Exercise 9.5

Find the value of x for which (8x + 4), (6x - 2) and (2x + 7) are in A.P. Solution:

Given,
(8x + 4), (6x – 2) and (2x + 7) are in A.P.
So, the common difference between the consecutive terms should be the same.
$(6x - 2) - (8x + 4) = (2x + 7) - (6x - 2)$
$\Rightarrow 6x - 2 - 8x - 4 = 2x + 7 - 6x + 2$
$\Rightarrow -2x - 6 = -4x + 9$
$\Rightarrow -2x + 4x = 9 + 6$
$\Rightarrow 2x = 15$
Therefore, $x = 15/2$

1. If x + 1, 3x and 4x + 2 are in A.P., find the value of x.
Solution:

Given,
x + 1, 3x and 4x + 2 are in A.P.
So, the common difference between the consecutive terms should be the same.
$3x - x - 1 = 4x + 2 - 3x$
$\Rightarrow 2x - 1 = x + 2$
$\Rightarrow 2x - x = 2 + 1$
$\Rightarrow x = 3$
Therefore, $x = 3$

2. Show that $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in A.P.
Solution:

If $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ have to be in A.P. then,
It should satisfy the condition,
$2b = a + c$ [for a, b, c are in A.P]
Thus,
$2(a^2 + b^2) = (a - b)^2 + (a + b)^2$
$2(a^2 + b^2) = a^2 + b^2 - 2ab + a^2 + b^2 + 2ab$
$2(a^2 + b^2) = 2a^2 + 2b^2 = 2(a^2 + b^2)$
LHS = RHS
Hence proved.

3. The sum of three terms of an A.P. is 21 and the product of the first and the third terms exceeds the second term by 6, find three terms.
Solution:

Let's consider the three terms of the A.P. to be a – d, a, a + d
so, the sum of three terms = 21

⇒ $a - d + a + a + d = 21$
⇒ $3a = 21$
⇒ $a = 7$

And, product of the first and 3rd = 2nd term + 6
⇒ $(a - d)(a + d) = a + 6$
$a^2 - d^2 = a + 6$
⇒ $(7)^2 - d^2 = 7 + 6$
⇒ $49 - d^2 = 13$
⇒ $d^2 = 49 - 13 = 36$
⇒ $d^2 = (6)^2$
⇒ $d = 6$

Hence, the terms are 7 – 6, 7, 7 + 6 ⇒ 1, 7, 13

4. Three numbers are in A.P. If the sum of these numbers be 27 and the product 648, find the numbers.
Solution:

Let the three numbers of the A.P. be a – d, a, a + d
From the question,
Sum of these numbers = 27
$a - d + a + a + d = 27$
⇒ $3a = 27$
$a = 27/3 = 9$

Now, product of these numbers = 648
$(a - d)(a)(a + d) = 648$
$a(a^2 - d^2) = 648$
$a^2 - 648/a = d^2$
$9^2 - (648/9) = d^2$
$9^3 - 648 = 9d^2$
$729 - 648 = 9d^2$
$81 = 9d^2$
$d^2 = 9$
$d = 3$ or -3

Hence, the terms are 9-3, 9 and 9+3 ⇒ 6, 9, 12 or 12, 9, 6 (for d = -3)

5. Find the four numbers in A.P., whose sum is 50 and in which the greatest number is 4 times the least.
Solution:

Let's consider the four terms of the A.P. to be $(a - 3d)$, $(a - d)$, $(a + d)$ and $(a + 3d)$.
From the question,
Sum of these terms = 50

⇒ (a – 3d) + (a – d) + (a + d) + (a + 3d) = 50
⇒ a – 3d + a – d + a + d + a – 3d= 50
⇒ 4a = 50
⇒ a = 50/4 = 25/2
And, also given that the greatest number = 4 x least number
⇒ a + 3d = 4 (a – 3d)
⇒ a + 3d = 4a – 12d
⇒ 4a – a = 3d + 12d
⇒3a = 15d
⇒a = 5d
Using the value of a in the above equation, we have
⇒25/2 = 5d
⇒ d = 5/2

So, the terms will be:
(a – 3d) = (25/2 – 3(5/2)), (a – d) = (25/2 – 5/2), (25/2 + 5/2) and (25/2 + 3(5/2)).
⇒ 5, 10, 15, 20