

1. Evaluate:

$$\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$$

**Sol:**

On substituting the values of various T-ratios, we get:

$$\begin{aligned} & \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ \\ &= \left( \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} \right) = \left( \frac{3}{4} + \frac{1}{4} \right) = \frac{4}{4} = 1 \end{aligned}$$

2. Evaluate:

$$\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$$

**Sol:**

On substituting the values of various T-ratios, we get:

$$\begin{aligned} & \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ \\ &= \left( \frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2} \right) = \left( \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} \right) = 0 \end{aligned}$$

3. Evaluate:

$$\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$

**Sol:**

On substituting the values of various T-ratios, we get:

$$\begin{aligned} & \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ \\ &= \left( \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} \right) = \left( \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} \right) = \left( \frac{\sqrt{3}+1}{2\sqrt{2}} \right) \end{aligned}$$

4. Evaluate:

$$\frac{\sin 30^\circ}{\cos 45^\circ} + \frac{\cot 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\tan 45^\circ} + \frac{\cos 30^\circ}{\sin 90^\circ}$$

**Sol:**

$$\begin{aligned} & \frac{\sin 30^\circ}{\cos 45^\circ} + \frac{\cot 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\tan 45^\circ} + \frac{\cos 30^\circ}{\sin 90^\circ} \\ &= \frac{\left(\frac{1}{2}\right)}{\left(\frac{1}{\sqrt{2}}\right)} + \frac{1}{2} - \frac{\left(\frac{\sqrt{3}}{2}\right)}{1} + \frac{\left(\frac{\sqrt{3}}{2}\right)}{1} \\ &= \frac{\sqrt{2}}{2} + \frac{1}{2} - \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \\ &= \frac{\sqrt{2} + 1}{2} \end{aligned}$$

5. Evaluate:

$$\frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

**Sol:**

$$\frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

$$\begin{aligned}
&= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \\
&= \frac{\left(\frac{5}{4} + \frac{4 \times 4}{3} - 1\right)}{\left(\frac{1}{4} + \frac{3}{4}\right)} \\
&= \frac{\left(\frac{5}{4} + \frac{16}{3} - \frac{1}{1}\right)}{\left(\frac{4}{4}\right)} \\
&= \frac{(15+64-12)}{4} \\
&= \frac{67}{4} \\
&= \frac{67}{12}
\end{aligned}$$

6. Evaluate:

$$2\cos^2 60^\circ + 3\sin^2 45^\circ - 3\sin^2 30^\circ + 2\cos^2 90^\circ$$

**Sol:**

On substituting the values of various T-ratios, we get:

$$2\cos^2 60^\circ + 3\sin^2 45^\circ - 3\sin^2 30^\circ + 2\cos^2 90^\circ$$

$$= 2 \times \left(\frac{1}{2}\right)^2 + 3 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 3 \times \left(\frac{1}{2}\right)^2 + 2 \times (0)^2$$

$$= 2 \times \frac{1}{4} + 3 \times \frac{1}{2} - 3 \times \frac{1}{4} + 0$$

$$= \left(\frac{1}{2} + \frac{3}{2} - \frac{3}{4}\right) = \left(\frac{2+6-3}{4}\right) = \frac{5}{4}$$

7. Evaluate:

$$\cot^2 30^\circ - 2\cos^2 30^\circ - \frac{3}{4}\sec^2 45^\circ + \frac{1}{4}\operatorname{cosec}^2 30^\circ$$

**Sol:**

On substituting the values of various T-ratios, we get:

$$\cot^2 30^\circ - 2\cos^2 30^\circ - \frac{3}{4}\sec^2 45^\circ + \frac{1}{4}\operatorname{cosec}^2 30^\circ$$

$$= (\sqrt{3})^2 - 2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4} \times (\sqrt{2})^2 + \frac{1}{4} \times (2)^2$$

$$= 3 - 2 \times \frac{3}{4} - \frac{3}{4} \times 2 + \frac{1}{4} \times 4$$

$$= 3 - \frac{3}{2} - \frac{3}{2} + 1$$

$$= 4 - \left(\frac{3}{2} + \frac{3}{2}\right)$$

$$= 4 - 3 = 1$$

8. Evaluate:

$$(\sin^2 30^\circ + 4 \cot^2 45^\circ - \sec^2 60^\circ) (\operatorname{cosec}^2 45^\circ \sec^2 30^\circ)$$

