EXERCISE 6.1

1. Find the remainder (without division) on dividing f(x) by (x - 2) where

(i) $f(x) = 5x^2 - 7x + 4$

Solutions:-

Let us assume $x - 2 = 0$

Then, $x = 2$

Given, $f(x) = 5x^2 - 7x + 4$

Now, substitute the value of x in f(x),

$f(2) = (5 \times 2^2) - (7 \times 2) + 4$

$= (5 \times 4) - 14 + 4$

$= 20 - 14 + 4$

$= 24 - 14$

$= 10$

Therefore, the remainder is 10.

(ii) $f(x) = 2x^3 - 7x^2 + 3$

Solution:-

Let us assume $x - 2 = 0$

Then, $x = 2$

Given, $f(x) = 2x^3 - 7x^2 + 3$

Now, substitute the value of x in f(x),

$f(2) = (2 \times 2^3) - (7 \times 2^2) + 3$

$= (2 \times 8) - (7 \times 4) + 3$

$= 16 - 28 + 3$

$= 19 - 28$

$= -9$

Therefore, the remainder is -9.

2. Using the remainder theorem, find the remainder on dividing f(x) by (x + 3) where

(i) $f(x) = 2x^2 - 5x + 1$

Solution:-

Let us assume $x + 3 = 0$

Then, $x = -3$

Given, $f(x) = 2x^2 - 5x + 1$

Now, substitute the value of x in f(x),

$f(-3) = (2 \times -3^2) - (5 \times (-3)) + 1$

$= (2 \times 9) - (-15) + 1$

= 18 + 15 + 1
= 34
Therefore, the remainder is 34.

(ii) $f(x) = 3x^3 + 7x^2 – 5x + 1$
Solution:-
Let us assume x + 3 = 0
Then, x = -3
Given, $f(x) = 3x^3 + 7x^2 – 5x + 1$
Now, substitute the value of x in f(x),
$f(-3)= (3 \times -3^3) + (7 \times -3^2) – (5 \times -3) + 1$
= (3 × -27) + (7 × 9) – (-15) + 1
= - 81 + 63 + 15 + 1
= -81 + 79
= -2
Therefore, the remainder is -2.

3. Find the remainder (without division) on dividing f(x) by (2x + 1) where,
(i) $f(x) = 4x^2 + 5x + 3$

Solution:-
Let us assume 2x + 1 = 0
Then, 2x = -1
X = -½
Given, $f(x) = 4x^2 + 5x + 3$
Now, substitute the value of x in f(x),
$f (-½) = 4 (-½)^2 + 5 (-½) + 3$
= (4 × ¼) + (-5/2) + 3
= 1 – 5/2 + 3
= 4 – 5/2
= (8 - 5)/2
= 3/2 = 1½
Therefore, the remainder is 1½.

(ii) $f(x) = 3x^3 – 7x^2 + 4x + 11$
Solution:-
Let us assume 2x + 1 = 0
Then, 2x = -1

$X = -½$

Given, $f(x) = 3x^3 – 7x^2 + 4x + 11$

Now, substitute the value of x in f(x),

$f(-½) = (3 \times (-½)^3) – (7 \times (-½)^2 + (4 \times -½) + 11$

$= 3 \times (-1/8) – (7 \times ¼) + (- 2) + 11$

$= -3/8 – 7/4 – 2 + 11$

$= - 3/8 – 7/4 + 9$

$= (-3 – 14 + 72)/8$

$= 55/8$

$= 6\frac{7}{8}$

4. Using remainder theorem, find the value of k if on dividing $2x^3 + 3x^2 – kx + 5$ by $x – 2$ leaves a remainder 7.

Solution:-

Let us assume, $x – 2 = 0$

Then, $x = 2$

Given, $2x^3 + 3x^2 – kx + 5$

Now, substitute the value of x in f(x),

$f(2) = (2 \times 2^3) + (3 \times 2^2) – (k \times 2) + 5$

$= (2 \times 8) + (3 \times 4) – 2k + 5$

$= 16 + 12 – 2k + 5$

$= 33 – 2k$

Form the question it is given that, remainder is 7.

So, $7 = 33 – 2k$

$2k = 33 – 7$

$2k = 26$

$K = 26/2$

$K = 13$

Therefore, the value of k is 13.

5. Using remainder theorem, find the value of 'a' if the division of $x^3 + 5x^2 – ax + 6$ by $(x - 1)$ leaves the remainder 2a.

Solution:-

Let us assume $x -1 = 0$

Then, $x = 1$

Given, $f(x) = x^3 + 5x^2 – ax + 6$

Now, substitute the value of x in f(x),

$f(1) = 1^3 + (5 \times 1^2) - (a \times 1) + 6$

$= 1 + 5 - a + 6$

$= 12 - a$

From the question it is given that, remainder is 2a

So, $2a = 12 - a$

$2a + a = 12$

$3a = 12$

$a = 12/3$

$a = 4$

Therefore, the value of a is 4.

6. (i) What number must be divided be subtracted from $2x^2 - 5x$ so that the resulting polynomial leaves the remainder 2, when divided by $2x + 1$?

Solution:-

let us assume 'p' be subtracted from $2x^2 - 5x$

So, dividing $2x^2 - 5x$ by $2x + 1$,

$2x + 1)\ 2x^2 - 5x - p\ (x - 3$

$2x^2 + x$

$-\quad -$

$-6x - p$

$-6x - 3$

$+\quad +$

$-p + 3$

Hence, remainder is $3 - p$

From the question it is given that, remainder is 2.

$3 - p = 2$

$p = 3 - 2$

$p = 1$

Therefore, 1 is to be subtracted.

(ii) What number must be added to $2x^3 - 7x^2 + 2x$ so that the resulting polynomial leaves the remainder – 2 when divided by $2x - 3$?

Solution:-

let us assume 'p' be subtracted from $2x^3 - 7x^2 + 2x$,

So, dividing it by $2x - 3$,

$2x - 3)\ 2x^3 - 7x^2 + 2x + p\ (x^2 - 2x - 2$

$2x^3 - 3x^2$

$-\quad -$

$-4x^2 + 2x$

$-4x^2 + 6x$

$+\quad -$

$-4x + p$

$-4x + 6$

$+\quad -$

$P - 6$

Hence, remainder is $p - 6$

From the question it is given that, remainder is - 2.

$P - 6 = -2$

$P = -2 + 6$

$P = 4$

Therefore, 4 is to be added.

7. (i) When divided by $x - 3$ the polynomials $x^3 - px^2 + x + 6$ and $2x^3 - x^2 - (p + 3)x - 6$ leave the same remainder. Find the value of 'p'.

Solution:-

From the question it is given that, by dividing $x^3 - px^2 + x + 6$ and $2x^3 - x^2 - (p + 3)x - 6$ by $x - 3 = 0$, then $x = 3$.

Let us assume $p(x) = x^3 - px^2 + x + 6$

Now, substitute the value of x in p(x),

$p(3) = 3^3 - (p \times 3^2) + 3 + 6$

$= 27 - 9p + 9$

$= 36 - 9p$

Then, $q(x) = 2x^3 - x^2 - (p + 3)x - 6$

Now, substitute the value of x in q(x),

$q(3) = (2 \times 3^3) - (3)^2 - (p + 3) \times 3 - 6$

$= (2 \times 27) - 9 - 3p - 9 - 6$

$= 54 - 24 - 3p$

$= 30 - 3p$

Given, the remainder in each case is same,

So, $36 - 9p = 30 - 3p$

$36 - 30 = 9p - 3p$

$6 = 6p$

p = 6/6
p = 1
Therefore, value of p is 1.

