$P(4,4) = \frac{4!}{(4-4)!}$

$= \frac{4!}{0!} = \frac{24}{1} = 24.$

Hence they can be arranged in 24 ways.

Q. 6. Six students are contesting the election for the president ship of the students, union. In how many ways can their names be listed on the ballot papers?

Answer : To find: number of arrangements of names on a ballot paper.

There are six contestants contesting in the elections.

Name of any 1 student out of six can appear first on the ballot paper.

2 position on the ballot paper can be filled by rest of the five names and so on.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, permutation of 6 different objects in 6 places is

$P(6,6) = \frac{6!}{(6-6)!}$

$= \frac{6!}{0!} = \frac{720}{1} = 720.$

Hence, their name can be arranged in 720 ways.

Q. 7. It is required to seat 5 men and 3 women in a row so that the women occupy the even places. How many such arrangements are possible?

Answer : To find: number of arrangements in which women sit in even places

Condition: women occupy even places

Here the total number of people is 8.

_ _ _ _ _ _ _ _

1 2 3 4 5 6 7 8

In this question first, the arrangement of women is required.

The positions where women can be made to sit is 2^{nd}, 4^{th}, 6^{th}, 8^{th}. There are 4 even places in which 3 women are to be arranged.

Women can be placed in P (4,3) ways. The rest 5 men can be arranged in 5! ways.

Therefore, the total number of arrangements is P (4,3) ×5!

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 4 different objects in 3 places and the arrangement of 5 men are

P (4,3) ×5! = $\frac{4!}{(4-3)!} \times 5!$

= $\frac{24}{1} \times 120$

= 2880.

Hence number of ways in which they can be seated is 2880.

Q. 8. There are 6 items in column A and 6 items in column B. A student is asked to match each item in column A with an item in column B. How many possible, correct or incorrect answers are there to this question?

Answer : To find: number of possibilities of a selection of answers.

Each item in column A can select another item in column B.

Therefore the question involves selecting each item from column A to each item in column B. this can be done in P(6,6)

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 6 different objects in 6 places is

$P(6,6) = \frac{6!}{(6-6)!}$

$= \frac{6!}{0!} = \frac{720}{1} = 720.$

Therefore, the possible number of selecting an incorrect or correct answer is 720.

Q. 9. Five letters F, K, R, R and V one in each were purchased from a plastic warehouse. How many ordered pairs of letters, to be used as initials, can be formed from them?

Answer : (i) The number of initials is 1

In this case, all letters have one chance (i.e. letters F, K, R, V).

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 4 different objects in 1 place is

$P(4,1) = \frac{4!}{(4-1)!}$

$= \frac{4!}{3!} = \frac{24}{6} = 4.$

So no of ways is 4.

(ii) The number of initials is 2

There are two cases here

(a) When two R do not occur in initials

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

$P(n,r) = n!/(n-r)!$

Therefore, a permutation of 4 different objects in 2 places is

$P(4,2) = \frac{4!}{(4-2)!}$

$= \frac{4!}{2!} = \frac{24}{2} = 12.$

A number of arrangements here are 12.

(b) When two R occurs in initials

When two R are chosen then 1 pair is included twice.

Selection of 0 letters remaining from 3 letters can be done in P(3,0) ways.

Formula:

A number of permutations of n objects in which p objects are alike of one kind are =n!/p!

Selections = $P(3,0) \times \frac{2!}{2!}$

$= \frac{3!}{3!} \times \frac{2!}{2!} = 1$

Therefore, the total number of pairs 13.

(iii) The number of initial is 3

(a) two R do not occur in initials

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

$P(n,r) = n!/(n-r)!$

Therefore, a permutation of 4 different objects in 3 places is

$P(4,3) = \frac{4!}{(4-3)!}$

$= \frac{4!}{1!} = \frac{24}{1} = 24.$

A number of arrangements here are 24.

(b) two R occurs in initials

When two R are chosen then 1 pair is included twice.

Selection of 1 letter from the remaining 3 letters is P(3,1)

Formula:

A number of permutations of n objects in which p objects are alike of one kind = n!/p!

Selections = $P(3,1) \times \frac{3!}{2!}$

$= \frac{3!}{2!} \times \frac{3!}{2!} = 9$

total number of arrangements for 3 initials are 33

(iv) The number of initials is 4

(a) Two R do not occur in initials

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 4 different objects in 4 places is

$P(4,4) = \frac{4!}{(4-4)!}$

$= \frac{4!}{0!} = \frac{24}{1} = 24.$

A number of arrangements here are 24.

(b) Two R occurs in the initials

When two R are chosen then 1 pair is included twice.

Selection of 2 letters from the remaining 3 letters is P(3,2)

Formula:

A number of permutations of n objects in which p objects are alike of one kind = n!/p!

