Exercise 16(C)

Solution 1

(i)
Ratio of area of triangles with same vertex and bases along the same line is equal to the ratio of their respective bases. So, we have:

$$\frac{\text{Area of } \Delta \text{ DOC}}{\text{Area of } \Delta \text{ BOC}} = \frac{\text{DO}}{\text{BO}} = 1 \text{ ----1}$$

Similarly

$$\frac{\text{Area of } \Delta \text{ DOA}}{\text{Area of } \Delta \text{ BOA}} = \frac{\text{DO}}{\text{BO}} = 1 \text{ ------2}$$

We know that area of triangles on the same base and between same parallel lines are equal.

Area of Δ ACD = Area of Δ BCD

Area of Δ AOD + Area of Δ DOC = Area of Δ DOC + Area of Δ BOC

=> Area of Δ AOD = Area of Δ BOC ------3

From 1, 2 and 3 we have

Area (Δ DOC) = Area (Δ AOB)

Hence Proved.
(ii)

Similarly, from 1, 2 and 3, we also have

Area of Δ DCB = Area of Δ DOC + Area of Δ BOC = Area of Δ AOB + Area of Δ BOC = Area of Δ ABC

So Area of Δ DCB = Area of Δ ABC

Hence Proved.

(iii)

We know that area of triangles on the same base and between same parallel lines are equal.

Given: triangles are equal in area on the common base, so it indicates AD|| BC.

So, ABCD is a parallelogram.

Hence Proved

Solution 2

Ratio of area of triangles with the same vertex and bases along the same line is equal to the ratio of their respective bases.

So, we have

$$\frac{\text{Area of } \Delta \text{ APD}}{\text{Area of } \Delta \text{ BPD}} = \frac{\text{AP}}{\text{BP}} = \frac{1}{2}$$

Area of parallelogram ABCD = 324 sq.cm

Area of the triangles with the same base and between the same parallels are equal.

We know that area of the triangle is half the area of the parallelogram if they lie on the same base and between the parallels.

Therefore, we have,

$$\begin{aligned} Area(\triangle ABD) &= \frac{1}{2} \times Area(||gm\ ABCD) \\ &= \frac{324}{2} \\ &= 162\ sq.\ cm \end{aligned}$$

From the diagram it is clear that,

$$Area(\triangle ABD) = Area(\triangle APD) + Area(\triangle BPD)$$
$$\Rightarrow 162 = Area(\triangle APD) + 2Area(\triangle APD)$$
$$\Rightarrow 162 = 3Area(\triangle APD)$$
$$\Rightarrow Area(\triangle APD) = \frac{162}{3}$$
$$\Rightarrow Area(\triangle APD) = 54\ sq.\ cm$$

(ii)

Consider the triangles $\triangle AOP$ *and* $\triangle COD$

$\angle AOP = \angle COD$ [vertically opposite angles]

$\angle CDO = \angle APD$ [AB and DC are parallel and DP is the transversal, alternate interior angles are equal]

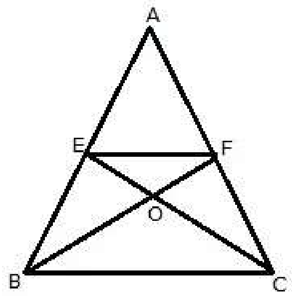
Thus, by Angle – Angle similarity, $\triangle AOP \sim \triangle COD$.

Hence the corresponding sides are proportional.

$$\begin{aligned} \frac{AP}{CD} = \frac{OP}{OD} &= \frac{AP}{AB} \\ &= \frac{AP}{AP + PB} \\ &= \frac{AP}{3AP} \\ &= \frac{1}{3} \end{aligned}$$

E and F are the midpoints of the sides AB and AC.

Consider the following figure.



Therefore, by midpoint theorem, we have, EF || BC

Triangles BEF and CEF lie on the common base EF and between the parallels, EF and BC

$Therefore, Ar.(\triangle BEF) = Ar.(\triangle CEF)$
$\Rightarrow Ar.(\triangle BOE) + Ar.(\triangle EOF) = Ar.(\triangle EOF) + Ar.(\triangle COF)$
$\Rightarrow Ar.(\triangle BOE) = Ar.(\triangle COF)$

Now BF and CE are the medians of the triangle ABC

Medians of the triangle divides it into two equal areas of triangles.

