to the plane 2x – y + 3z + 7 = 0.

**Answer**

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same.

Therefore ,

Parallel Plane 2x – y + 3z + 7 = 0

Normal vector = (2i - j + 3k)

∴ Normal vector of required plane = (2i - j + 3k)

Equation of required plane r . (2i - j + 3k) = d

In cartesian form 2x - y + 3z = d

Plane passes through point (1,4, - 2) therefore it will satisfy it.

2(1) - (4) + 3( - 2) = d

d = 2 – 4 – 6 = - 8

Equation of required plane 2x - y + 3z = - 8

2x - y + 3z + 8 = 0

**6. Question**

Find the equations of the plane passing through the origin and parallel to the plane 2x – 3y + 7z + 13 = 0.

**Answer**

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same.

Therefore ,

Parallel Plane 2x – 3y + 7z + 13 = 0

Normal vector = (2i - 3j + 7k)

∴ Normal vector of required plane = (2i - 3j + 7k)

Equation of required plane r . (2i - 3j + 7k) = d

In cartesian form 2x - 3y + 7z = d

Plane passes through point (0,0,0) therefore it will satisfy it.

2(0) - (0) + 3(0) = d

d = 0

Equation of required plane 2x - 3y + 7z = 0

**7. Question**

Find the equations of the plane passing through the point ( - 1, 0, 7) and parallel to the plane 3x – 5y + 4z = 11.

**Answer**

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same.

Therefore ,

Parallel Plane 3x – 5y + 4z = 11

Normal vector = (3i - 5j + 4k)

∴ Normal vector of required plane = (3i - 5j + 4k)

Equation of required plane r . (3i - 5j + 4k) = d

In cartesian form 3x - 5y + 4z = d

Plane passes through point ( - 1,0,7) therefore it will satisfy it.

3( - 1) - 5(0) + 4(7) = d

d = - 3 + 28 = 25

Equation of required plane 3x - 5y + 4z = 25

**8. Question**

Find the equations of planes parallel to the plane x - 2y + 2z = 3 which are at a unit distance from the point (1, 2, 3).



**Answer**

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same

Therefore ,

Parallel Plane x – 2y + 2z – 3 = 0

Normal vector = (i - 2j + 2k)

∴ Normal vector of required plane = (i - 2j + 2k)

Equation of required planes r . (i - 2j + 2k) = d

In cartesian form x – 2y + 2y = d

It should be at unit distance from point (1,2,3)

Distance = $\frac{|(1\times1)+(2\times-2)+(3\times2)-(d)|}{\sqrt{(1)^2+(-2)^2+(2)^2}}$

$= \frac{|1-4+6-d|}{\sqrt{1+4+4}}$

$= \frac{|3-d|}{\sqrt{9}}$

$1 = \frac{\pm(3-d)}{3}$

$3 = \pm(3-d)$

For + sign = > 3 = 3 - d = > d = 0

For - sign = > 3 = - 3 + d = > d = 6

Therefore equations of planes are : -

For d = 0 For d = 6

x - 2y + 2y = d x - 2y + 2y = d

x - 2y + 2y = 0 x - 2y + 2y = 6

Required planes = x - 2y + 2y = 0

x - 2y + 2y - 6 = 0

**9. Question**

Find the distance between the planes x + 2y + 3z + 7 = 0 and 2x + 4y + 6z + 7 = 0.

**Answer**

Formula : The distance between two parallel planes, say

Plane 1:ax + by + cz + d1 = 0 &

Plane 2:ax + by + cz + d2 = 0 is given by the formula

$$Distance = \frac{|d_2 - d_1|}{\sqrt{a^2 + b^2 + c^2}}$$

where $(d_1, d_2)$ are constants of the planes

Therefore ,

First Plane x + 2y + 3z + 7 = 0

2(x + 2y + 3z + 7) = 0

2x + 4y + 6z + 14 = 0 ...... (1)

Second plane 2x + 4y + 6z + 7 = 0 ...... (2)

Using equation (1) and (2)

Distance between both planes = $\frac{|7-(14)|}{\sqrt{(2)^2+(4)^2+(6)^2}}$

$$= \frac{|-7|}{\sqrt{4 + 16 + 36}}$$

$$= \frac{|-7|}{\sqrt{56}}$$

$$= \frac{7}{\sqrt{56}}\ units$$

## Exercise 28E

**1. Question**

Find the equation of the plane through the line of intersection of the planes x + y + z = 6 and 2x + 2y + 4z + 5 = 0, and passing through the point (1, 1, 1).

**Answer**

Equation of plane through the line of intersection of planes in Cartesian form is

$$A_1x + B_1y + C_1z + D_1 + \lambda(A_2x + B_2y + C_2z + D_2) = 0 \ (1)$$

For the standard equation of planes,

$$A_1x + B_1y + C_1z + D_1 \text{ and } A_2x + B_2y + C_2z + D_2$$

So, putting in equation (1), we have

x + y + z-6 + λ(2x + 2y + 4z + 5)=0

(1 + 2λ)x + (1 + 2λ)y + (1 + 4λ)z-6 + 5λ=0 (2)

Now plane passes through (1,1,1) then it must satisfy the plane equation,

(1 + 2λ).1 + (1 + 2λ).1 + (1 + 4λ).1-6 + 5λ=0

1 + 2λ + 1 + 2λ + 1 + 4λ-6 + 5λ=0

3 + 8λ-6 + 5λ=0

13λ=3

$$\lambda = \frac{3}{13}$$

Putting in equation (2)

$$\left(1+2.\frac{3}{13}\right)x+\left(1+2.\frac{3}{13}\right)y+\left(1+4.\frac{3}{13}\right)z-6+5.\frac{3}{13}=0$$

$$\left(\frac{13+6}{13}\right)x+\left(\frac{13+6}{13}\right)y+\left(\frac{13+12}{13}\right)z+\frac{-78+15}{13}=0$$

19x + 19y + 25z-63=0

So, the required equation of plane is 19x + 19y + 25z=63.

**2. Question**

Find the equation of the plane through the line of intersection of the planes x - 3y + z + 6 = 0 and x + 2y + 3z + 5 = 0, and passing through the origin.