**Sol:**

On substituting the values of various T-ratios, we get:

$$(\sin^2 30^\circ + 4 \cot^2 45^\circ - \sec^2 60^\circ) (\operatorname{cosec}^2 45^\circ \sec^2 30^\circ)$$

$$= \left[ \left(\frac{1}{2}\right)^2 + 4 \times (1)^2 - (2)^2 \right] \left[ (\sqrt{2})^2 \left(\frac{2}{\sqrt{3}}\right)^2 \right]$$

$$= \left(\frac{1}{4} + 4 - 4\right) \left(2 \times \frac{4}{3}\right)$$

$$= \frac{1}{4} \times \frac{8}{3} = \frac{2}{3}$$

9. Evaluate:

$$\frac{4}{\cot^2 30^\circ} + \frac{1}{\sin^2 30^\circ} - 2 \cos^2 45^\circ - \sin^2 0^\circ$$

**Sol:**

On substituting the values of various T-ratios, we get:

$$\frac{4}{\cot^2 30^\circ} + \frac{1}{\sin^2 30^\circ} - 2 \cos^2 45^\circ - \sin^2 0^\circ$$

$$= \frac{4}{(\sqrt{3})^2} + \frac{1}{\left(\frac{1}{2}\right)^2} - 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 - (0)^2$$

$$= \frac{4}{3} + \frac{1}{\frac{1}{4}} - 2 \times \frac{1}{2} - 0$$

$$= \frac{4}{3} + 4 - 1$$

$$= \frac{4}{3} + 3 = \frac{4+9}{3} = \frac{13}{3}$$

10. Show that:

$$(i) \frac{1 - \sin 60^\circ}{\cos 60^\circ} = \frac{\tan 60^\circ - 1}{\tan 60^\circ + 1}$$

$$(ii) \frac{\cos 30^\circ + \sin 60^\circ}{1 + \sin 30^\circ + \cos 60^\circ} = \cos 30^\circ$$

**Sol:**

(i)

$$\text{LHS} = \frac{1 - \sin 60^\circ}{\cos 60^\circ} = \frac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\left(\frac{2 - \sqrt{3}}{2}\right)}{\frac{1}{2}} = \left(\frac{2 - \sqrt{3}}{2}\right) \times 2 = 2 - \sqrt{3}$$

$$\text{RHS} = \frac{\tan 60^\circ - 1}{\tan 60^\circ + 1} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1} = \frac{(\sqrt{3} - 1)^2}{(\sqrt{3})^2 - 1^2} = \frac{3 + 1 - 2\sqrt{3}}{3 - 1} = \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$$

Hence, LHS = RHS

$$\therefore \frac{1 - \sin 60^\circ}{\cos 60^\circ} = \frac{\tan 60^\circ - 1}{\tan 60^\circ + 1}$$

(ii)

$$\text{LHS} = \frac{\cos 30^\circ + \sin 60^\circ}{1 + \sin 30^\circ + \cos 60^\circ} = \frac{\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right)}{1 + \frac{1}{2} + \frac{1}{2}} = \frac{\frac{\sqrt{3} + \sqrt{3}}{2}}{\frac{2 + 1 + 1}{2}} = \frac{\sqrt{3}}{2}$$

$$\text{Also, RHS} = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

Hence, LHS = RHS

$$\therefore \frac{\cos 30^\circ + \sin 60^\circ}{1 + \sin 30^\circ + \cos 60^\circ} = \cos 30^\circ$$

11. Verify each of the following:

(i)  $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

(ii)  $\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$

(iii)  $2 \sin 30^\circ \cos 30^\circ$

(iv)  $2 \sin 45^\circ \cos 45^\circ$

**Sol:**

(i)  $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

$$= \left(\frac{\sqrt{3}}{2}\right) \times \left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

Also,  $\sin 30^\circ = \frac{1}{2}$

$$\therefore \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ = \sin 30^\circ$$