(ii) Find 'a' if the two polynomials $ax^3 + 3x^2 - 9$ and $2x^3 + 4x + a$, leaves the same remainder when divided by x + 3.
Solution:-
Let us assume $p(x) = ax^3 + 3x^2 - 9$ and $q(x) = 2x^3 + 4x + a$
From the question it is given that, both p(x) and q(x) leaves the same remainder when divided by x + 3.
Let us assume that, x + 3 = 0
Then, x = -3
Now, substitute the value of x in p(x) and in q(x),
So, p(-3) = q(-3)
$a(-3)^3 + 3(-3)^2 - 9 = 2(-3)^3 + 4(-3) + a$
-27a + 27 – 9 = - 54 – 12 + a
-27a + 18 = - 66 + a
-27a – a = -66 – 18
-28 a = -84
a = 84/28
Therefore, a = 3

(iii) The polynomials $ax^3 + 3x^2 - 3$ and $2x^3 - 5x + a$ when divided by x – 4 leave the remainder r_1 and r_2 respectively. If $2r_1 = r_2$, then find the value of a.
Solution:
Let us assume $p(x) = ax^3 + 3x^2 - 3$ and $q(x) = 2x^3 - 5x + a$
From the question it is given that, both p(x) and q(x) leaves the remainder r_1 and r_2 respectively when divided by x - 4.
Also, given relation $2r_1 = r_2$
Let us assume that, x - 4 = 0
Then, x = 4
Now, substitute the value of x in p(x) and in q(x),
By factor theorem, $r_1 = p(x)$ and $r_2 = q(x)$
So, 2 × p(4) = q(4)
$2[a(4)^3 + 3(4)^2 - 3] = 2(4)^3 - 5(4) + a$
2[64a + 48 - 3] = 128 – 20 + a
128a + 96 – 6 = 128 – 20 + a

128a + 90 = 108 + a
128a – a = 108 – 90
127a = 18
a = 18/127
Therefore, the value of a = 18/127.

8. Using remainder theorem, find the remainders obtained when $x^3 + (kx + 8)x + k$ Is divided by x + 1 and x – 2. Hence, find k if the sum of two remainders is 1.
Solution:
Let us assume $p(x) = x^3 + (kx + 8)x + k$
From the question it is given that, the sum of the remainders when p(x) is divided by (x + 1) and (x – 2) is 1.
Let us assume that, x + 1 = 0
Then, x = -1
Also, when x – 2 = 0
Then, x = 2
Now, by remainder theorem we have
p(-1) + p(2) = 1
$(-1)^3 + [k(-1) + 8](-1) + k + (2)^3 + [k(2) + 8](2) + k = 1$

-1 + k – 8 + k + 8 + 4k + 16 + k = 1
7k + 15 = 1
7k = 1 – 15
k = -14/7
k = -2
Therefore, k = -2.

9. By factor theorem, show that (x + 3) and (2x - 1) are factors of $2x^2 + 5x – 3$.
Solution:-
Let us assume, x + 3 = 0
Then, x = - 3
Given, $f(x) = 2x^2 + 5x – 3$
Now, substitute the value of x in f(x),
$f(-3) = (2 \times (-3)^2) + (5 \times -3) – 3$
$= (2 \times 9) + (-15) – 3$
= 18 – 15 – 3
= 18 – 18
= 0

Now, $2x – 1 = 0$
Then, $2x = 1$
$x = ½$
Given, $f(x) = 2x^2 + 5x – 3$
Now, substitute the value of x in f(x),
$f(½) = (2 × (½)^2) + (5 × ½) – 3$
$= (2 × (¼)) + 5/2 – 3$
$= ½ + 5/2 – 3$
$= (1 + 5)/2 – 3$
$= 6/2 – 3$
$= 3 -3$
$= 0$
Hence, it is proved that, (x + 3) and (2x - 1) are factors of $2x^2 + 5x – 3$.

10. Without actual division, prove that $x^4 + 2x^3 – 2x^2 + 2x + 3$ is exactly divisible by $x^2 + 2x – 3$.
Solution:-
Consider $x^2 + 2x - 3$
By factor method, $x^2 + 3x – x – 3$
$= x (x + 3) – 1(x + 3)$
$= (x - 1) (x + 3)$
So, $f(x) = x^4 + 2x^3 – 2x^2 + 2x + 3$
Now take, $x + 3 = 0$
$X = -3$
Then, $f(-3) = (-3)^4 + 2 × -(3^3) – (2 × (-3)^2) + (2 × -3) + 3$
$= 81 – 54 – 18 – 6 – 3$
$= 0$
Therefore, (x + 3) is a factor of f(x)
And also, take $x – 1 = 0$
$X = 1$
Then, $f(1) = 1^4 + 2(1)^3 – 2(1)^2 + 2(1) – 3$
$= 0$
Therefore, (x - 1) is a factor of f(x)
By comparing both results, p(x) is exactly divisible by $x^2 + 2x – 3$.

11. Show that (x - 2) is a factor of $3x^2 – x – 10$. Hence factories $3x^2 – x – 10$.
Solution:-

Let us assume x – 2 = 0
Then, x = 2
Given, $f(x) = 3x^2 – x – 10$
Now, substitute the value of x in f(x),
$f(2) = (3 \times 2^2) – 2 – 10$
$= (3 \times 4) – 2 – 10$
$= 12 – 2 – 10$
$= 12 – 12$
$= 0$
Therefore, (x - 2) is a factor of f(x)
Then, dividing $(3x^2 – x - 10)$ by (x - 2), we get

$x-2)\ 3x^2 – x – 10\ (3x + 5$
$3x^2 + x$
$- \quad +$
$5x – 10$
$5x – 10$
$- \quad +$
0

Therefore, $3x^2 – x – 10 = (x - 2)(3x + 5)$

12. Using the factor theorem, show that (x - 2) is a factor of $x^3 + x^2 - 4x – 4$. Hence factorize the polynomial completely.
Solution:-
Let us assume, x – 2 = 0
Then, x = 2
Given, $f(x) = x^3 + x^2 - 4x – 4$
Now, substitute the value of x in f(x),
$f(2) = (2)^3 + (2)^2 – 4(2) – 4$
$= 8 – 4 – 8 – 4$
$= 0$
Therefore, by factor theorem (x - 2) is a factor of $x^3 + x^2 - 4x – 4$
Then, dividing f(x) by (x - 2), we get

$$
\begin{array}{r|llll}
 & x^2 & +3x & +2 & \\
\hline
x-2 & x^3 & +x^2 & -4x & -4 \\
 & - & & & \\
 & x^3 & -2x^2 & & \\
\hline
 & & 3x^2 & -4x & -4 \\
 & & - & & \\
 & & 3x^2 & -6x & \\
\hline
 & & & 2x & -4 \\
 & & & - & \\
 & & & 2x & -4 \\
\hline
 & & & & 0
\end{array}
$$

Therefore, $x^3 + x^2 - 4x – 4 = (x - 2) (x^2 + 3x + 2)$
$= (x – 2) (x^2 + 2x + x + 2)$
$= (x – 2) (x(x + 2) + 1(x + 2))$
$= (x – 2) (x + 2) (x + 1)$

13. Show that 2x + 7 is a factor of $2x^3 + 5x^2 – 11x – 14$. Hence factorize the given expression completely, using the factor theorem.