**Answer**

Equation of plane through the line of intersection of planes in Cartesian form is

$A_1x+B_1y+C_1z+D_1+\lambda(A_2x+B_2y+C_2z+D_2)=0$ (1)

For the standard equation of planes,

$A_1x+B_1y+C_1z+D_1$ and $A_2x+B_2y+C_2z+D_2$

So, putting in equation (1), we have

x-3y + z + 6 + λ(x + 2y + 3z + 5)=0

(1 + λ)x + (-3 + 2λ)y + (1 + 3λ)z + 6 + 5λ=0 (2)

Now plane passes through (0,0,0) then it must satisfy the plane equation,

(1 + λ).0 + (-3 + 2λ).0 + (1 + 3λ).0 + 6 + 5λ=0

5λ=-6

$$\lambda=\frac{-6}{5}$$

Putting in equation (2)

$$\left(1+\frac{-6}{5}\right)x+\left(-3+2.\frac{-6}{5}\right)y+\left(1+3.\frac{-6}{5}\right)z+6+5.\frac{-6}{5}=0$$

$$\left(\frac{5+(-6)}{5}\right)x+\left(\frac{-15-12}{5}\right)y+\left(\frac{5+(-18)}{5}\right)z+\frac{30+(-30)}{5}=0$$

-x-27y-13z=0

x + 27y + 13z=0

So, required equation of plane is x + 27y + 13z=0.

**3. Question**

Find the equation of the plane passing through the intersection of the planes 2x + 3y - z + 1 = 0 and x + y - 2z + 3 = 0, and perpendicular to the plane 3x - y -2z -4 = 0.

**Answer**

Equation of plane through the line of intersection of planes in Cartesian form is

$A_1x+B_1y+C_1z+D_1+\lambda(A_2x+B_2y+C_2z+D_2)=0$ (1)

For the standard equation of planes,

$A_1x+B_1y+C_1z+D_1$ and $A_2x+B_2y+C_2z+D_2$

So, putting in equation (1), we have

2x + 3y-z + 1 + λ(x + y-2z + 3)=0

(2 + λ)x + (3 + λ)y + (-1-2λ)z + 1 + 3λ=0 (2)

Now as the plane 3x-y-2z-4=0 is perpendicular the given plane,

For θ=90°, cos90°=0

$A_1A_2+B_1B_2+C_1C_2=0$ (3)

On comparing with standard equations in Cartesian form,

$A_1=2+\lambda, B_1=3+\lambda, C_1=-1-2\lambda$ and $A_2=3, B_2=-1, C_2=-2$

Putting values in equation (3), we have

(2 + λ).3 + (3 + λ).(-1) + (-1-2λ).(-2)=0

6 + 3λ-3-λ + 2 + 4λ=0

5 + 6λ=0

$$\lambda = \frac{-5}{6}$$

Putting in equation(2)

$$\left(2+\frac{-5}{6}\right)x+\left(3+\frac{-5}{6}\right)y+\left(-1-2.\frac{-5}{6}\right)z+1+3.\frac{-5}{6}=0$$

$$\left(\frac{12-5}{6}\right)x+\left(\frac{18-5}{6}\right)y+\left(\frac{-6+10}{6}\right)z+\frac{6-15}{6}=0$$

7x + 13y + 4z-9=0

7x + 13y + 4z=9

So, required equation of plane is 7x + 13y + 4z=9.

**4. Question**

Find the equation of the plane passing through the line of intersection of the planes 2x - y = 0 and 3z - y = 0, and perpendicular to the plane 4x + 5y - 3z = 9.

**Answer**

Equation of plane through the line of intersection of planes in Cartesian form is

$$A_1x+B_1y+C_1z+D_1+\lambda(A_2x+B_2y+C_2z+D_2)=0 \ (1)$$

For the standard equation of planes,

$$A_1x+B_1y+C_1z+D_1 \text{ and } A_2x+B_2y+C_2z+D_2$$

So, putting in equation (1), we have

2x-y + λ(3z-y)=0

2x + (-1-λ)y + 3λz=0 (2)

Now as the plane is perpendicular the given plane,

For θ=90°, cos90°=0

$$A_1A_2+B_1B_2+C_1C_2=0 \ (3)$$

On comparing with standard equations in Cartesian form,

$$A_1=2, B_1=-1-\lambda, C_1=3\lambda \text{ and } A_2=4, B_2=5, C_2=-3$$

Putting values in equation(3),

2.4 + (-1-λ).5 + 3λ.-3=0

8-5-5λ-9λ=0

-14λ=-3

$$\lambda=\frac{3}{14}$$

Putting in equation(2)

$$2x+\left(-1-\frac{3}{14}\right)y+3\left(\frac{3}{14}\right)z=0$$

$$2x+\left(\frac{-14-3}{14}\right)y+\frac{9}{14}.z=0$$

28x-17y + 9z=0

So, required equation of plane is 28x-17y + 9z=0.

**5. Question**

Find the equation of the plane passing through the intersection of the planes x - 2y + z = 1 and 2x + y + z = 8, and parallel to the line with direction ratios 1, 2, 1. Also, find the perpendicular distance of (1, 1, 1) from the plane.

**Answer**

Equation of plane through the line of intersection of planes in Cartesian form is

$$A_1x+B_1y+C_1z+D_1+\lambda(A_2x+B_2y+C_2z+D_2)=0 \ (1)$$

For the standard equation of planes,

$$A_1x+B_1y+C_1z+D_1 \text{ and } A_2x+B_2y+C_2z+D_2$$

So, putting in equation (1), we have

x-2y + z-1 + λ(2x + y + z-8)=0

(1 + 2λ)x + (-2 + λ)y + (1 + λ)z-1-8λ=0 (2)

For plane the normal is perpendicular to line given parallel to this i.e.

$$A_1A_2+B_1B_2+C_1C_2=0$$

Where $A_1$, $B_1$, $C_1$ are direction ratios of plane and $A_2$, $B_2$, $C_2$ are of line.