(ii)  $\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$

$$= \left(\frac{1}{2}\right) \times \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right) \times \left(\frac{1}{2}\right) = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

Also,  $\cos 30^\circ = \frac{\sqrt{3}}{2}$

$$\therefore \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ = \cos 30^\circ$$

(iii)  $2 \sin 30^\circ \cos 30^\circ$

$$= 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

Also,  $\sin 60^\circ = \frac{\sqrt{3}}{2}$

$$\therefore 2 \sin 30^\circ \cos 30^\circ = \sin 60^\circ$$

(iv)  $2 \sin 45^\circ \cos 45^\circ$

$$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1$$

Also,  $\sin 90^\circ = 1$

$$\therefore 2 \sin 45^\circ \cos 45^\circ = \sin 90^\circ$$

12. If  $A = 45^\circ$ , verify that:

(i)  $\sin 2A = 2 \sin A \cos A$

(ii)  $\cos 2A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$

**Sol:**

$$A = 45^\circ$$

$$\Rightarrow 2A = 2 \times 45^\circ = 90^\circ$$

(i)  $\sin 2A = \sin 90^\circ = 1$

$$2 \sin A \cos A = 2 \sin 45^\circ \cos 45^\circ = 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1$$

$$\therefore \sin 2A = 2 \sin A \cos A$$

$$(ii) \cos 2A = \cos 90^\circ = 0$$

$$2 \cos^2 A - 1 = 2 \cos^2 45^\circ - 1 = 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 1 = 2 \times \frac{1}{2} - 1 = 1 - 1 = 0$$

$$\text{Now, } 1 - 2 \sin^2 A = 1 - 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 1 = 1 - 2 \times \frac{1}{2} = 1 - 1 = 0$$

$$\therefore \cos 2A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

13. If  $A = 30^\circ$ , verify that:

$$(i) \sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$$

$$(ii) \cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$(iii) \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

**Sol:**

$$A = 30^\circ$$

$$\Rightarrow 2A = 2 \times 30^\circ = 60^\circ$$

$$(i) \sin 2A = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\frac{2 \tan A}{1 + \tan^2 A} = \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} = \frac{2 \times \left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{1 + \frac{1}{3}} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{\frac{4}{3}} = \left(\frac{2}{\sqrt{3}}\right) \times \frac{3}{4} = \frac{\sqrt{3}}{2}$$

$$\therefore \sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$$

$$(ii) \cos 2A = \cos 60^\circ = \frac{1}{2}$$

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ} = \frac{1 - \left(\frac{1}{\sqrt{3}}\right)^2}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\left(1 - \frac{1}{3}\right)}{1 + \frac{1}{3}} = \frac{\left(\frac{2}{3}\right)}{\frac{4}{3}} = \left(\frac{2}{3}\right) \times \frac{3}{4} = \frac{1}{2}$$

$$\therefore \cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$(iii) \tan 2A = \tan 60^\circ = \sqrt{3}$$

$$\frac{2 \tan A}{1 - \tan^2 A} = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = \frac{2 \times \left(\frac{1}{\sqrt{3}}\right)}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{1 - \frac{1}{3}} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{\frac{2}{3}} = \left(\frac{2}{\sqrt{3}}\right) \times \frac{3}{2} = \sqrt{3}$$

$$\therefore \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

14. If  $A = 60^\circ$  and  $B = 30^\circ$ , verify that:

- (i)  $\sin(A + B) = \sin A \cos B + \cos A \sin B$   
 (ii)  $\cos(A + B) = \cos A \cos B - \sin A \sin B$