Solution:-

Let us assume 2x + 7 = 0

Then, 2x = -7

X = -7/2

Given, $f(x) = 2x^3 + 5x^2 – 11x – 14$

Now, substitute the value of x in f(x),

$f(-7/2) = 2(-7/2)^3 + 5(-7/2)^2 + 11(-7/2) – 14$
$= 2(-343/8) + 5(49/4) + (-77/2) – 14$
$= -343/4 + 245/4 – 77/2 – 14$
$= (-343 + 245 + 154 - 56)/4$
$= -399 + 399/4$
$= 0$

Therefore, (2x + 7) is a factor of $2x^3 + 5x^2 – 11x – 14$

Then, dividing f(x) by (2x + 1), we get

$$
\begin{array}{r|l}
 & x^2 \quad -x \quad -2 \\
\hline
2x+7 & 2x^3 \quad +5x^2 \quad -11x \quad -14 \\
 & - \\
 & 2x^3 \quad +7x^2 \\
\hline
 & \quad -2x^2 \quad -11x \quad -14 \\
 & - \\
 & \quad -2x^2 \quad -7x \\
\hline
 & \quad \quad -4x \quad -14 \\
 & - \\
 & \quad \quad -4x \quad -14 \\
\hline
 & \quad \quad \quad 0
\end{array}
$$

Therefore, $2x^3 + 5x^2 – 11x – 14 = (2x + 7) (x^2 – x - 2)$

$= (2x + 7) (x^2 – 2x + x - 2)$

$= (2x + 7) (x(x - 2) + 1 (x - 2))$

$= (x + 1) (x - 2) (2x + 7)$

14. Use factor theorem to factorize the following polynomials completely.

(i) $x^3 + 2x^2 – 5x – 6$

Solution:-

Let us assume x = -1,

Given, $f(x) = x^3 + 2x^2 – 5x – 6$

Now, substitute the value of x in f(x),

$f(-1) = (-1)^3 + 2(-1)^2 – 5(-1) – 6$

$= -1 +2 (1) + 5 – 6$

$= -1 +2 + 5 – 6$

$= -7 + 7$

$= 0$

Then, dividing f(x) by (x + 1), we get

$$
\begin{array}{r|rrrr}
 & x^2 & +x & -6 & \\
\hline
x+1 & x^3 & +2x^2 & -5x & -6 \\
 & - & & & \\
 & x^3 & +x^2 & & \\
\hline
 & & x^2 & -5x & -6 \\
 & & - & & \\
 & & x^2 & +x & \\
\hline
 & & & -6x & -6 \\
 & & & - & \\
 & & & -6x & -6 \\
\hline
 & & & & 0
\end{array}
$$

Therefore, $x^3 + 2x^2 – 5x – 6 = (x + 1) (x^2 + 3x – 2x - 6)$

$= (x + 1) (x(x + 3) – 2(x + 3))$

$= (x + 1) (x - 2) (x + 3)$

(ii) $x^3 – 13x – 12$

Solution:-

Let us assume x = -1,

Given, $f(x) = x^3 – 13x – 12$

Now, substitute the value of x in f(x),

$f(-1) = (-1)^3 - 13(-1) – 12$

$= -1 + 13 - 12$

$= - 13 + 13$

$= 0$

Then, dividing f(x) by (x + 1), we get

$$
\begin{array}{r|l}
 & x^2 \quad -x \quad -12 \\
\hline
x+1 & x^3 \quad +0x^2 \quad -13x \quad -12 \\
 & - \\
 & x^3 \quad +x^2 \\
\hline
 & \quad -x^2 \quad -13x \quad -12 \\
 & \quad - \\
 & \quad -x^2 \quad -x \\
\hline
 & \quad\quad -12x \quad -12 \\
 & \quad\quad - \\
 & \quad\quad -12x \quad -12 \\
\hline
 & \quad\quad\quad 0
\end{array}
$$

Therefore, $x^3 - 13x - 12 = (x + 1)(x^2 - x - 12)$

$= (x + 1)(x^2 - 4x + 3x - 12)$

$= (x + 1)(x(x - 4)) + 3(x - 4))$

$= (x + 1)(x + 3)(x - 4)$

15. Use the remainder theorem to factorize the following expression.

(i) $2x^3 + x^2 - 13x + 6$

Solution:-

Let us assume x = 2,

Then, $f(x) = 2x^3 + x^2 - 13x + 6$

Now, substitute the value of x in f(x),

$f(2) = (2 \times 2^3) + 2^2 - 13 \times 2 + 6$

$= (2 \times 8) + 4 - 26 + 6$

$= 16 + 4 - 26 + 6$

$= 26 - 26$

$= 0$

Then, dividing f(x) by (x - 2), we get

$$
\begin{array}{r|llll}
 & 2x^2 & +5x & -3 & \\
\hline
x-2 & 2x^3 & +x^2 & -13x & +6 \\
 & - & & & \\
 & 2x^3 & -4x^2 & & \\
\hline
 & & 5x^2 & -13x & +6 \\
 & & - & & \\
 & & 5x^2 & -10x & \\
\hline
 & & & -3x & +6 \\
 & & & - & \\
 & & & -3x & +6 \\
\hline
 & & & & 0
\end{array}
$$

Therefore, $2x^3 + x^2 - 13x + 6 = (x - 2)(2x^2 + 5x - 3)$

$= (x - 2)(2x^2 + 6x - x - 3)$

$= (x - 2)(2x(x + 3) - 1(x + 3))$

$= (x - 2)(x + 3)(2x - 1)$

(ii) $3x^2 + 2x^2 - 19x + 6$

Solution:-

Given, $f(x) = 3x^3 + 2x^2 - 19x + 6$

Let us assume $x = 1$

Then, $f(1) = 3(1)^3 + 2(1)^2 - (19 \times 1) + 6$

$= 3 + 2 - 19 + 6$

$= 11 - 19$

$= -8$

So, $-8 \neq 0$

Let us assume $x = -1$

Then, $f(-1) = 3(-1)^3 + 2(-1)^2 - (19 \times (-1)) + 6$

$= -3 + 2 + 19 + 6$

$= -3 + 27$

$= 24$

So, $24 \neq 0$

Now, assume $x = 2$

Then, $f(2) = 3(2)^3 + 2(2)^2 - (19 \times (2)) + 6$

$= 24 + 8 - 38 + 6$

= 38 - 38

= 0

So, 0 = 0

Therefore, (x - 2) is a factor of f(x).

$f(x) = 3x^3 + 2x^2 – 19x + 6$

$= 3x^3 – 6x^2 + 8x^2 – 16x – 3x + 6$

$= 3x^2 (x - 2) + 8x (x - 2) – 3(x - 2)$

$= (x - 2) (3x^2 + 8x - 3)$

$= (x - 2) (3x^2 + 9x – x – 3)$

$= (x - 2) (3x(x + 3) – 1(x + 3)$

$= (x - 2) (x + 3) (3x - 1)$

(iii) $2x^3 + 3x^2 – 9x – 10$

Solution:-

Given, $f(x) = 2x^3 + 3x^2 – 9x – 10$

Let us assume, x = -1

$= 2(-1)^3 + 3(-1)^2 – 9 (-1) – 10$

$= -2 + 3 + 9 – 10$

$= 12 – 12$

$= 0$

Therefore, (x + 1) is the factor of $2x^3 + 3x^2 – 9x – 10$

Then, dividing f(x) by (x + 1), we get

$$2x^2 \quad +x \quad -10$$

$$x+1 \;\big)\; 2x^3 \quad +3x^2 \quad -9x \quad -10$$

$$-$$

$$2x^3 \quad +2x^2$$

$$x^2 \quad -9x \quad -10$$

$$-$$

$$x^2 \quad +x$$

$$-10x \quad -10$$

$$-$$

$$-10x \quad -10$$

$$0$$

Therefore, $2x^3 + 3x^2 – 9x – 10 = 2x^2 + 5x – 4x – 10$

= x(2x + 5) – 2 (2x + 5) – (2x + 5) (x - 2)

Hence the factors are (x + 1) (x - 2) (2x + 5)