(1 + 2λ).1 + (-2 + λ).2 + (1 + λ).1=0

1 + 2λ-4 + 2λ + 1 + λ=0

-2 + 5λ=0

$$\lambda=\frac{2}{5}$$

Putting the value of λ in equation (2)

$$\left(1+2.\left(\frac{2}{5}\right)\right).x+\left(-2+\frac{2}{5}\right).y+\left(1+\frac{2}{5}\right).z-1-8.\left(\frac{2}{5}\right)=0$$

$$\left(\frac{5+4}{5}\right)x+\left(\frac{-10+2}{5}\right)y+\left(\frac{5+2}{5}\right)z+\frac{-5-16}{5}=0$$

9x-8y + 7z-21=0

9x-8y + 7z=21

For the equation of plane Ax + By + Cz=D and point (x1,y1,z1), a distance of a point from a plane can be calculated as

$$\left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right|$$

$$\left|\frac{9.1-8.1+7.1-21}{\sqrt{(9)^2+(-8)^2+(7)^2)}}\right| \Rightarrow \left|\frac{9-8+7-21}{\sqrt{81+64+49}}\right|=\left|\frac{13}{\sqrt{194}}\right|$$

So, the required equation of the plane is 9x-8y + 7z=21, and distance of the plane from (1,1,1) is

$$d=\frac{13}{\sqrt{194}}$$

**6. Question**

Find the equation of the plane passing through the line of intersection of the planes x + 2y + 3z – 5 = 0 and 3x - 2y –z + 1 = 0 and cutting off equal intercepts on the x-axis and z-axis.

**Answer**

Equation of plane through the line of intersection of planes in Cartesian form is

$$A_1x+B_1y+C_1z+D_1+\lambda(A_2x+B_2y+C_2z+D_2)=0 \ (1)$$

For the standard equation of planes in Cartesian form

$$A_1x+B_1y+C_1z+D_1 \text{ and } A_2x+B_2y+C_2z+D_2$$

So, putting in equation 1 we have

x + 2y + 3 z-5 + λ(3x -2y -z + 1)=0

(1 + 3λ)x + (2-2λ)y + (3-λ)z-5 + λ=0

Now equation of plane in intercept form

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

As given equal intercept means a=c

First, we transform equation of a plane in intercept form

$$\frac{x}{\frac{1}{(1+3\lambda)}}+\frac{y}{\frac{1}{(2-2\lambda)}}+\frac{z}{\frac{1}{(3-\lambda)}}=5-\lambda$$

$$\frac{x}{\frac{5-\lambda}{(1+3\lambda)}}+\frac{y}{\frac{5-\lambda}{(2-2\lambda)}}+\frac{z}{\frac{5-\lambda}{(3-\lambda)}}=1$$

On comparing with the standard equation of a plane in intercept form

$$a=\frac{5-\lambda}{(1+3\lambda)},c=\frac{5-\lambda}{(3-\lambda)}$$

Now as a=b=c

$$\frac{5-\lambda}{(1+3\lambda)}=\frac{5-\lambda}{(3-\lambda)}\Rightarrow 3-\lambda=1+3\lambda$$

$$4\lambda=2\Rightarrow\lambda=\frac{1}{2}$$

Putting in equation (2), we have

$$\left(1+3.\frac{1}{2}\right)x+\left(2-2.\frac{1}{2}\right)y+\left(3-\frac{1}{2}\right)z-5+\frac{1}{2}=0$$

$$\left(\frac{2+3}{2}\right)x+\left(\frac{4-2}{2}\right)y+\left(\frac{6-1}{2}\right)z+\frac{-10+1}{2}=0$$

5x + 2y + 5z-9=0

5x + 2y + 5z=9

So, required equation of plane is 5x + 2y + 5z=9.



**7. Question**

Find the equation of the plane through the intersection of the planes 3x – 4y + 5z =10 and 2x + 2y - 3z = 4 and parallel to the line x = 2y = 3z.

**Answer**

Equation of plane through the line of intersection of planes in Cartesian form is

$$A_1x+B_1y+C_1z+D_1+\lambda(A_2x+B_2y+C_2z+D_2)=0 \text{ (1)}$$

For the standard equation of planes in Cartesian form

$$A_1x+B_1y+C_1z+D_1 \text{ and } A_2x+B_2y+C_2z+D_2$$

So, putting in equation (1), we have

3x – 4y + 5z -10 + λ(2x + 2y - 3z - 4)=0

(3 + 2λ)x + (-4 + 2λ)y + (5-3λ)z-10-4λ=0

Given line is parallel to plane then the normal of plane is perpendicular to line,

$$A_1A_2+B_1B_2+C_1C_2=0$$

Where $A_1$, $B_1$, $C_1$ are direction ratios of plane and $A_2$, $B_2$, $C_2$ are of line.

(3 + 2λ).6 + (-4 + 2λ).3 + (5-3λ).2=0

18 + 12λ-12 + 6λ + 10-6λ=0

16 + 12λ=0

$$\lambda=\frac{-16}{12}\Rightarrow\frac{-4}{3}$$

Putting the value of λ in equation (2)

$$\left(3+2.\left(\frac{-4}{3}\right)\right)x+\left(-4+2.\left(\frac{-4}{3}\right)\right)y+\left(5-3\left(\frac{-4}{3}\right)\right)z-10-4.\left(\frac{-4}{3}\right)=0$$

$\left(\frac{9-8}{3}\right)x+\left(\frac{-12-8}{3}\right)y+\left(\frac{15+12}{3}\right)z+\frac{-30+16}{3}=0$

x-20y + 27z-14=0

So, required equation of plane is x-20y + 27z-14=0.

**8. Question**

Find the vector equation of the plane through the intersection of the planes $\vec{r}.\left(\hat{i}+3\hat{j}-\hat{k}\right)=0$ and $\vec{r}.\left(\hat{j}+2\hat{k}\right)=0$, and passing through the point (2, 1, -1).

**Answer**

Equation of plane through the line of intersection of two planes in vector form is

$\vec{r}.(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ (1)

Where the standard equation of planes are

$\vec{r}.\vec{n}_1=d_1$ and $\vec{r}.\vec{n}_2=d_2$

Putting values in equation(1)

$\vec{r}(\hat{i}+3\hat{j}-\hat{k}+\lambda\left(\hat{j}+2\hat{k}\right)=0+\lambda.0$

$\vec{r}\left(\hat{i}+(3+\lambda)\hat{j}+(-1+2\lambda)\hat{k}\right)=0$ (2)

Now as the plane passes through (2,1,-1)

$\vec{r}=2\hat{i}+\hat{j}-\hat{k}$

Putting in equation (2)

$\left(2\hat{i}+\hat{j}-\hat{k}\right)(\left(\hat{i}+(3+\lambda)\hat{j}+(-1+2\lambda)\hat{k}\right)=0$

2.1 + 1.(3 + λ) + (-1)(-1 + 2λ)=0

2 + 3 + λ + 1-2λ=0

λ=6

Putting the value of λ in equation (2)

$\vec{r}\left(\hat{i}+(3+6)\hat{j}+(-1+2(6))\hat{k}\right)=0$

$\vec{r}\left(\hat{i}+9\hat{j}+11\hat{k}\right)=0$

So, required equation of plane is $\vec{r}\left(\hat{i}+9\hat{j}+11\hat{k}\right)=0$.

