EXERCISE 19.25

Evaluate the following integrals:

1. $\int x \cos x \, dx$

Solution:

Let $I = \int x \cos x \, dx$

We know that, $\int u\, v\, dx = u \int v\, dx - \int \left(\frac{du}{dx} \int v\, dx\right) dx$

Using integration by parts,

$$I = x \int \cos x \, dx - \int (\frac{d}{dx} x \int \cos x \, dx) \, dx$$

We have, $\int \sin x = -\cos x, \int \cos x = \sin x$

$$= x \times \sin x - \int \sin x \, dx$$

$$= x \sin x + \cos x + c$$

2. $\int \log(x+1) \, dx$

Solution:

Let $I = \int \log(x+1) \, dx$

That is,

$$I = \int 1 \times \log(x+1) \, dx$$

Using integration by parts,

$$I = \log(x+1) \int 1 \, dx - \int \frac{d}{dx} \log(x+1) \int 1 \, dx$$

We know that, $\int 1 \, dx = x$ and $\int \log x = \frac{1}{x}$

$$= \log(x+1) \times x - \int \frac{1}{x+1} \times x \, dx$$

Now,

$$\frac{x}{x+1} = 1 - \frac{1}{x+1}$$

$$= x\log(x+1) - \int \left(1 - \frac{1}{x+1}\right) dx$$

$$= x\log(x+1) - x + \log(x+1) + c$$

3. $\int x^3 \log x \, dx$

Solution:

Let $I = \int x^3 \log x \, dx$

Using integration by parts,

$$I = \log x \int x^3 \, dx - \int \frac{d}{dx} \log x \int x^3 \, dx$$

We have, $\int x^n dx = \frac{x^{n+1}}{n+1}$ and $\int \log x = \frac{1}{x}$

$$= \log x \times \frac{x^4}{4} - \int \frac{1}{x} \times \frac{x^4}{4} \, dx$$

$$= \log x \times \frac{x^4}{4} - \frac{1}{4} \int x^3 dx$$

$$= \frac{x^4}{4} \log x - \frac{1}{4} \times \frac{x^4}{4}$$

$$= \frac{x^4}{4} \log x - \frac{x^4}{16} + c$$

4. $\int x e^x \, dx$

Solution:

Let $I = \int xe^x dx$

Using integration by parts,

$$I = x\int e^x dx - \int(\frac{d}{dx}x\int e^x dx)dx$$

We know that, $\int e^x dx = e^x$ and $\frac{d}{dx}x = 1$

$$= xe^x - \int e^x dx$$

$$= xe^x - e^x + c$$

5. $\int xe^{2x}\, dx$

Solution:

Let $I = \int xe^{2x} dx$

Using integration by parts,

$$I = x\int e^{2x} dx - \int(\frac{d}{dx}x\int e^{2x} dx)dx$$

We know that, $\int e^{nx} dx = \frac{e^x}{n}$ and $\frac{d}{dx}x = 1$

$$= \frac{xe^{2x}}{2} - \int \frac{e^{2x}}{2} dx$$

$$= \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + c$$

$$I = \left(\frac{x}{2} - \frac{1}{4}\right)e^{2x} + c$$