(iv) $x^3 + 10x^2 - 37x + 26$

Solution:-

Given, f(x) = $x^3 + 10x^2 - 37x + 26$

Let us assume, x = 1

Then, f(1) = $1^3 + 10(1)^2 - 37(1) + 26$

= 1 + 10 – 37 + 26

= 37 – 37

= 0

Therefore, x – 1is a factor of $x^3 + 10x^2 - 37x + 26$

Then, dividing f(x) by (x - 1), we get

$$
\begin{array}{r|rrrr}
 & x^2 & +11x & -26 & \\
\hline
x-1 & x^3 & +10x^2 & -37x & +26 \\
 & - & & & \\
 & x^3 & -x^2 & & \\
\hline
 & & 11x^2 & -37x & +26 \\
 & & - & & \\
 & & 11x^2 & -11x & \\
\hline
 & & & -26x & +26 \\
 & & & - & \\
 & & & -26x & +26 \\
\hline
 & & & & 0
\end{array}
$$

Therefore, $x^3 + 10x^2 - 37x + 26 = (x - 1)(x^2 + 11x - 26)$

$= (x - 1)(x^2 + 13x - 2x - 26)$

$= (x - 1)(x(x + 13) - 2(x + 13))$

$= (x - 1)((x - 2)(x + 13))$

16. If (2x + 1) is a factor of $6x^3 + 5x^2 + ax - 2$ find the value of a.

Solution:-

Let us assume 2x + 1 = 0

Then, 2x = - 1

X = -½

Given, $f(x) = 6x^3 + 5x^2 + ax - 2$
Now, substitute the value of x in f(x),
$f(-½) = 6(-½)^3 + 5(-½)^2 + a(-½) - 2$
$= 6(-1/8) + 5(¼) - ½a - 2$
$= -3/4 + 5/4 - a/2 - 2$
$= (-3 + 4 - 2a - 8)/4$
$= (-6 - 2a)/4$
From the question, $(2x + 1)$ is a factor of $6x^3 + 5x^2 + ax - 2$
Then, remainder is 0.
So, $(-6 - 2a)/4 = 0$
$-6 - 2a = 4 \times 0$
$-6 - 2a = 0$
$-2a = 6$
$a = -6/2$
$a = -3$
Therefore, the value of a is – 3.

17. If (3x - 2) is a factor of $3x^3 - kx^2 + 21x - 10$, find the value of k.
Solution:-

Let us assume $3x - 2 = 0$
Then, $3x = 2$
$X = 2/3$
Given, $f(x) = 3x^3 - kx^2 + 21x - 10$
Now, substitute the value of x in f(x),
$f(2/3) = 3(2/3)^3 - k(2/3)^2 + 21(2/3) - 10$
$= 3(8/27) - k(4/9) + 14 - 10$
$= 8/9 - 4k/9 + 14 - 10$
$= 8/9 - 4k/9 + 4$
$= (8 - 4k + 36)/9$
$= (44 - 4k)/9$
From the question, $(3x - 2)$ is a factor of $3x^3 - kx^2 + 21x - 10$
Then, remainder is 0
So, $(44 - 4k)/9 = 0$
$44 - 4k = 0 \times 9$
$44 = 4k$
$K = 44/4$
$K = 11$

18. If (x – 2) is a factor of $2x^3 – x^2 + px – 2$, then (i) find the value of p. (ii) with this value of p, factorize the above expression completely.

Solution:-

Let us assume x -2 = 0

Then, x = 2

Given, $f(x) = 2x^3 – x^2 + px – 2$

Now, substitute the value of x in f(x),

$f(2) = (2 \times 2^3) – 2^2 + (p \times 2) – 2$

$= (2 \times 8) – 4 + 2p – 2$

$= 16 – 4 + 2p – 2$

$= 16 – 6 + 2p$

$= 10 + 2p$

From the question, (x – 2) is a factor of $2x^3 – x^2 + px – 2$

Then, remainder is 0.

10 + 2p = 0

2p = - 10

P = -10/2

P = -5

So, (x – 2) is a factor of $2x^3 – x^2 + 5x – 2$

$$
\begin{array}{r|llll}
 & 2x^2 & +3x & +11 & \\
\hline
x-2 & 2x^3 & -x^2 & +5x & -2 \\
- & 2x^3 & -4x^2 & & \\
\hline
 & & 3x^2 & +5x & -2 \\
 & - & 3x^2 & -6x & \\
\hline
 & & & 11x & -2 \\
 & & - & 11x & -22 \\
\hline
 & & & & 20
\end{array}
$$

Therefore, $2x^3 – x^2 + 5x – 2 = (x - 2) (2x^2 + 3x + 1)$

$= (x - 2) (2x^2 + 2x + x + 1)$

$= (x - 2) (2x(x + 1) + 1(x + 1))$

= (x + 1) (x - 2) (2x + 1)

19. What number should be subtracted from $2x^3 - 5x^2 + 5x$ so that the resulting polynomial has $2x - 3$ as a factor?

Solution:-

Let us assume the number to be subtracted from $2x^3 - 5x^2 + 5x$ be p.

Then, $f(x) = 2x^3 - 5x^2 + 5x - p$

Given, $2x - 3 = 0$

$x = 3/2$

$f(3/2) = 0$

So, $f(3/2) = 2(3/2)^3 - 5(3/2)^2 + 5(3/2) - p = 0$

$2(27/8) - 5(9/4) + 15/2 - p = 0$

$27/4 - 45/4 + 15/2 - p = 0$ [multiply by 4 for all numerator]

$27 - 45 + 30 - 4p = 0$

$57 - 45 - 4p = 0$

$12 - 4p = 0$

$P = 12/4$

$P = 3$

Therefore, 3 is the number should be subtracted from $2x^3 - 5x^2 + 5x$.

20. (i) Find the value of the constants a and b, if (x – 2) and (x + 3) are both factors of the expression $x^3 + ax^2 + bx - 12$.

Solution:-

Let us assume $x - 2 = 0$

Then, $x = 2$

Given, $f(x) = x^3 + ax^2 + bx - 12$

Now, substitute the value of x in f(x),

$f(2) = 2^3 + a(2)^2 + b(2) - 12$

$= 8 + 4a + 2b - 12$

$= 4a + 2b - 4$

From the question, (x - 2) is a factor of $x^3 + ax^2 + bx - 12$.

So, $4a + 2b - 4 = 0$

$4a + 2b = 4$

By dividing both the side by 2 we get,

$2a + b = 2$ … [equation (i)]

Now, assume $x + 3 = 0$

Then, $x = -3$

Given, $f(x) = x^3 + ax^2 + bx - 12$
Now, substitute the value of x in f(x),
$f(-3) = (-3)^3 + a(-3)^2 + b(-3) - 12$
$= -27 + 9a - 3b - 12$
$= 9a - 3b - 39$
From the question, (x - 3) is a factor of $x^3 + ax^2 + bx - 12$.
So, $9a - 3b - 39 = 0$
$9a - 3b = 39$
By dividing both the side by 3 we get,
$3a - b = 13$ … [equation (ii)]
Now, adding both equation (i) and equation (ii) we get,
$(2a + b) + (3a - b) = 2 + 13$
$2a + 3a + b - b = 15$
$5a = 15$
$a = 15/5$
$a = 3$
Consider the equation (i) to find out 'b'.
$2a + b = 2$
$2(3) + b = 2$

$6 + b = 2$
$b = 2 - 6$
$b = -4$

(ii) If (x + 2) and (x + 3) are factors of $x^3 + ax + b$, Find the values of a and b.
Solution:-
Let us assume $x + 2 = 0$
Then, $x = -2$
Given, $f(x) = x^3 + ax + b$
Now, substitute the value of x in f(x),
$f(-2) = (-2)^3 + a(-2) + b$
$= -8 - 2a + b$
From the question, (x + 2) is a factor of $x^3 + ax + b$.
Therefore, remainder is 0.
$f(x) = 0$
$- 8 - 2a + b = 0$
$2a - b = - 8$ … [equation (i)]
Let us assume $x + 3 = 0$

Then, $x = -3$

Given, $f(x) = x^3 + ax + b$

Now, substitute the value of x in f(x),

$f(-2) = (-3)^3 + a(-3) + b$

$= -27 – 3a + b$

From the question, $(x + 3)$ is a factor of $x^3 + ax + b$.