**9. Question**

Find the vector equation of the plane through the point (1, 1, 1), and passing through the intersection of the planes $\vec{r}.\left(\hat{i}-\hat{j}+3\hat{k}\right)+1=0$ and $\vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)-5=0$.

**Answer**

Equation of plane through the line of intersection of two planes in vector form is

$\vec{r}.(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ (1)

Where the standard equation of planes are

$\vec{r}.\vec{n}_1=d_1$ and $\vec{r}.\vec{n}_2=d_2$

Putting values in equation(1)

$\vec{r}(\hat{i}-\hat{j}+3\hat{k}+\lambda\left(2\hat{i}+\hat{j}-\hat{k}\right)=-1+\lambda.5$

$\vec{r}\left((1+2\lambda)\hat{i}+(-1+\lambda)\hat{j}+(3-\lambda)\hat{k}\right)=-1+5\lambda$ (2)

Now as the plane passes through (1,1,1)

$\vec{r}=\hat{i}+\hat{j}+\hat{k}$

Putting in equation (2)

$$\left(\hat{i}+\hat{j}+\hat{k}\right)\left(\left((1+2\lambda)\hat{i}+(-1+\lambda)\hat{j}+(3-\lambda)\hat{k}\right)=-1+5\lambda\right.$$

1.(1 + 2λ) + 1.(-1 + λ) + 1.(3-λ)=-1 + 5λ

1 + 2λ-1 + λ + 3- λ + 1-5λ=0

-3λ + 4=0

$$\lambda=\frac{4}{3}$$

Putting the value of λ in equation (2)

$$\vec{r}\left(\left(1+2.\frac{4}{3}\right)\hat{i}+\left(-1+\frac{4}{3}\right)\hat{j}+\left(3-\frac{4}{3}\right)\hat{k}\right)=-1+5.\frac{4}{3}$$

$$\vec{r}\left(\left(\frac{3+8}{3}\right)\hat{i}+\left(\frac{-3+4}{3}\right)\hat{j}+\left(\frac{9-4}{3}\right)\hat{k}\right)=\frac{-3+20}{3}$$

$$\vec{r}\left(11\hat{i}+\hat{j}+5\hat{k}\right)=17$$

So, required equation of plane is $\vec{r}\left(11\hat{i}+\hat{j}+5\hat{k}\right)=17.$

**10. Question**

Find the vector equation of the plane passing through the intersection of the planes $\vec{r}.\left(2\hat{i}-7\hat{j}+4\hat{k}\right)=3$ and $\vec{r}.\left(3\hat{i}-5\hat{j}+4\hat{k}\right)+11=0,$ and passing through the point (-2, 1, 3).

**Answer**

Equation of plane through the line of intersection of two planes in vector form is

$\vec{r}.(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ (1)

Where the standard equation of planes are

$\vec{r}.\vec{n}_1=d_1$ and $\vec{r}.\vec{n}_2=d_2$

Putting values in equation(1)

$$\vec{r}(2\hat{i}-7\hat{j}+4\hat{k}+\lambda\left(3\hat{i}-5\hat{j}+4\hat{k}\right)=3-\lambda.11$$

$\vec{r}\left((2+3\lambda)\hat{i}+(-7-5\lambda)\hat{j}+(4+4\lambda)\hat{k}\right)=3-11\lambda$ (2)

Now as the plane passes through (-2,1,3)

$$\vec{r}=-2\hat{i}+\hat{j}+3\hat{k}$$

Putting in equation (2)

$$\left(-2\hat{i}+\hat{j}+3\hat{k}\right)\left(\left((2+3\lambda)\hat{i}+(-7-5\lambda)\hat{j}+(4+4\lambda)\hat{k}\right)=3-11\lambda\right.$$

-2.(2 + 3λ) + 1.(-7-5λ) + 3.(4 + 4λ)=3-11λ

-4-6λ-7-5λ + 12 + 12λ-3 + 11λ=0

-14 + 12 + 12λ=0

$$\lambda=\frac{1}{6}$$

Putting the value of λ in equation (2)

$$\vec{r}\left(\left(2+3.\frac{1}{6}\right)\hat{i}+\left(-7-5.\frac{1}{6}\right)\hat{j}+\left(4+4\frac{1}{6}\right)\hat{k}\right)=3-11.\frac{1}{6}$$

$$\vec{r}\left(\left(\frac{12+3}{6}\right)\hat{i}+\left(\frac{-42-5}{6}\right)\hat{j}+\left(\frac{24+4}{6}\right)\hat{k}\right)=\frac{18-11}{6}$$

$$\vec{r}\left(15\hat{i}-47\hat{j}+28\hat{k}\right)=7$$

So, required equation of plane is $\vec{r}\left(15\hat{i}-47\hat{j}+28\hat{k}\right)=7.$

**11. Question**

Find the equation of the plane through the line of intersection of the planes $\vec{r}.\left(2\hat{i}-3\hat{j}+4\hat{k}\right)=1$ and $\vec{r}.\left(\hat{i}-\hat{j}\right)+4=0$ and perpendicular to the plane $\vec{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)+8=0.$

**Answer**

Equation of plane through the line of intersection of two planes in vector form is

$\vec{r}.(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ (1)

Where the standard equation of planes are

$\vec{r}.\vec{n}_1=d_1$ and $\vec{r}.\vec{n}_2=d_2$

Putting values in equation (1), we have

$\vec{r}(2\hat{i}-3\hat{j}+4\hat{k}+\lambda\left(\hat{i}-\hat{j}\right)=1-\lambda.4$

$\vec{r}\left((2+\lambda)\hat{i}+(-3-\lambda)\hat{j}+4\hat{k}\right)=1-4\lambda$ (2)

