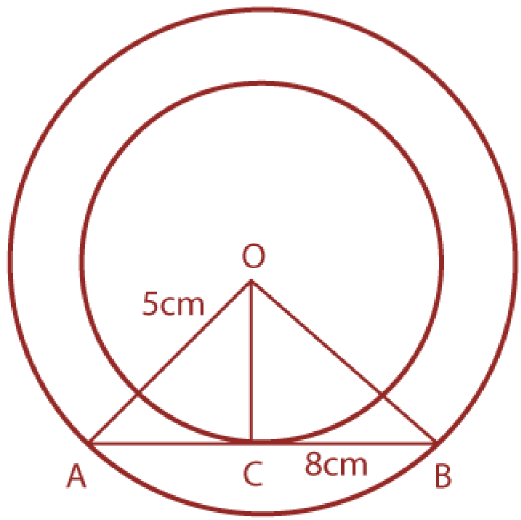
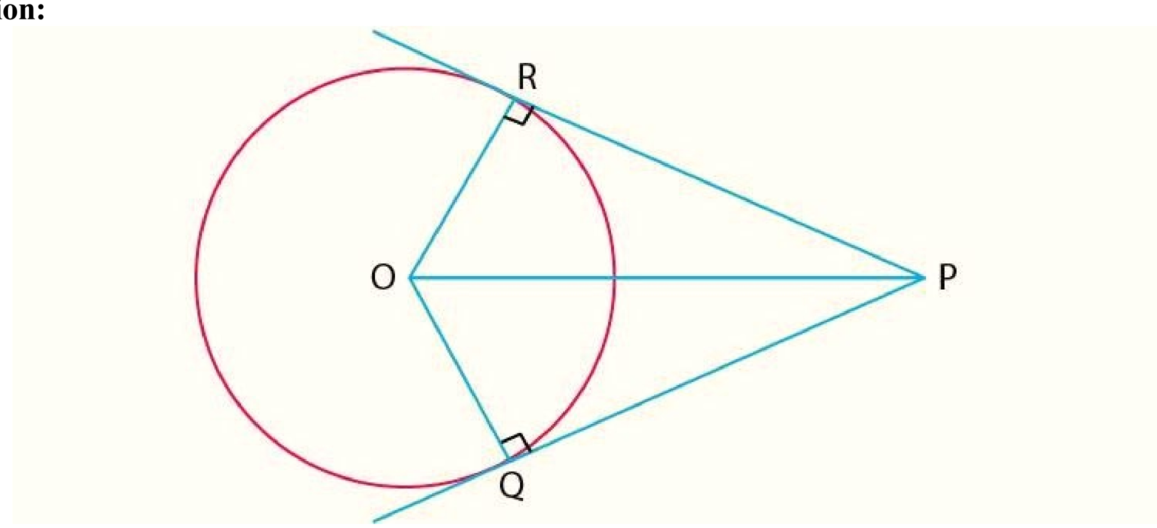
EXERCISE 9.3

Out of the two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle.
Solution:



From the figure,
Chord AB = 8 cm
OC is perpendicular to the chord AB
AC = CB = 4 cm
In right triangle OCA
$OC^2 + CA^2 = OA^2$
$OC^2 = 5^2 - 4^2 = 25 - 16 = 9$
OC = 3 cm

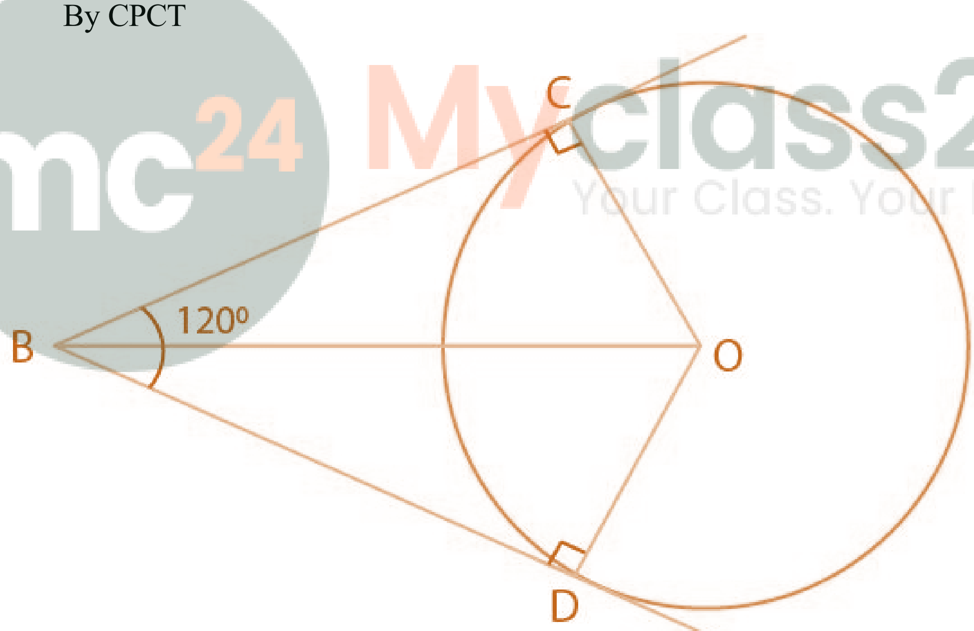
1. Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral.
Solution:



We know that,
Radius ⊥ Tangent = OR ⊥ PR
i.e., ∠ORP = 90°
Likewise,
Radius ⊥ Tangent = OQ ⊥PQ
∠OQP = 90°
In quadrilateral ORPQ,
Sum of all interior angles = 360º
∠ORP + ∠RPQ+ ∠PQO + ∠QOR = 360º
90º + ∠RPQ + 90º + ∠QOR = 360º
Hence, ∠O + ∠P = 180º
PROQ is a cyclic quadrilateral.

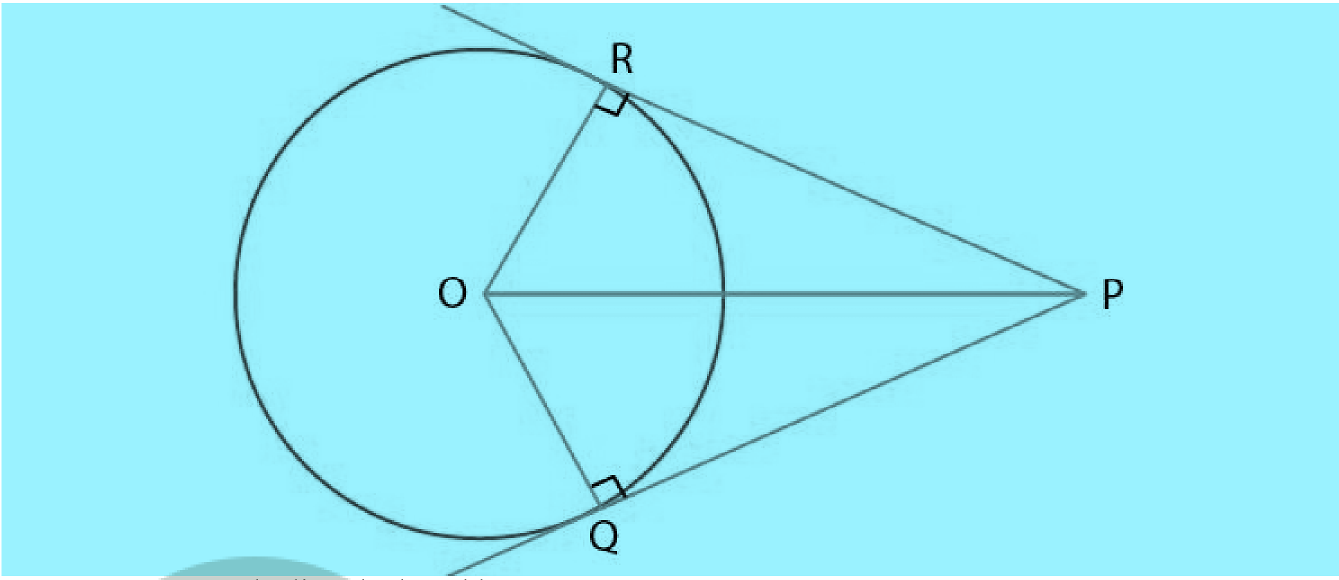
2. If from an external point B of a circle with centre O, two tangents BC and BD are drawn such that angle DBC = 120°, prove that BC + BD = BO, i.e., BO = 2BC.
Solution:

According to the question,
By RHS rule,
ΔOBC and ΔOBD are congruent
By CPCT



∠OBC and ∠ OBD are equal
Therefore,
∠OBC = ∠OBD =60°
In triangle OBC,
cos 60°=BC/OB
½ =BC/OB
OB=2BC
Hence proved

3. Prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines.
Solution:



Let the lines be l_1 and l_2.
Assume that O touches l_1 and l_2 at M and N,
We get,
OM = ON (Radius of the circle)
Therefore,
From the centre "O" of the circle, it has equal distance from l_1 & l_2.
In Δ OPM & OPN,
OM = ON (Radius of the circle)
∠OMP = ∠ONP (As, Radius is perpendicular to its tangent)
OP = OP (Common sides)

Therefore,
Δ OPM = ΔOPN (SSS congruence rule)
By C.P.C.T,
∠MPO = ∠NPO
So, l bisects ∠MPN.
Therefore, O lies on the bisector of the angle between l_1 & l_2 .
Hence, we prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines.

4. In Fig. 9.13, AB and CD are common tangents to two circles of unequal radii. Prove that AB = CD.

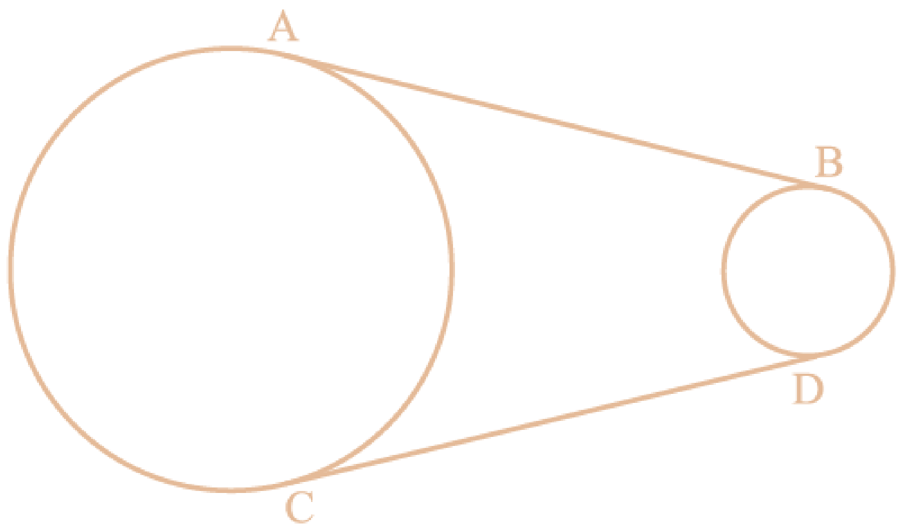
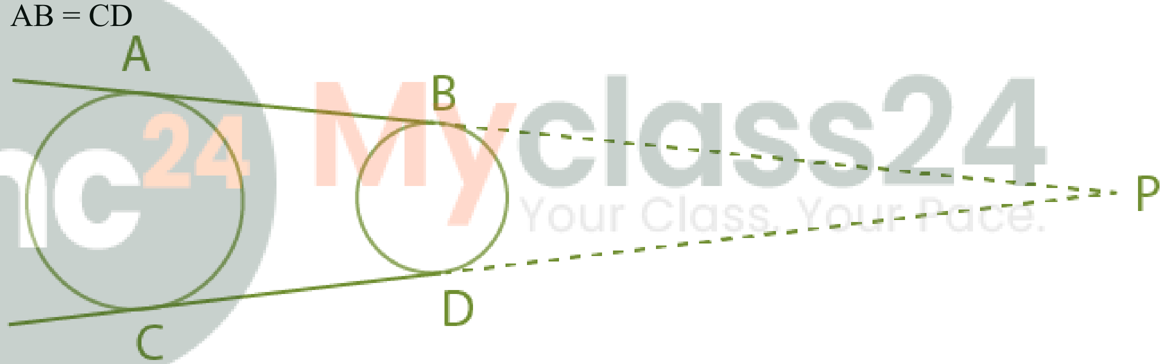


Fig. 9.13

Solution:

According to the question,

AB = CD



Construction: Produce AB and CD, to intersect at P.

Proof:

Consider the circle with greater radius.

Tangents drawn from an external point to a circle are equal

AP = CP …(1)

Also,

Consider the circle with smaller radius.

Tangents drawn from an external point to a circle are equal

BP = BD …(2)

Subtract Equation (2) from (1). We Get

AP - BP = CP – BD

AB = CD

Hence Proved.