Therefore, remainder is 0.

$f(x) = 0$

$- 27 – 3a + b = 0$

$3a - b = - 27$ … [equation (i)]

Now, subtracting both equation (i) and equation (ii) we get,

$(2a – b) - (3a - b) = -8 – (-27)$

$2a - 3a – b + b = - 8 + 27$

$-a = 19$

$a = -19$

Consider the equation (i) to find out 'b'.

$2a - b = - 8$

$2(-19) – b = -8$

$-38 – b = - 8$

$b = -38 +8$

$b = -30$

21 . If (x + 2) and (x – 3) are factors of $x^3 + ax + b$, find the values of a and b. With these values of a and b, factorize the given expression.

Solution:-

Let us assume $x + 2 = 0$

Then, $x = -2$

Given, $f(x) = x^3 + ax + b$

Now, substitute the value of x in f(x),

$f(-2) = (-2)^3 + a(-2) + b$

$= -8 – 2a + b$

From the question, $(x + 2)$ is a factor of $x^3 + ax + b$.

Therefore, remainder is 0.

$f(x) = 0$

$- 8 – 2a + b = 0$

$2a - b = - 8$ … [equation (i)]

Now, assume $x – 3 = 0$

Then, $x = 3$

Given, $f(x) = x^3 + ax + b$

Now, substitute the value of x in f(x),

$f(3) = (3)^3 + a(3) + b$

$= 27 + 3a + b$

From the question, $(x - 3)$ is a factor of $x^3 + ax + b$.

Therefore, remainder is 0.

$f(x) = 0$

$27 + 3a + b = 0$

$3a + b = - 27$ … [equation (ii)]

Now, adding both equation (i) and equation (ii) we get,

$(2a - b) + (3a + b) = - 8 – 27$

$2a – b + 3a + b = -35$

$5a = -35$

$a = -35/5$

$a = -7$

Consider the equation (i) to find out 'b'.

$2a – b = - 8$

$2(-7) – b = -8$

$-14 – b = -8$

$b = - 14 + 8$

$b = -6$

Therefore, value of $a = -7$ and $b = -6$.

Then, $f(x) = x^3 – 7x – 6$

$(x + 2) (x - 3)$

$= x(x - 3) + 2(x - 3)$

$= x^2 – 3x + 2x – 6$

$= x^2 – x – 6$

Dividing f(x) by $x^2 – x – 6$ we get,

$$
\begin{array}{r|rrrr}
 & x & +1 & & \\
\hline
x^2 - x - 6 & x^3 & +0x^2 & -7x & -6 \\
- & & & & \\
 & x^3 & -x^2 & -6x & \\
\hline
 & & x^2 & -x & -6 \\
 & & - & & \\
 & & x^2 & -x & -6 \\
\hline
 & & & & 0
\end{array}
$$

Therefore, $x^3 – 7x – 6 = (x + 1) (x + 2) (x - 3)$

22. (x – 2) is a factor of the expression x³ + ax² + bx + 6. When this expression is divided by (x – 3), it leaves the remainder 3. Find the values of a and b.

Solution:-

From the question it is given that, $(x – 2)$ is a factor of the expression $x^3 + ax^2 + bx + 6$

Then, $f(x) = x^3 + ax^2 + bx + 6$ … [equation (i)]

Let assume $x – 2 = 0$

Then, $x = 2$

Now, substitute the value of x in f(x),

$f(2) = 2^3 + a(2)^2 + 2b + 6$

$= 8 + 4a + 2b + 6$

$= 14 + 4a + 2b$

By dividing the numbers by 2 we get,

$= 7 + 2a + b$

From the question, $(x – 2)$ is a factor of the expression $x^3 + ax^2 + bx + 6$.

So, remainder is 0.

$f(x) = 0$

$7 + 2a + b = 0$

$2a + b = -7$ … [equation (ii)]

Now, expression is divided by $(x – 3)$, it leaves the remainder 3.

$$\begin{array}{r|l} & x^2 \quad +x(a+3) \quad +3a+b+9 \\ x-3 & x^3 \quad +ax^2 \quad +bx \quad +6 \\ & - \\ & x^3 \qquad -3x^2 \\ \hline & x^2(a+3) \qquad +bx \qquad +6 \\ & - \\ & x^2(a+3) \quad +x(-3a-9) \\ \hline & x(3a+b+9) \qquad +6 \\ & - \\ & x(3a+b+9) \quad +-9a-3b-27 \\ \hline & 9a+3b+33 \end{array}$$

So, remainder = 33 + 9a + 3b = 3

9a + 3b = 3 – 33

9a + 3b = -30

By dividing the numbers by 3 we get,

= 3a + b = - 10 … [equation (iii)]

Now, subtracting equation (iii) from equation (ii) we get,

(3a + b) – (2a + b) = - 10 – (-7)

3a – 2a + b – b = - 10 + 7

a = -3

Consider the equation (ii) to find out ‘b’.

2a + b = - 7

2(-3) + b = - 7

-6 + b = - 7

b = - 7 + 6

b = - 1

23. If (x – 2) is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by (x – 3), it leaves a remainder 52, find the values of a and b.

Solution:-

From the question it is given that, (x – 2) is a factor of the expression $2x^3 + ax^2 + bx - 14$

Then, $f(x) = 2x^3 + ax^2 + bx - 14$ … [equation (i)]

Let assume x – 2 = 0

Then, x = 2

Now, substitute the value of x in f(x),

$f(2) = 2(2)^3 + a(2)^2 + 2b - 14$

$= 16 + 4a + 2b - 14$

$= 2 + 4a + 2b$

By dividing the numbers by 2 we get,

$= 1 + 2a + b$

From the question, (x – 2) is a factor of the expression $2x^3 + ax^2 + bx - 14$.

So, remainder is 0.

f(x) = 0

1 + 2a + b = 0

2a + b = -1 … [equation (ii)]

Now, expression is divided by (x – 3), it leaves the remainder 52.

$$2x^2 \quad +x(a+6)$$

$$x - 3 \;\big)\; 2x^3 \quad +ax^2 \quad +bx \quad -14$$

$$-$$

$$2x^3 \qquad -6x^2$$

$$x^2(a+6) \qquad +bx \qquad -14$$

$$-$$

$$x^2(a+6) \quad +x(-3a-18)$$

$$-14$$

$$-$$

$$x(3a+b+18) \quad +-9a-3b-54$$

$$9a+3b+40$$

So, remainder = 9a + 3b + 40 = 52

9a + 3b = 52 – 40

9a + 3b = 12

By dividing the numbers by 3 we get,

= 3a + b = 4 … [equation (iii)]

Now, subtracting equation (iii) from equation (ii) we get,

(3a + b) – (2a + b) = 4 – (-1)

3a – 2a + b – b = 4 + 1

a = 5

a = 5

Consider the equation (ii) to find out 'b'.

$2a + b = -1$

$2(5) + b = -1$

$10 + b = -1$

$b = -1 - 10$

$b = -11$

24. If $ax^3 + 3x^2 + bx - 3$ has a factor $(2x + 3)$ and leaves remainder -3 when divided by $(x + 2)$, find the values of a and b. With these values of a and b, factorize the given expression.