Given a plane perpendicular to this plane, So if n1 and n2 are normal

vectors of planes

$\vec{n}_1.\vec{n}_2=0$

$(2\hat{i}-\hat{j}+\hat{k}).\left((2+\lambda)\hat{i}+(-3-\lambda)\hat{j}+4\hat{k}\right)=0$

$2.(2+\lambda)+(-1).(-3-\lambda)+1.4=0$

$4+2\lambda+3+\lambda+4=0$

$11+3\lambda=0$

$\lambda=\frac{-11}{3}$

Putting the value of λ in equation (2)

$\vec{r}\left(\left(2+\frac{-11}{3}\right)\hat{i}+\left(-3-\frac{-11}{3}\right)\hat{j}+4\hat{k}\right)=1-4.\frac{-11}{3}$

$\vec{r}\left(\left(\frac{6-11}{3}\right)\hat{i}+\left(\frac{-9+11}{3}\right)\hat{j}+4\hat{k}\right)=\frac{3+44}{3}$

$\vec{r}\left(-5\hat{i}-2\hat{j}+12\hat{k}\right)=47$

So required equation of plane is $\vec{r}\left(-5\hat{i}-2\hat{j}+12\hat{k}\right)=47.$

**12. Question**

Find the Cartesian and vector equations of the planes through the line of intersection of the planes $\vec{r}.\left(\hat{i}-\hat{j}\right)+6=0$ and $\vec{r}.\left(3\hat{i}+3\hat{j}-4\hat{k}\right)=0,$ which are at a unit distance from the origin.

**Answer**

Equation of plane through the line of intersection of two planes in vector form is

$\vec{r}.(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ (1)

Where the standard equation of planes are

$\vec{r}.\vec{n}_1=d_1$ and $\vec{r}.\vec{n}_2=d_2$

Putting values in equation (1)

$\vec{r}(\hat{i}-\hat{j}+\lambda\left(3\hat{i}+3\hat{j}-4\hat{k}\right)=6+\lambda.0$

$\vec{r}\left((1+3\lambda)\hat{i}+(-1+3\lambda)\hat{j}+(-4\lambda)\hat{k}\right)=6$ (2)

For the equation of plane Ax + By + Cz=D and point (x1,y1,z1), a distance of a point from a plane can be calculated as

$$\left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right|$$

$$\left|\frac{(1+3\lambda)0+(-1+3\lambda).0+(-4\lambda).0-6}{\sqrt{(1+3\lambda)^2+(-1+3\lambda)^2+(-4\lambda)^2)}}\right|=1$$

$$\left|\frac{-6}{\sqrt{1+9\lambda^2+6\lambda+1+9\lambda^2-6\lambda+16\lambda^2}}\right|=1$$

$$\sqrt{2+34\lambda^2}=-6$$

$$2+34\lambda^2=(-6)^2$$

$$34\lambda^2=36-2$$

$$34\lambda^2=34$$

$$\lambda^2=1\Rightarrow\lambda=1,-1$$

Putting value of λ in equation (2)

λ=1

$$\vec{r}\left((1+3.1)\hat{i}+(-1+3.1)\hat{j}+(-4.1)\hat{k}\right)=6$$

$$\vec{r}\left(4\hat{i}+2\hat{j}-4\hat{k}\right)=6\Rightarrow\vec{r}.\left(2\hat{i}+\hat{j}-2\hat{k}\right)=3$$

λ=-1

$$\vec{r}\left((1+3.(-1))\hat{i}+(-1+3(-1))\hat{j}+(-4(-1))\hat{k}\right)=6$$

$$\vec{r}\left(-2\hat{i}-4\hat{j}+4\hat{k}\right)=6\Rightarrow\vec{r}.\left(\hat{i}+2\hat{j}-2\hat{k}\right)=-3$$

For equations in Cartesian form put

$$\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$$

For λ=1

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right).\left(2\hat{i}+\hat{j}-2\hat{k}-3\right)=0$$

x.2 + y.1 + z.(-2)-3=0

2x + y-2z-3=0

For λ=-1

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right).\left(\hat{i}+2\hat{j}-2\hat{k}+3\right)=0$$

x.1 + y.2 + z.(-2) + 3=0

x + 2y-2z + 3=0

So, required equation of plane

in vector form are $\vec{r}.\left(2\hat{i}+\hat{j}-2\hat{k}\right)=3$ for $\lambda=1$

$\vec{r}.\left(\hat{i}+2\hat{j}-2\hat{k}\right)=-3$ for $\lambda=-1$

In Cartesian form are 2x + y-2z-3=0 & x + 2y-2z + 3=0

## Exercise 28F

**1. Question**

Find the acute angle between the following planes :

(i) $\vec{r}\cdot\left(\hat{i}+\hat{j}-2\hat{k}\right)=5$ and $\vec{r}\cdot\left(2\hat{i}+2\hat{j}-\hat{k}\right)=9$

(ii) $\vec{r}\cdot\left(\hat{i}+2\hat{j}-\hat{k}\right)=6$ and $\vec{r}\cdot\left(2\hat{i}-\hat{j}-\hat{k}\right)+3=0$

(iii) $\vec{r}\cdot\left(2\hat{i}-3\hat{j}+4\hat{k}\right)=1$ and $\vec{r}\cdot\left(-\hat{i}+\hat{j}\right)=4$

(iv) $\vec{r}\cdot\left(2\hat{i}-3\hat{j}+6\hat{k}\right)=8$ and $\vec{r}\cdot\left(3\hat{i}+4\hat{j}-12\hat{k}\right)+7=0$

**Answer**

To find the angle between two planes, we simply find the angle between the normal vectors of planes. So if n1 and n2 are normal vectors and θ is the angle between both then,

$$\cos\theta=\left|\frac{\vec{n}_1\overline{n_2}}{\left|\overline{n_1}\right|\left|\vec{n}_2\right|}\right|$$

(i)On comparing with the standard equation of planes in vector form

$$\vec{r}.\vec{n}_1=d_1 \text{ and } \vec{r}.\vec{n}_2=d_2$$

$$\vec{n}_1=\hat{i}+\hat{j}-2\hat{k} \text{ and } \vec{n}_2=2\hat{i}+2\hat{j}-\hat{k}$$