Thus, we have, Ar. $\triangle$ ABF=Ar. $\triangle$ CBF

Subtracting Ar. $\triangle$ BOE on the both the sides, we have

Ar. $\triangle$ ABF - Ar. $\triangle$ BOE=Ar. $\triangle$ CBF - Ar. $\triangle$ BOE

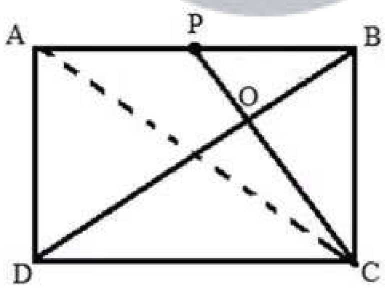
Since, Ar.($\triangle$ BOE)= Ar.($\triangle$ COF),

Ar. $\triangle$ ABF- Ar. $\triangle$ BOE=Ar. $\triangle$ CBF- Ar. $\triangle$ COF

Ar. (quad. AEOF)=Ar.($\triangle$ OBC), hence proved

Solution 4:

(i) Joining AC we have the following figure



Consider the triangles $\triangle POB$ and $\triangle COD$
$\angle POB = \angle DOC$ [vertically opposite angles]
$\angle OPB = \angle ODC$ [AB and DC are parallel, CP and BD are the transversals, alternate interior angles are equal]
Therefore, by Angle – Angle similarity criterion of congruence,
$\triangle POB \sim \triangle COD$
Since P is the midpoint $AP = BP$, and $AB = CD$, we have $CD = 2BP$
Therefore, we have,

$$\frac{BP}{CD} = \frac{OP}{OC} = \frac{OB}{OD} = \frac{1}{2}$$

$\Rightarrow OP : OC = 1:2$

(ii)

Since from part (i), we have

$$\frac{BP}{CD} = \frac{OP}{OC} = \frac{OB}{OD} = \frac{1}{2},$$

Ratio between the areas of two similar triangles is equal to the ratio between the squares of the corresponding sides.

Here, $\triangle DOC$ and $\triangle POB$ are similar triangles.

Thus, we have,

$$\frac{Ar.(\triangle DOC)}{Ar.(\triangle POB)} = \frac{DC^2}{PB^2}$$

$$\Rightarrow \frac{Ar.(\triangle DOC)}{Ar.(\triangle POB)} = \frac{(2PB)^2}{PB^2}$$

$$\Rightarrow \frac{Ar.(\triangle DOC)}{Ar.(\triangle POB)} = \frac{4PB^2}{PB^2}$$

$$\Rightarrow \frac{Ar.(\triangle DOC)}{Ar.(\triangle POB)} = 4$$

$$\Rightarrow Ar.(\triangle DOC) = 4Ar.(\triangle POB)$$
$$= 4 \times 40$$
$$= 160\ cm^2$$

Now consider $Ar.(\triangle DBC) = Ar.(\triangle DOC) + Ar.(\triangle BOC)$
$$= 160 + 80$$
$$= 240\ cm^2$$

Two triangles are equal in area if they are on the equal bases and between the same parallels.

Therefore, $Ar.(\triangle DBC) = Ar.(\triangle ABC) = 240\ cm^2$

Median divides the triangle into areas of two equal triangles.

Thus, CP is the median of the triangle ABC.

Hence, $Ar.(\triangle ABC) = 2Ar.(\triangle PBC)$

$$\Rightarrow Ar.(\triangle PBC) = \frac{Ar.(\triangle ABC)}{2}$$

$$\Rightarrow Ar.(\triangle PBC) = 120\ cm^2$$

(iii)

From part(ii) we have,

$Ar.(\triangle ABC) = 2Ar.(PBC) = 240\ cm^2$

Area of a triangle is half the area of the Parallelogram if both are on equal bases and between the same parallels.