Solution:-

Let us assume, $2x + 3 = 0$

Then, $2x = -3$

$x = -3/2$

Given, $f(x) = ax^3 + 3x^2 + bx - 3$

Now, substitute the value of x in f(x),

$f(-3/2) = a(-3/2)^3 + 3(-3/2)^2 + b(-3/2) - 3$

$= a(-27/8) + 3(9/4) - 3b/2 - 3$

$= -27a/8 + 27/4 - 3b/2 - 3$

From the question it is given that, $ax^3 + 3x^2 + bx - 3$ has a factor $(2x + 3)$.

So, remainder is 0.

$-27a/8 + 27/4 - 3b/2 - 3 = 0$

$-27a + 54 - 12b - 24 = 0$

$-27a - 12b = -30$

By dividing the numbers by - 3 we get,

$9a + 4b = 10$ [equation (i)]

Now, let us assume $x + 2 = 0$

Then, $x = -2$

Given, $f(x) = ax^3 + 3x^2 + bx - 3$

Now, substitute the value of x in f(x),

$f(2) = a(-2)^3 + 3(-2)^2 + b(-2) - 3$

$= -8a + 12 - 2b - 3$

$= -8a - 2b + 9$

Leaves the remainder -3

So, $-8a - 2b + 9 = -3$

$-8a - 2b = -3 - 9$

$-8a - 2b = -12$

By dividing both sides by -2 we get,
4a + b = 6 [equation (ii)]
By multiplying equation (ii) by 4,
16a + 4b = 24
Now, subtracting equation (ii) from equation (i) we get,

(16a + 4b) - (9a + 4b) = 24 – 10
16a - 9a + 4b – 4b = 14
7a = 14
a = 14/7
a = 2

Consider the equation (i) to find out ‘b’.

9a + 4b = 10
9(2) + 4b = 10
18 + 4b = 10
4b = 10 – 18
4b = -8
b = -8/4
b = -2

Therefore, $f(x) = ax^3 + 3x^2 + bx – 3$
$= 2x^3 + 3x^2 – 2x – 3$

Given, 2x + 3 is a factor of f(x)
So, divide f(x) by 2x + 3

$$
\begin{array}{r|llll}
 & x^2 & -1 & & \\
\hline
2x+3 & 2x^3 & +3x^2 & -2x & -3 \\
- & 2x^3 & +3x^2 & & \\
\hline
 & & 0 & -2x & -3 \\
 & & - & & \\
 & & -2x & -3x & \\
\hline
 & & 0 & & \\
\end{array}
$$

Therefore, $2x^3 + 3x^2 – 2x – 3 = (2x + 3)(x^2 - 1)$
$= (2x + 3)(x + 1)(x - 1)$

25. Given $f(x) = ax^2 + bx + 2$ and $g(x) = bx^2 + ax + 1$. If $x – 2$ is a factor of f(x) but leaves the remainder – 15 when it divides g(x), find the values of a and b. With these values

of a and b, factorize the expression. $f(x) + g(x) + 4x^2 + 7x$.

Solution:-

From the question it is given that, $f(x) = ax^2 + bx + 2$ and $g(x) = bx^2 + ax + 1$ and $x – 2$ is a factor of f(x),

So, x = 2

Now, substitute the value of x in f(x),

f(2) = 0

$a(2)^2 + b(2) + 2 = 0$

4a + 2b + 2 = 0

By dividing both sides by 2 we get,

2a + b + 1 = 0 … [equation (i)]

Given, g(x) divide by (x - 2), leaves remainder – 15

$g(x) = bx^2 + ax + 1$

So, g(2) = -15

$b(2)^2 + 2a + 1 = -15$

4b + 2a + 1 + 15 = 0

4b + 2a + 16 = 0

By dividing both sides by 2 we get,

2b + a + 8 = 0 … [equation (ii)]

Now, subtracting equation (ii) from equation (i) multiplied by 2,

(4a + 2b + 2) – (a + 2b + 8) = 0 – 0

4a – a + 2b – 2b + 2 – 8 = 0

3a – 6 = 0

3a = 6

a = 6/3

a = 2

Consider the equation (i) to find out ‘b’.

2a + b + 1 = 0

2(2) + b = - 1

4 + b = - 1

b = - 1 - 4

b = - 5

Now, $f(x) = ax^2 + bx + 2 = 2x^2 – 5x + 2$

$g(x) = bx^2 + ax + 1 = -5x^2 + 2x + 1$

then, $f(x) + g(x) + 4x^2 + 7x$

$= 2x^2 – 5x + 2 – 5x^2 + 2x + 1 + 4x^2 + 7x$

$= x^2 + 4x + 3$

$= x^2 + 3x + x + 3$
$= x(x + 3) + 1(x + 3)$
$= (x + 1)(x + 3)$

CHAPTER TEST

1. Find the remainder when $2x^3 - 3x^2 + 4x + 7$ is divided by

(i) x – 2

(ii) x + 3

(iii) 2x + 1

Solution:-

From the question it is given that, $f(x) = 2x^3 - 3x^2 + 4x + 7$

(i) Consider x -2

let us assume x – 2 = 0

Then, x = 2

Now, substitute the value of x in f(x),

$f(2) = 2(2)^3 - 3(2)^2 + 4(2) + 7$

= 16 – 12 + 8 + 7

= 31 -12

= 19

Therefore, the remainder is 19

(ii) consider x + 3

let us assume x + 3 = 0

Then, x = -3

Now, substitute the value of x in f(x),

$f(2) = 2(-3)^3 - 3(-3)^2 + 4(-3) + 7$

= 2(-27) – 3(9) - 12 + 7

= - 54 – 27 – 12 + 7

= - 93 + 7

= - 86

Therefore, remainder is -86.

(iii) consider 2x + 1

Let us assume, 2x + 1 = 0

Then, 2x = -1

X = -½

Now, substitute the value of x in f(x),

$f(-½) = 2(-½)^3 - 3(-½)^2 + 4(-½) + 7$

= 2(-1/8) – 3 (¼) + 4 (-½) + 7

= -¼ - ¾ - 2 + 7

= -1 – 2 + 7

= 4

Therefore, remainder is 4.

2. When $2x^3 - 9x^2 + 10x - p$ is divided by $(x + 1)$, the remainder is – 24. Find the value of p.

Solution:-

Let us assume x + 1 = 0

Then, x = -1

Given, $f(x) = 2x^3 - 9x^2 + 10x - p$

Now, substitute the value of x in f(x),

$f(-1) = 2(-1)^3 - 9(-1)^2 + 10(-1) - p$

$= -2 - 9 - 10 + p$

$= -21 + p$

From the question it is given that, the remainder is – 24,

So, -21 + p = -24

p = - 24 + 21

p = -3

So, $f(x) = 2x^3 - 9x^2 + 10x - (-3)$

$= 2x^3 - 9x^2 + 10x + 3$

Therefore, the value of p is 3.

3. If (2x – 3) is a factor of $6x^2 + x + a$, find the value of a. With this value of a, factorise the given expression.

Solution:-

Let us assume 2x – 3 = 0

Then, 2x = 3

X = 3/2

Given, $f(x) = 6x^2 + x + a$

Now, substitute the value of x in f(x),

$f(3/2) = 6(3/2)^2 + (3/2) + a$

$= 6(9/4) + (3/2) + a$

$= 3(9/2) + (3/2) + a$

$= 27/2 + 3/2 + a$

$= 30/2 + a$

$= 15 + a$

From the question, (2x – 3) is a factor of $6x^2 + x + a$.

So, remainder is 0.

Then, 15 + a = 0

a = -15

Therefore, $f(x) = 6x^2 + x - 15$

Dividing f(x) by 2x – 3 we get,

$$\begin{array}{r|l} & 3x \quad +5 \\ 2x-3 & 6x^2 \quad +x \quad -15 \\ & -\ (6x^2 \quad -9x) \\ \hline & \qquad 10x \quad -15 \\ & \qquad -\ (10x \quad -15) \\ \hline & \qquad\qquad 0 \end{array}$$

Therefore, $6x^2 + x – 15 = (2x - 3) (3x + 5)$

4. When $3x^2 – 5x + p$ is divided by $(x – 2)$, the remainder is 3. Find the value of p. Also factorize the polynomial $3x^2 – 5x + p – 3$.