Then

$$\cos\theta=\left|\frac{\left(\hat{i}+\hat{j}-2\hat{k}\right).\left(2\hat{i}+2\hat{j}-\hat{k}\right)}{\left|\hat{i}+\hat{j}-2\hat{k}\right|\left|2\hat{i}+2\hat{j}-\hat{k}\right|}\right|\Rightarrow\left|\frac{1.2+1.2+(-2).(-1)}{\left(\sqrt{1^2+1^2+(-2)^2}\right).\left(\sqrt{2^2+2^2+(-1)^2}\right)}\right|=\left|\frac{2+2+2}{\sqrt{1+1+4}\sqrt{4+4+1}}\right|$$

$$\Rightarrow\left|\frac{6}{\sqrt{6}.\sqrt{9}}\right|=\left|\frac{\sqrt{6}}{3}\right|$$

$$\theta=\cos^{-1}\left(\frac{\sqrt{6}}{3}\right)$$



(ii) On comparing with the standard equation of planes in vector form

$$\vec{r}.\vec{n}_1=d_1 \text{ and } \vec{r}.\vec{n}_2=d_2$$

$$\vec{n}_1=\hat{i}+2\hat{j}-\hat{k} \text{ and } \vec{n}_2=2\hat{i}-\hat{j}-\hat{k}$$

Then

$$\cos\theta=\left|\frac{\left(\hat{i}+2\hat{j}-\hat{k}\right).\left(2\hat{i}-\hat{j}-\hat{k}\right)}{\left|\hat{i}+2\hat{j}-\hat{k}\right|\left|2\hat{i}-\hat{j}-\hat{k}\right|}\right|\Rightarrow\left|\frac{1.2+2.(-1)+(-1).(-1)}{\left(\sqrt{1^2+2^2+(-1)^2}\right).\left(\sqrt{2^2+(-1)^2+(-1)^2}\right)}\right|=\left|\frac{2-2+1}{\sqrt{1+4+1}\sqrt{4+1+1}}\right|$$

$$\Rightarrow\left|\frac{1}{\sqrt{6}.\sqrt{6}}\right|=\left|\frac{1}{6}\right|$$

$$\theta=\cos^{-1}\left(\frac{1}{6}\right)$$

(iii) On comparing with the standard equation of planes in vector form

$$\vec{r}.\vec{n}_1=d_1 \text{ and } \vec{r}.\vec{n}_2=d_2$$

$$\vec{n}_1=2\hat{i}-3\hat{j}+4\hat{k} \text{ and } \vec{n}_2=-\hat{i}+\hat{j}$$

Then

$$\cos\theta=\left|\frac{\left(2\hat{i}-3\hat{j}+4\hat{k}\right).\left(-\hat{i}+\hat{j}\right)}{\left|2\hat{i}-3\hat{j}+4\hat{k}\right|\left|-\hat{i}+\hat{j}\right|}\right|\Rightarrow\left|\frac{2.(-1)+(-3).1+4.0}{\left(\sqrt{2^2+(-3)^2+4^2}\right).\left(\sqrt{(-1)^2+1^2}\right)}\right|=\left|\frac{-2+(-3)}{\left(\sqrt{4+9+16}\right)\left(\sqrt{1+1}\right)}\right|$$

$$\Rightarrow\left|\frac{-5}{\sqrt{29}\sqrt{2}}\right|=\left|\frac{-5}{\sqrt{58}}\right|$$

$\theta=\cos^{-1}\left(\frac{5}{\sqrt{58}}\right)$

(iv)On comparing with the standard equation of planes in vector for

$\vec{r}.\vec{n}_1=d_1$ and $\vec{r}.\vec{n}_2=d_2$

$\vec{n}_1=2\hat{i}-3\hat{j}+6\hat{k}$ and $\vec{n}_2=3\hat{i}+4\hat{j}-12\hat{k}$

Then

$$\cos\theta=\left|\frac{\left(2\hat{i}-3\hat{j}+6\hat{k}\right).\left(3\hat{i}+4\hat{j}-12\hat{k}\right)}{\left|2\hat{i}-3\hat{j}+6\hat{k}\right|\left|3\hat{i}+4\hat{j}-12\hat{k}\right|}\right|\Rightarrow\left|\frac{2.3+(-3).4+6.(-12)}{\left(\sqrt{2^2}+(-3)^2+6^2\right).(\sqrt{3^2}+4^2+(-12)^2}\right|$$

$$=\left|\frac{6+(-12)+(-72)}{\left(\sqrt{4}+9+36\right)\left(\sqrt{9}+16+144\right)}\right|$$

$$\Rightarrow\left|\frac{-78}{\sqrt{49}\sqrt{169}}\right|=\left|\frac{-78}{7.13}\right|$$

$$\theta=\cos^{-1}\left(\frac{6}{7}\right)$$

**2. Question**

Show that the following planes are at right angles:

(i) $\vec{r}\cdot\left(4\hat{i}-7\hat{j}-8\hat{k}\right)=5$ and $\vec{r}\cdot\left(3\hat{i}-4\hat{j}+5\hat{k}\right)+10=0$

(ii) $\vec{r}\cdot\left(2\hat{i}+6\hat{j}+6\hat{k}\right)=13$ and $\vec{r}\cdot\left(3\hat{i}+4\hat{j}-5\hat{k}\right)+7=0$

**Answer**

To show the right angle between two planes, we simply find the angle between the normal vectors of planes. So if n1 and n2 are normal vectors and θ is the angle between both then

$\cos\theta=\left|\frac{\vec{n}_1\overline{n_2}}{\left|\vec{n}_1\right|\left|\vec{n}_2\right|}\right|$.for right angle θ=90°

Cos90°=0

$\vec{n}_1.\vec{n}_2=0$ (1)

(i)On comparing with standard equation

$\vec{n}_1=4\hat{i}-7\hat{j}-8\hat{k}$ and $\vec{n}_2=3\hat{i}-4\hat{j}+5\hat{k}$

$$\text{LHS}=\vec{n}_1.\vec{n}_2\Rightarrow\left(4\hat{i}-7\hat{j}-8\hat{k}\right).\left(3\hat{i}-4\hat{j}+5\hat{k}\right)=4.3+(-7).(-4)+(-8).5$$

$$\Rightarrow 12+28-40=40-40\Rightarrow 0=\text{RHS}$$

Hence proved planes at right angles.