Thus, $Ar.(\triangle ABC) = \frac{1}{2} Ar.(||gm\ ABCD)$

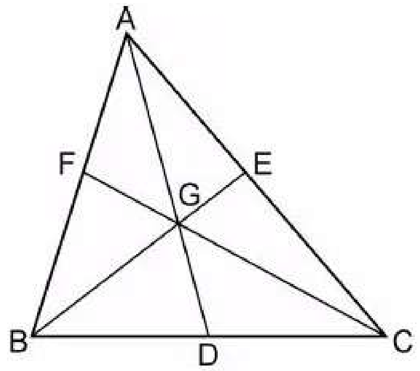
$$\Rightarrow Ar.(||gm\ ABCD) = 2\ Ar.(\triangle ABC)$$

$$\Rightarrow Ar.(||gm\ ABCD) = 2 \times 240$$

$$\Rightarrow Ar.(||gm\ ABCD) = 480\ cm^2$$

Solution 5:

(i) The figure is shown below



Medians intersect at centroid.

Given that G is the point of intersection of medians and hence G is the centroid of the triangle ABC.

Centroid divides the medians in the ratio 2:1

That is $AG:GD = 2:1$

Since BG divides AD in the ratio 2:1, *we have,*

$$\frac{Area(\triangle AGB)}{Area(\triangle BGD)} = \frac{2}{1}$$

$\Rightarrow Area(\triangle AGB) = 2Area(\triangle BGD)$

From the figure, it is clear that,

$Area(\triangle ABD) = Area(\triangle AGB) + Area(\triangle BGD)$

$\Rightarrow Area(\triangle ABD) = 2Area(\triangle BGD) + Area(\triangle BGD)$

$\Rightarrow Area(\triangle ABD) = 3Area(\triangle BGD)....(1)$

(ii)

Medians intersect at centroid.

Given that G is the point of intersection of medians and hence G is the centroid of the triangle ABC.

Centroid divides the medians in the ratio 2:1

That is $AG:GD = 2:1$

Similarly CG divides AD in the ratio 2:1, *we have,*

$$\frac{Area(\triangle AGC)}{Area(\triangle CGD)} = \frac{2}{1}$$

$\Rightarrow Area(\triangle AGC) = 2Area(\triangle CGD)$

From the figure, it is clear that,

$Area(\triangle ACD) = Area(\triangle AGC) + Area(\triangle CGD)$

$\Rightarrow Area(\triangle ACD) = 2Area(\triangle CGD) + Area(\triangle CGD)$

$\Rightarrow Area(\triangle ACD) = 3Area(\triangle CGD)....(2)$

(iii)

Adding equations (1) *and* (2), *we have,*

$Area(\triangle ABD) + Area(\triangle ACD) = 3Area(\triangle BGD) + 3Area(\triangle CGD)$

$\Rightarrow Area(\triangle ABC) = 3[Area(\triangle BGD) + Area(\triangle CGD)]$

$\Rightarrow Area(\triangle ABC) = 3[Area(\triangle BGC)]$

$$\Rightarrow \frac{Area(\triangle ABC)}{3} = [Area(\triangle BGC)]$$

$$\Rightarrow Area(\triangle BGC) = \frac{1}{3}Area(\triangle ABC)$$

Solution 6:

Consider that the sides be x cm, y cm and (37-x-y) cm. also, consider that the lengths of altitudes be 6a cm, 5a cm and 4a cm.

$\therefore$ Area of a triangle= $\frac{1}{2} \times$ base $\times$ altitude

$$\therefore \frac{1}{2} \times x \times 6a = \frac{1}{2} \times y \times 5a = \frac{1}{2} \times (37 - x - y) \times 4a$$

$$6x = 5y = 148 - 4x - 4y$$

$6x = 5y$ and $6x = 148 - 4x - 4y$

$6x - 5y = 0$ and $10x + 4y = 148$

Solving both the equations, we have

X=10 cm, y=12 cm and (37-x-y)cm=15 cm

Solution 7:

(i)

Consider the triangles $\triangle AFE$ and $\triangle DFC$.