Solution:-

Let us assume x – 2 = 0

Then, x = 2

Given, $f(x) = 3x^2 – 5x + p$

Now, substitute the value of x in f(x),

So, $f(2) = 3(2)^2 – 5(2) + p$

= 3(4) – 10 + p

= 12 – 10 + p

= 2 + p

From the question it is given that, remainder is 3.

So, 2 + p = 3

p = 3 – 2

p = 1

Therefore, $f(x) = 3x^2 – 5x + 1$

Consider the polynomial, $3x^2 – 5x + p – 3$

Now, substitute the value of p in polynomial,

$= 3x^2 – 5x + 1 – 3$

$= 3x^2 – 5x – 2$

Now, by factorizing the polynomial $3x^2 – 5x – 2$,

Dividing $3x^2 – 5x – 2$ by x – 2 we get,

$$
\begin{array}{r|l}
 & 3x \quad +1 \\
x-2 & 3x^2 \quad -5x \quad -2 \\
 & -\ \ 3x^2 \quad -6x \\
\hline
 & \qquad\quad x \quad -2 \\
 & \qquad -\ \ x \quad -2 \\
\hline
 & \qquad\qquad 0
\end{array}
$$

Therefore, $3x^2 - 5x - 2 = (x - 2)(3x + 1)$

5. Prove that (5x + 4) is a factor of $5x^3 + 4x^2 - 5x - 4$. Hence factorize the given polynomial completely.

Solution:-

Let us assume (5x + 4) = 0

Then, 5x = -4

x = -4/5

Given, $f(x) = 5x^3 + 4x^2 - 5x - 4$

Now, substitute the value of x in f(x),

So, $f(-4/5) = 5(-4/5)^3 + 4(-4/5)^2 - 5(-4/5) - 4$

= 5(-64/125) + 4 (16/25) + 4 – 4

= -64/25 + 64/25

= (-64 + 64)/25

= 0/25

= 0

Hence, (5x + 4) is a factor of $5x^3 + 4x^2 - 5x - 4$.

So, dividing $5x^3 + 4x^2 - 5x - 4$ by 5x + 4 we get,

$$
\begin{array}{r|l}
 & x^2 \quad -1 \\
5x+4 & 5x^3 \quad +4x^2 \quad -5x \quad -4 \\
 & -\ \ 5x^3 \quad +4x^2 \\
\hline
 & \qquad\qquad 0 \quad -5x \quad -4 \\
 & \qquad -\ \ -5x \quad -4x \\
\hline
 & \qquad\qquad 0
\end{array}
$$

Therefore, $5x^3 + 4x^2 – 5x – 4 = (5x + 4) (x^2 - 1)$
$= (5x + 4) (x^2 – 1^2)$
$= (5x + 4) (x + 1) (x - 1)$

6. Use factor theorem to factorize the following polynomials completely:
(i) $4x^3 + 4x^2 – 9x – 9$
Solution:-
Let us assume x = -1,
Given, $f(x) = 4x^3 + 4x^2 – 9x – 9$
Now, substitute the value of x in f(x),
$f(-1) = 4(-1)^3 + 4(-1)^2 – 9(-1) – 9$
$= -4 + 4 + 9 – 9$
$= 0$
Therefore, x + 1 is the factor of $4x^3 + 4x^2 – 9x – 9$.
Now, dividing $4x^3 + 4x^2 – 9x – 9$ by x + 1 we get,

$$
\begin{array}{r|rrrr}
 & 4x^2 & & -9 & \\
\hline
x+1 & 4x^3 & +4x^2 & -9x & -9 \\
- & 4x^3 & +4x^2 & & \\
\hline
 & & 0 & -9x & -9 \\
 & & - & -9x & -9x \\
\hline
 & & & 0 &
\end{array}
$$

Therefore, $4x^3 + 4x^2 – 9x – 9 = (x + 1) (4x^2 - 9)$
$= (x + 1) ((2x)^2 – (3)^2)$
$= (x + 1) (2x + 3) (2x - 3)$

(ii) $x^3 – 19x – 30$
Solution:-
Let us assume x = -2,
Given, $f(x) = x^3 – 19x – 30$
Now, substitute the value of x in f(x),
$f(-1) = (-2)^3 – 19(-2) – 30$
$= -8 + 38 – 30$

= -38 + 38

= 0

Therefore, x + 2 is the factor of $x^3 - 19x - 30$.

Now, dividing $x^3 - 19x - 30$ by x + 2 we get,

$$
\begin{array}{r|llll}
 & x^2 & -2x & -15 & \\
x+2 & x^3 & +0x^2 & -19x & -30 \\
 & - & & & \\
 & x^3 & +2x^2 & & \\
\hline
 & & -2x^2 & -19x & -30 \\
 & & - & & \\
 & & -2x^2 & -4x & \\
\hline
 & & & -15x & -30 \\
 & & & - & \\
 & & & -15x & -30 \\
\hline
 & & & & 0
\end{array}
$$

Therefore, $x^3 - 19x - 30 = (x + 2)(x^2 - 2x - 15)$

$= (x + 2)(x^2 - 5x + 3x - 15)$

$= (x + 2)(x - 5)(x + 3)$

7. If $x^3 - 2x^2 + px + q$ has a factor (x + 2) and leaves a remainder 9, when divided by (x + 1), find the values of p and q. With these values of p and q, factorize the given polynomial completely.

Solution:-

From the question it is given that, (x + 2) is a factor of the expression $x^3 - 2x^2 + px + q$

Then, $f(x) = x^3 - 2x^2 + px + q$

Let assume x + 2 = 0

Then, x = -2

Now, substitute the value of x in f(x),

$f(-2) = (-2)^3 - 2(-2)^2 + p(-2) + q$

$= -8 - 8 - 2p + q$

$= -16 - 2p + q$

$2p - q = -16$ … [equation (i)]

Now, consider (x + 1)

Then, $f(x) = x^3 - 2x^2 + px + q$
Let assume $x + 1 = 0$
Then, $x = -1$
Now, substitute the value of x in f(x),
$f(-1) = (-1)^3 - 2(-1)^2 + p(-1) + q$
$= -1 - 2 - p + q$
$= -3 - p + q$
Given, remainder is 9
So, $-3 - p + q = 9$
$-p + q = 9 + 3$
$-p + q = 12$ … [equation (ii)]
Now, adding equation (i) and equation (ii) we get,
$(2p - q) + (-p + q) = -16 + 12$
$2p - q - p + q = -4$
$P = -4$
Consider the equation (ii) to find out 'b'.
$-p + q = 12$
$-(-4) + q = 12$
$4 + q = 12$
$q = 12 - 4$
$q = 8$
Therefore, by substituting the value of p and q $f(x) = x^3 - 2x^2 - 4x + 8$
Dividing f(x) be (x + 2) we get,

$$
\begin{array}{r|rrrr}
 & x^2 & -4x & +4 & \\
\hline
x+2 & x^3 & -2x^2 & -4x & +8 \\
 & - & & & \\
 & x^3 & +2x^2 & & \\
\hline
 & & -4x^2 & -4x & +8 \\
 & & - & & \\
 & & -4x^2 & -8x & \\
\hline
 & & & 4x & +8 \\
 & & & - & \\
 & & & 4x & +8 \\
\hline
 & & & & 0
\end{array}
$$

$x^3 - 2x^2 - 4x + 8 = (x + 2)(x^2 - 4x + 4)$

$= (x + 2)(x^2 - 2 \times x(-2) + 2^2)$

$= (x + 2)(x - 2)^2$

8. If (x + 3) and (x – 4) are factors of x³ + ax² – bx + 24, find the values of a and b: With these values of a and b, factorize the given expression.