(ii) On comparing with the standard equation of a plane

$\vec{n}_1=2\hat{i}+6\hat{j}+6\hat{k}$ and $\vec{n}_2=3\hat{i}+4\hat{j}-5\hat{k}$

$$\text{LHS}=\vec{n}_1.\vec{n}_2\Rightarrow\left(2\hat{i}+6\hat{j}+6\hat{k}\right).\left(3\hat{i}+4\hat{j}-5\hat{k}\right)=2.3+6.4+6.(-5)$$

$$\Rightarrow 6+24-30=30-30\Rightarrow 0=\text{RHS}$$

Hence proved planes at right angles.

**3. Question**

Find the value of λ for which the given planes are perpendicular to each other:

(i) $\vec{r}\cdot\left(2\hat{i}-\hat{j}-\lambda\hat{k}\right)=7$ and $\vec{r}\cdot\left(3\hat{i}+2\hat{j}+2\hat{k}\right)=9$

(ii) $\vec{r}\cdot\left(\lambda\hat{i}+2\hat{j}+3\hat{k}\right)=5$ and $\vec{r}\cdot\left(\hat{i}+2\hat{j}-7\hat{k}\right)+11=0$

**Answer**

For planes perpendicular Cos90°=0

$\vec{n}_1.\vec{n}_2 = 0$ (1)

(i)On comparing with the standard equation of a plane

$\vec{n}_1 = 2\hat{i} - \hat{j} - \lambda\hat{k}$ and $\vec{n}_2 = 3\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{n}_1.\vec{n}_2 = \left(2\hat{i} - \hat{j} - \lambda\hat{k}\right).\left(3\hat{i} + 2\hat{j} + 2\hat{k}\right) = 0$

2.3 + (-1).2 + (-λ).2=0

6-2-2λ=0

2λ=4

λ=2

(ii) On comparing with the standard equation of a plane

$\vec{n}_1 = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{n}_2 = \hat{i} + 2\hat{j} - 7\hat{k}$

$\vec{n}_1.\vec{n}_2 = \left(\lambda\hat{i} + 2\hat{j} + 3\hat{k}\right).\left(\hat{i} + 2\hat{j} - 7\hat{k}\right) = 0$

λ.1 + 2.2 + 3.(-7)=0 λ + 4-21=0 λ=17

**4. Question**

Find the acute angle between the following planes:

(i) 2X – y + z = 5 and x + y + 2z = 7

(ii) x + 2y + 2z = 3 and 2x - 3y + 6z = 8

(iii) x + y - z = 4 and x + 2y + z = 9

(iv) x + y - 2z = 6 and 2x - 2y + z = 11

**Answer**

To find angle in Cartesian form, for standard equation of planes

$A_1x + B_1y + C_1z + D_1 = 0$ and $A_1x + B_2y + C_2z + D_2 = 0$

$$\cos\theta = \left|\frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{\left(A_1^2 + B_1^2 + C_1^2\right)}\sqrt{\left(A_2^2 + B_2^2 + C_2^2\right)}}\right|$$

(i)On comparing with the standard equation of planes

$A_1 = 2, B_1 = -1, C_1 = 1$ and $A_2 = 1, B_2 = 1, C_2 = 2$

$$\cos\theta = \left|\frac{2.1 + (-1).1 + 1.2}{\sqrt{2^2 + (-1)^2 + 1^2}\sqrt{1^2 + 1^2 + 2^2}}\right| \Rightarrow \left|\frac{2 + (-1) + 2}{\sqrt{4 + 1 + 1}\sqrt{1 + 1 + 4}}\right| = \left|\frac{3}{\sqrt{6}\sqrt{6}}\right|$$

$$= \frac{3}{6} \Rightarrow \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) \Rightarrow \frac{\pi}{3}$$

(ii)On comparing with the standard equation of planes

$A_1 = 1, B_1 = 2, C_1 = 2$ and $A_2 = 2, B_2 = -3, C_2 = 6$

$$\cos\theta = \left|\frac{1.2 + 2.(-3) + 2.6}{\sqrt{1^2 + 2^2 + 2^2}\sqrt{2^2 + (-3)^2 + 6^2}}\right| \Rightarrow \left|\frac{2 + (-6) + 12}{\sqrt{1 + 4 + 4}\sqrt{4 + 9 + 36}}\right| = \left|\frac{8}{\sqrt{9}\sqrt{49}}\right|$$

$$= \frac{8}{3.7} \Rightarrow \frac{8}{21}$$

$$\theta = \cos^{-1}\left(\frac{8}{21}\right)$$

(iii) On comparing with standard equation of planes

$A_1 = 1, B_1 = 1, C_1 = -1 \text{ and } A_2 = 1, B_2 = 2, C_2 = 1$

$$\cos\theta = \left|\frac{1.1+1.2+(-1).1}{\sqrt{1^2+1^2+(-1)^2}\sqrt{1^2+2^2+1^2}}\right| \Rightarrow \left|\frac{1+2+(-1)}{\sqrt{1+1+1}\sqrt{1+4+1}}\right| = \left|\frac{2}{\sqrt{3}\sqrt{6}}\right|$$

$$= \frac{\sqrt{2}}{3}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$$

(iv)On comparing with the standard equation of planes

$A_1 = 1, B_1 = 1, C_1 = -2 \text{ and } A_2 = 2, B_2 = -2, C_2 = 1$

$$\cos\theta = \left|\frac{1.2+1.(-2)+(-2).1}{\sqrt{1^2+1^2+(-2)^2}\sqrt{2^2+(-2)^2+1^2}}\right| \Rightarrow \left|\frac{2+(-2)+(-2)}{\sqrt{1+1+4}\sqrt{4+4+1}}\right| = \left|\frac{-2}{\sqrt{6}\sqrt{9}}\right|$$

$$= \frac{2}{\sqrt{6}.3}$$

$$\theta = \cos^{-1}\left(\frac{2}{3\sqrt{6}}\right)$$

**5. Question**

Show that each of the following pairs of planes are at right angles:

(i) 3x + 4y - 5z = 7 and 2x + 6y + 6z + 7 = 0

(ii) x - 2y + 4z = 10 and 18x + 17y + 4z = 49

**Answer**

To find angle in Cartesian form, for standard equation of planes

$$A_1x + B_1y + C_1z + D_1 = 0 \text{ and } A_1x + B_2y + C_2z + D_2 = 0 \quad \cos\theta = \left|\frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{\left(A_1^2 + B_1^2 + C_1^2\right)}\sqrt{\left(A_2^2 + B_2^2 + C_2^2\right)}}\right|$$



For θ=90°, cos90°=0

$$A_1A_2 + B_1B_2 + C_1C_2 = 0$$

(i)On comparing with the standard equation of a plane

$A_1 = 3, B_1 = 4, C_1 = -5 \text{ and } A_2 = 2, B_2 = 6, C_2 = 6$

$$LHS = A_1A_2 + B_1B_2 + C_1C_2 \Rightarrow 3.2 + 4.6 + (-5).6 = 6 + 24 - 30$$

=0=RHS

Hence proved that the angle between planes is 90°.