$\angle AFE = \angle DEC$ [Vertically opposite angles]

$\angle FAE = \angle DCF$ [AB and DC are parallel lines, AC is a transversal, alternate interior angles are equal]

Thus, by Angle – Angle similarity, we have,

$\triangle AFE \sim \triangle DFC$

Therefore, we have,

$$\frac{DF}{FE} = \frac{DC}{AE} = \frac{CF}{AF} = \frac{2}{1}$$

$\Rightarrow DF : FE = 2 : 1$

(ii)

Since from part(i) we have DF:FE = 2:1, therefore,

$Area(\triangle DCF) = 4Area(\triangle AFE)...(1)$

Also we know that,

$Area(\triangle ADF) + Area(\triangle AFE) = Area(\triangle ADE)$

$\Rightarrow 60 + Area(\triangle AFE) = Area(\triangle ADE) \quad [Area(\triangle ADF) = 60\ cm^2]$

$\Rightarrow 2Area(\triangle ADE) = 2[60 + Area(\triangle AFE)]$

Median divides the triangle into two equal areas of triangle.

Therefore, $2Area(\triangle ADE) = Area(\triangle ABD)$

$\Rightarrow Area(\triangle ABD) = 2[60 + Area(\triangle AFE)]$

$\Rightarrow Area(\triangle ABD) = 120 + 2Area(\triangle AFE)...(2)$

Triangles with equal bases and between the parallels are of equal area.

$Area(\triangle ABD) = Area(\triangle ACD)$

Thus, Equation (2), becomes,

$Area(\triangle ACD) = 120 + 2Area(\triangle AFE)...(3)$

From the figure, it is clear that,

$Area(\triangle ACD) = Area(\triangle DCF) + Area(\triangle ADF)$

$\Rightarrow Area(\triangle ACD) = Area(\triangle DCF) + 60$

$\Rightarrow Area(\triangle ACD) = 4Area(\triangle AEF) + 60...(4)$

Equating equations (3) and (4), we have,

$120 + 2Area(\triangle AFE) = 4Area(\triangle AEF) + 60$

$\Rightarrow 2Area(\triangle AFE) = 60$

$\Rightarrow Area(\triangle AFE) = \frac{60}{2}$

$\Rightarrow Area(\triangle AFE) = 30$

$\Rightarrow Arear(\triangle ADE) = Area(\triangle ADF) + Area(\triangle AFE)$

$\Rightarrow Arear(\triangle ADE) = 60 + 30$

$\Rightarrow Arear(\triangle ADE) = 90\ cm^2$

(iii)

Median of a trianlge divides it intot two equal areas of triangle.

$Arear(\triangle ADB) = 2Arear(\triangle ADE)$

$\Rightarrow Arear(\triangle ADB) = 2Arear(\triangle ADE)$

$\Rightarrow Arear(\triangle ADB) = 2 \times 90\ cm^2$

$\Rightarrow Arear(\triangle ADB) = 180\ cm^2$

(iv)

Since DB divides the parallelogram ABCD into two equal triangle, therefore Area of $\triangle$DBC= Area of $\triangle$ADB

$=180\ cm^2$

Thus the area of the parallelogram ABCD = Area of $\triangle$ADB+ Area of $\triangle$DBC

$=180\ cm^2 + 180\ cm^2$

$=360\ cm^2$

Solution 8:

Here BCED is a parallelogram, since BD = CE and BD || CE.
ar.(ΔDBC) = ar.(ΔEBC)...(Since they have the same base and are between the same parallels)
In ΔABC,
BE is the median,
So, ar.(ΔEBC) = $\frac{1}{2}$ar.(ΔABC)
Now, ar.(ΔABC) = ar.(ΔEBC) + ar.(ΔABE)
Also, ar.(ΔABC) = 2ar.(ΔEBC)
⇒ ar.(ΔABC) = 2ar.(ΔDBC)

Solution 9:

Given :
ΔCAD = 140 cm^2
ΔODC = 172 cm^2
AB || CD
As Triangle DBC and ΔCAD have same base CD and between the same parallel lines Hence,
Area of ΔDBC = Area of ΔCAD = 140 cm^2
Area of ΔOAC = Area of ΔCAD + Area of ΔODC = 140 cm^2 + 172 cm^2 = 312 cm^2
Area of ΔODB = Area of ΔDBC + Area of ΔODC = 140 cm^2 + 172 cm^2 = 312 cm^2