Selections = P(3,2) × $\frac{4!}{2!}$

= $\frac{3!}{1!} \times \frac{4!}{2!}$ = 36

Total number of arrangements for 4 initials are 60

(v) The number of initials is 5

Formula:

A number of permutations of n objects in which p objects are alike of one kind = n!/p!

Selections = $\frac{5!}{2!}$ = 60.

Total number of arrangements are 4 + 13 + 33 + 60 + 60 = 170

Q. 10. Ten students are participating in a race. In how many ways can the first three prizes be won?

Answer : To find: number of ways of winning the first three prizes.

The first price can go to any of the 10 students.

The second price can go to any of the remaining 9 students.

The third price can go to any of the remaining 8 students.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 10 different objects in 3 places is

P(10,3) = $\frac{10!}{(10-3)!}$

= $\frac{10!}{7!}$ = $\frac{3628800}{5040}$ = 720.

Therefore, there are 10 × 9 × 8 = 720 ways to win first three prizes.

Q. 11. If there are 6 periods on each working day of a school, in how many ways can one arrange 5 subjects such that each subject is allowed at least one period?

Answer : To find: number of ways of arranging 5 subjects in 6 periods.

Condition: at least 1 period for each subject.

5 subjects in 6 periods can be arranged in P (6,5).

Remaining 1 period can be arranged in P (5,1)

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Total arrangements = P(6,5) × P(5,1) = $\frac{6!}{(6-5)!}$ × $\frac{5!}{(5-1)!}$

= $\frac{6!}{1!}$ × $\frac{5!}{4!}$ = 720 × 5 = 3600.

Total number of ways is 3600 ways.

Q. 12. In how many ways can 6 pictures be hung from 4 picture nails on a wall?

Answer : To find: number of ways of hanging 6 pictures on 4 picture nails.

There are 6 pictures to be placed in 4 places.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 6 different objects in 4 places is

$P(6,4) = \frac{6!}{(6-4)!} = \frac{6!}{2!} = \frac{720}{2} = 360$

This can be done by 360 ways.

Q. 13. Find the number of words formed (may be meaningless) by using all the letters of the word 'EQUATION', using each letter exactly once.

Answer : There are 8 alphabets in the word EQUATION.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 8 different objects in 8 places is

$P(8,8) = \frac{8!}{(8-8)!} = \frac{8!}{0!} = \frac{40320}{1} = 40320$

Hence there are 40320 words formed.

Q. 14. Find the number of different 4-letter words (may be meaningless) that can be formed from the letters of the word 'NUMBERS',

Answer : To find: 4 lettered word from letters of word NUMBERS

There are 7 alphabets in word NUMBERS.

The word is a 4 different letter word.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 7 different objects in 4 places is

$P(7,4) = \frac{7!}{(7-4)!} = \frac{7!}{3!} = \frac{5040}{6} = 840.$

Hence, they can be arranged in 840 words.

Q. 15. How many words can be formed from the letters of the word 'SUNDAY'? How many of these begin with D?

Answer : There are 6 letters in the word SUNDAY.

Different words formed using 6 letters of the word SUNDAY is P(6,6)

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 6 different objects in 6 places is

$P(6,6) = \frac{6!}{(6-6)!} = \frac{6!}{0!} = \frac{720}{1} = 720.$

720 words can be formed using letters of the word SUNDAY.

When a word begins with D.

Its position is fixed, i.e. the first position.

Now rest 5 letters are to be arranged in 5 places.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 5 different objects in 5 places is

$P(5,5) = \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{120}{1} = 120.$

Therefore, the total number of words starting with D are 120.

Q. 16. How many words beginning with C and ending with Y can be formed by using the letters of the word 'COURTESY'?

Answer : To find: number of words starting with C and end with Y

There are 8 letters in word COURTESY.

Here the position of the letters C and Y are fixed which is 1st and 8th.

_ _ _ _ _ _ _ _

C ? ? ? ? ? ? Y

Rest 6 letters are to be arranged in 6 places which can be done in P(6,6).

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 6 different objects in 6 places is

$P(6,6) = \frac{6!}{(6-6)!} = \frac{6!}{0!} = \frac{720}{1} = 720.$

Therefore, total number of words starting with C and ending with Y is 720.

Q. 17. Find the number of permutations of the letters of the word 'ENGLISH'. How many of these begin with E and end with I?

Answer : There are 7 letters in the word ENGLISH.

Permutation of 7 letters in 7 places can be done in P(7,7) ways.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 7 different objects in 7 places is

$P(7,7) = \frac{7!}{(7-7)!} = \frac{7!}{0!} = \frac{5040}{1} = 5040.$

Hence, the total number of permutations is P 5040.

To find a number of words starting with E and ending with I, let us consider their position which is 1st and 7th.