(ii) On comparing with the standard equation of a plane

$A_1 = 1, B_1 = -2, C_1 = 4 \text{ and } A_2 = 18, B_2 = 17, C_2 = 4$

$$LHS = A_1A_2 + B_1B_2 + C_1C_2 \Rightarrow 1.18 + (-2).17 + 4.4 = 18 + (-34) + 16$$

=0=RHS

Hence proved that angle between planes is 90°.

**6. Question**

Prove that the plane 2x + 2y + 4z = 9 is perpendicular to each of the planes x + 2y + 2z - 7 = 0 and 5x + 6y + 7z = 23.

**Answer**

To show that planes are perpendicular

$$A_1A_2 + B_1B_2 + C_1C_2 = 0$$

Where $A_1$, $B_1$, $C_1$ are direction ratios of plane and $A_2$, $B_2$, $C_2$ are of other

plane.

2.1 + 2.2 + 4.2=2 + 4 + 8=14≠0

Hence, planes are not perpendicular.

Similarly for the other plane

2.5 + 2.6 + 2.7=10 + 12 + 14=36≠0

Hence, planes are not perpendicular.

**7. Question**

Show that the planes 2x – 2y + 4z + 5 = 0 and 3x – 3y + 6z - 1 = 0 are parallel.

**Answer**

To show that planes are parallel

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}$$

On comparing with the standard equation of a plane

$$A_1=2, B_1=-2, C_1=4 \text{ and } A_2=3, B_2=-3, C_2=6$$

$$\frac{A_1}{A_2}=\frac{2}{3}, \frac{B_1}{B_2}=\frac{-2}{-3}\Rightarrow\frac{2}{3}, \frac{C_1}{C_2}=\frac{4}{6}\Rightarrow\frac{2}{3}$$

So,

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}=\frac{2}{3}$$

Hence proved that planes are parallel.

**8. Question**

Find the value of λ for which the planes x – 4y + λz + 3 = 0 and 2x + 2y + 3z = 5 are perpendicular to each other.

**Answer**

To find an angle in Cartesian form, for the standard equation of planes

$$A_1x+B_1y+C_1z+D_1=0 \text{ and } A_1x+B_2y+C_2z+D_2=0$$

$$\cos\theta=\left|\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{\left(A_1^2+B_1^2+C_1^2\right)}\sqrt{\left(A_2^2+B_2^2+C_2^2\right)}}\right|$$

For θ=90°, cos90°=0

$$A_1A_2+B_1B_2+C_1C_2=0$$

On comparing with the standard equation of the plane,

$$A_1=1, B_1=-4, C_1=\lambda \text{ and } A_2=2, B_2=2, C_2=3$$

$$A_1A_2+B_1B_2+C_1C_2\Rightarrow 1.2+(-4).2+\lambda.3=0$$

2 + (-8) + 3λ=0

-6 + 3λ=0

λ=2

**9. Question**

Write the equation of the plane passing through the origin and parallel to the plane 5x - 3y + 7z + 11 = 0.

**Answer**

Let the equation of plane be

$$A_1x+B_1y+C_1z+D_1=0$$

Direction ratios of parallel planes are related to each other as

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}=k\,(\text{constant})$$

Putting the values from the equation of a given parallel plane,

$$\frac{A_1}{5}=\frac{B_1}{-3}=\frac{C_1}{7}=k$$

$$A_1=5k, B_1=-3k, C_1=7k$$

Putting in equation plane

$$5kx-3ky+7kz+D_1=0$$

As the plane is passing through (0,0,0), it must satisfy the plane,

$$5k.0-3k.0+7k.0+D_1=0$$

$$D_1=0$$

5kx-3ky + 7kz=0

5x-3y + 7z=0

So, required equation of plane is 5x-3y + 7z=0.

**10. Question**

Find the equation of the plane passing through the point (a, b, c) and parallel to the plane $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=2$.

**Answer**

Let the equation of a plane

$$\vec{r}.(x_1\hat{i}+y_1\hat{j}+z_1\hat{k})=d \ (1)$$

Direction ratios of parallel planes are related to each other as

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}=\lambda(\text{constant})$$

Putting the values from the equation of a given parallel plane,

$$\frac{x_1}{1}=\frac{x_1}{1}=\frac{z_1}{1}=\lambda$$

$$x_1=y_1=z_1=\lambda$$

Putting values in equation (1), we have

$$\vec{r}.(\lambda\hat{i}+\lambda\hat{j}+\lambda\hat{k})=d \ (2)$$

A plane passes through (a,b,c) then it must satisfy the equation of a plane

$$(a\hat{i}+b\hat{j}+c\hat{k}).(\lambda\hat{i}+\lambda\hat{j}+\lambda\hat{k})=d$$

$$\lambda(a\hat{i}+b\hat{j}+c\hat{k})(\hat{i}+\hat{j}+\hat{k})=d$$

λ(a.1 + b.1 + c.1)=d

λ(a + b + c)=d

Putting value in equation (2)

$$\vec{r}.(\hat{i}+\hat{j}+\hat{k})\lambda=\lambda(a+b+c)$$

$$\vec{r}.(\hat{i}+\hat{j}+\hat{k})=a+b+c$$

So, required equtaion of plane is $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=a+b+c$.

**11. Question**

Find the equation of the plane passing through the point (1, -2, 7) and parallel to the plane 5x + 4y - 11z = 6.

**Answer**

Let the equation of plane be

$$A_1x+B_1y+C_1z+D_1=0$$

Direction ratios of parallel planes are related to each other as