Solution:-

Let us assume $x + 3 = 0$

Then, $x = -3$

Given, $f(x) = x^3 + ax^2 - bx + 24$

Now, substitute the value of x in f(x),

$f(-3) = (-3)^3 + a(-3)^2 - b(-3) + 24$

$= -27 + 9a + 3b + 24$

$= 9a + 3b - 3$

Dividing all terms by 3 we get,

$= 3a + b - 1$

From the question, $(x + 3)$ is a factor of $x^3 + ax^2 - bx + 24$.

Therefore, remainder is 0.

$f(x) = 0$

$3a + b - 1 = 0$

$3a + b = 1$ … [equation (i)]

Now, assume $x - 4 = 0$

Then, $x = 4$

Given, $f(x) = x^3 + ax^2 - bx + 24$

Now, substitute the value of x in f(x),

$f(4) = 4^3 + a(4)^2 - b(4) + 24$

$= 64 + 16a - 4b + 24$

$= 88 + 16a - 4b$

Dividing all terms by 4 we get,

$= 22 + 4a - b$

From the question, $(x - 4)$ is a factor of $x^3 + ax^2 - bx + 24$.

Therefore, remainder is 0.

$f(x) = 0$

$22 + 4a - b = 0$

$4a - b = -22$ … [equation (ii)]

Now, adding both equation (i) and equation (ii) we get,

$(3a + b) + (4a - b) = 1 - 22$

3a + b + 4a – b = - 21
7a = - 21
a = -21/7
a = -3
Consider the equation (i) to find out ‘b’.
3a + b = 1
3(-3) + b = 1
-9 + b = 1
b = 1 + 9
b = 10
Therefore, value of a = -3 and b = 10.
Then, by substituting the value of a and b f(x) = $x^3 - 3x^2 - 10x + 24$
(x + 3) (x - 4)
= x(x - 4) + 3(x - 4)
= $x^2 - 4x + 3x - 12$
= $x^2 - x - 12$
Dividing f(x) by $x^2 - x - 12$ we get,

$$\begin{array}{r|rrrr} & x & -2 & & \\ \hline x^2 - x - 12 & x^3 & -3x^2 & -10x & +24 \\ & - & & & \\ & x^3 & -x^2 & -12x & \\ \hline & & -2x^2 & +2x & +24 \\ & & - & & \\ & & -2x^2 & +2x & +24 \\ \hline & & & & 0 \end{array}$$

Therefore, $x^3 - 3x^2 - 10x + 24 = (x^2 - x - 12)(x - 2)$
= (x + 3) (x - 4) (x - 2)

9. If (2x + 1) is a factor of both the expressions $2x^2 - 5x + p$ and $2x^2 + 5x + q$, find the value of p and q. Hence find the other factors of both the polynomials.
Solution:-
Let us assume 2x + 1 = 0
Then, 2x = -1
x = -½

Given, $p(x) = 2x^2 – 5x + p$
Now, substitute the value of x in p(x),
$p(-½) = 2(-½)^2 – 5(-½) + p$
$= 2(1/4) + 5/2 + p$
$= ½ + 5/2 + p$
$= 6/2 + p$
$= 3 + p$
From the question it is given that, (2x + 1) is a factor of both the expressions $2x^2 – 5x + p$
So, remainder is 0.
Then, $3 + p = 0$
$p = - 3$
Now consider $q(x) = 2x^2 + 5x + q$
Substitute the value of x in q(x)
$q(-½) = 2(-½)^2 + 5(-½) + q$
$= 2(1/4) - 5/2 + q$
$= ½ - 5/2 + q$
$= (1 - 5)/2 + q$
$= -4/2 + q$
$= q - 2$

From the question it is given that, (2x + 1) is a factor of both the expressions $2x^2 + 5x + q$
So, remainder is 0.
$q – 2 = 0$
$q = 2$
Therefore, p = - 3 and q = 2
$P(x) = 2x^2 – 5x – 3$
$q(x) = 2x^2 + 5x + 2$
Then, divide p(x) by 2x + 1

$$\begin{array}{r|rrr} & x & -3 & \\ \hline 2x+1 & 2x^2 & -5x & -3 \\ - & 2x^2 & +x & \\ \hline & & -6x & -3 \\ & & - & \\ & & -6x & -3 \\ \hline & & & 0 \end{array}$$

Therefore, $2x^2 - 5x - 3 = (2x + 1)(x - 3)$

Now, divide q(x) by 2x + 1

$$\begin{array}{r|l} & x \quad +2 \\ 2x+1 & 2x^2 \quad +5x \quad +2 \\ & -\; 2x^2 \quad +x \\ \hline & \quad 4x \quad +2 \\ & -\; 4x \quad +2 \\ \hline & \quad 0 \end{array}$$

Therefore, $2x^2 + 5x + 2 = (2x + 1)(x + 2)$

10. If a polynomial $f(x)= x^4-2x^3 + 3x^2- ax + b$ leaves reminder 5 and 19 when divided by (x - 1) and (x + 1) respectively, Find the values of a and b. Hence determined the reminder when f(x) is divided by (x-2).

Solution:-

From the question it is given that,

$f(x) = x^4 - 2x^3 + 3x^2 - ax + b$

Factor (x - 1) leaves remainder 5,

Factor (x + 1) leaves remainder 19,

Where x = 1 and x = - 1

$f(-1) = (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + b = 19$

$1 - 2(-1) + 3(1) - a(-1) + b = 19$

$1 + 2 + 3 + a + b = 19$

$6 + a + b = 19$

$a + b = 19 - 6$

$a + b = 13$ … [equation (i)]

$f(1) = (1)^4 - 2(1)^3 + 3(1)^2 - a(1) + b = 5$

$1 - 2(1) + 3(1) - a(1) + b = 5$

$1 - 2 + 3 - a + b = 5$

$2 - a + b = 5$

$-a + b = 5 - 2$

$-a + b = 3$ … [equation (ii)]

Now, subtracting equation (ii) from equation (i) we get,

$(a + b) - (-a + b) = 13 - 3$

$a + b + a - b = 10$

2a = 10
a = 10/2
a = 5
To find out the value of b, substitute the value of a in equation (i) we get,
a + b = 13
5 + b = 13
b = 13 – 5
b = 8
Therefore, value of a = 5 and b = 8

11. When a polynomial f(x) is divided by (x – 1), the remainder is 5 and when it is, divided by (x – 2), the remainder is 7. Find the remainder when it is divided by (x – 1) (x – 2).

Solution:-
From the question it is given that,
Polynomial f(x) is divided by (x – 1),
Remainder = 5
Let us assume x – 1 = 0
x = 1

f(1) = 5
and the divided be (x - 2), remainder = 7
let us assume x – 2 = 0
x = 2
Therefore, f(2) = 7
So, f(x) = (x - 1) (x - 2) q(x) + ax + b
Where, q(x) is the quotient and ax + b is remainder,
Now put x = 1, we get,
f(1) = (1 - 1)(1 - 2)q(1) + (a × 1) + b
a + b = 5 … [equation (i)]
x = 2,
f(2) = (2 - 1)(2 - 2)q(2) + (a × 2) + b
2a + b = 7 … [equation (ii)]
Now subtracting equation (i) from equation (ii) we get,
(2a + b) – (a + b) = 7 – 5
2a + b – a – b = 2
a = 2
To find out the value of b, substitute the value of a in equation (i) we get,

a + b = 5
2 + b = 5
b = 5 – 2
b = 3
Therefore, the remainder = ax + b = 2x + 3