_ _ _ _ _ _ _

E ? ? ? ? ? I

The rest 5 letters are to be arranged in 5 places which can be done in P (5,5)

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 5 different objects in 5 places is

$P(5,5) = \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{120}{1} = 120.$

Therefore, there are 120 words starting with E and ending with I.

Q. 18. In how many ways can the letters of the word 'HEXAGON' be permuted? In how many words will the vowels be together?

Answer : There are 7 letters in the word HEXAGON.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 7 different objects in 7 places is

$P(7,7) = \frac{7!}{(7-7)!} = \frac{7!}{0!} = \frac{5040}{1} = 5040.$

They can be permuted in P (7,7) = 5040 ways.

The vowels in the word are E, A, O.

Consider this as a single group.

Now considering vowels as a single group, there are total 5 groups (4 letters and 1 vowel group) can be permuted in P (5,5)

Now vowel can be arranged in 3! Ways.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, the arrangement of 5 groups and vowel group is

$P(5,5)\times 3! = \frac{5!}{(5-5)!} \times 3! = \frac{5!}{0!} \times 3! = \frac{120}{1} \times 6 = 720.$

Hence total number of arrangements possible is 720.

Q. 19. How many words can be formed out of the letters of the word 'ORIENTAL' so that the vowels always occupy the odd places?

Answer : To find: number of words formed

Condition: vowels occupy odd places

There are 8 letters in the word ORIENTAL and vowels are 4 which are O, I, E,A respectively.

O E O E O E O E

There is 4 odd places in which 4 vowels are to be arranged.

The rest 4 letters can be arranged in 4! Ways.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, the total arrangement is

$P(4,4) \times 4! = \frac{4!}{(4-4)!} \times 4! = \frac{4!}{0!} \times 4! = \frac{24}{1} \times 24 = 576.$

Therefore, total number of words formed are 576.

Q. 20. In how many ways can the letters of the word 'FAILURE' be arranged so that the consonants may occupy only odd positions?

Answer : To find: number of words

Condition: consonants occupy odd places

There are total of 7 letters in the word FAILURE.

There are 3 consonants, i.e. F, L, R which are to be arranged in 4 places.

The rest 5 letters can be arranged in 4! Ways.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, the total number of words are

$P(4,3) \times 4! = \frac{4!}{(4-3)!} \times 4! = \frac{4!}{1!} \times 4! = \frac{24}{1} \times 24 = 576.$

Hence total number of arrangements is 576.

Q. 21. In how many arrangements of the word 'GOLDEN' will the vowels never occur together?

Answer : To find: number of words

Condition: vowels should never occur together.

There are 6 letters in the word GOLDEN in which there are 2 vowels.

Total number of words in which vowels never come together =

Total number of words – total number of words in which the vowels come together.

A total number of words is 6! = 720 words.

Consider the vowels as a group.

Hence there are 5 groups that can be arranged in P(5,5) ways, and vowels can be arranged in P(2,2,) ways.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Total arrangements = P(5,5) × P(2,2) = $\frac{5!}{(5-5)!} \times \frac{2!}{(2-2)!}$

= $\frac{5!}{0!} \times \frac{2!}{0!}$ = 120 × 2 = 240.

Hence a total number of words having vowels together is 240.

Therefore, the number of words in which vowels don't come together is 720 – 240 = 480 words.

Q. 22. Find the number of ways in which the letters of the word 'MACHINE' can be arranged such that the vowels may occupy only odd positions.

Answer : To find: number of words

Condition: vowels occupy odd positions.

There are 7 letters in the word MACHINE out of which there are 3 vowels namely A C E.

There are 4 odd places in which 3 vowels are to be arranged which can be done P(4,3).

The rest letters can be arranged in 4! ways

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, the total number of words is

$P(4,3)4!\times = \frac{4!}{(4-3)!} \times 4!$

$= \frac{4!}{1!} \times 4! = \frac{24}{1} \times 24 = 576.$

Hence the total number of word in which vowel occupy odd positions only is 576.

Q. 23. How many permutations can be formed by the letters of the word 'VOWELS', when

(i) there is no restriction on letters;
(ii) each word begins with E;
(iii) each word begins with O and ends with L;
(iv) all vowels come together;
(v) all consonants come together?

Answer : (i) There is no restriction on letters

The word VOWELS contain 6 letters.

The permutation of letters of the word will be 6! = 720 words.

(ii) Each word begins with E

Here the position of letter E is fixed.

Hence, the rest 5 letters can be arranged in 5! = 120 ways.

(iii) Each word begins with O and ends with L

The position of O and L are fixed.

Hence the rest 4 letters can be arranged in 4! = 24 ways.

(iv) All vowels come together

There are 2 vowels which are O, E.

Consider this group.

Therefore, the permutation of 5 groups is 5! = 120

The group of vowels can also be arranged in 2! = 2 ways.