**Sol:**

$$A = 60^\circ \text{ and } B = 30^\circ$$

$$\text{Now, } A + B = 60^\circ + 30^\circ = 90^\circ$$

$$\text{Also, } A - B = 60^\circ - 30^\circ = 30^\circ$$

$$(i) \sin(A + B) = \sin 90^\circ = 1$$

$$\begin{aligned} \sin A \cos B + \cos A \sin B &= \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ \\ &= \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}\right) = \left(\frac{3}{4} + \frac{1}{4}\right) = 1 \end{aligned}$$

$$\therefore \sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$(ii) \cos(A + B) = \cos 90^\circ = 0$$

$$\begin{aligned} \cos A \cos B - \sin A \sin B &= \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ \\ &= \left(\frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2}\right) = \left(\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}\right) = 0 \end{aligned}$$

$$\therefore \cos(A + B) = \cos A \cos B - \sin A \sin B$$

15. If  $A = 60^\circ$  and  $B = 30^\circ$ , verify that:

- (i)  $\sin(A - B) = \sin A \cos B - \cos A \sin B$   
 (ii)  $\cos(A - B) = \cos A \cos B + \sin A \sin B$   
 (iii)  $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

**Sol:**

$$(i) \sin(A - B) = \sin 30^\circ = \frac{1}{2}$$

$$\begin{aligned} \sin A \cos B - \cos A \sin B &= \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ \\ &= \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}\right) = \left(\frac{3}{4} - \frac{1}{4}\right) = \frac{2}{4} = \frac{1}{2} \end{aligned}$$

$$\therefore \sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$(ii) \cos(A - B) = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\begin{aligned} \cos A \cos B + \sin A \sin B &= \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ \\ &= \left(\frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}\right) = \left(\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}\right) = 2 \times \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2} \end{aligned}$$

$$\therefore \cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$(iii) \tan(A - B) = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\frac{\tan A - \tan B}{1 + \tan A \tan B} = \frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ} = \frac{\sqrt{3} - \left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\sqrt{3} \times \frac{1}{\sqrt{3}}\right)} = \frac{1}{2} \times \frac{3-1}{\sqrt{3}} = \frac{3}{\sqrt{3}}$$

$$\therefore \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

16. If A and B are acute angles such that  $\tan A = \frac{1}{3}$ ,  $\tan B = \frac{1}{2}$  and  $\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ , show that  $A + B = 45^\circ$ .

**Sol:**

Given:

$$\tan A = \frac{1}{3} \text{ and } \tan B = \frac{1}{2}$$

$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

On substituting these values in RHS of the expression, we get:

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\left(\frac{1}{3} + \frac{1}{2}\right)}{\left(1 - \frac{1}{3} \times \frac{1}{2}\right)} = \frac{\left(\frac{5}{6}\right)}{\left(1 - \frac{1}{6}\right)} = \frac{\left(\frac{5}{6}\right)}{\left(\frac{5}{6}\right)} = 1$$

$$\Rightarrow \tan (A + B) = 1 = \tan 45^\circ \quad [\because \tan 45^\circ = 1]$$

$$\therefore A + B = 45^\circ$$

17. Using the formula,  $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$ , find the value of  $\tan 60^\circ$ , it being given that  $\tan 30^\circ = \frac{1}{\sqrt{3}}$ .

**Sol:**

$$A = 30^\circ$$

$$\Rightarrow 2A = 2 \times 30^\circ = 60^\circ$$

By substituting the value of the given T-ratio, we get:

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\Rightarrow \tan 60^\circ = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = \frac{2 \times \left(\frac{1}{\sqrt{3}}\right)}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{1 - \frac{1}{3}} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{\frac{2}{3}} = \left(\frac{2}{\sqrt{3}}\right) \times \frac{3}{2} = \sqrt{3}$$

$$\therefore \tan 60^\circ = \sqrt{3}$$

18. Using the formula,  $\cos A = \sqrt{\frac{1 + \cos 2A}{2}}$ , find the value of  $\cos 30^\circ$ , it being given that  $\cos 60^\circ = \frac{1}{2}$ .

**Sol:**

$$A = 30^\circ$$

$$\Rightarrow 2A = 2 \times 30^\circ = 60^\circ$$

By substituting the value of the given T-ratio, we get:

$$\cos A = \sqrt{\frac{1 + \cos 2A}{2}}$$

$$\cos 30^\circ = \sqrt{\frac{1 + \cos 60^\circ}{2}} = \sqrt{\frac{1 + \frac{1}{2}}{2}} = \sqrt{\frac{\frac{3}{2}}{2}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

$$\therefore \cos A = \frac{\sqrt{3}}{2}$$

19. Using the formula,  $\sin A = \sqrt{\frac{1 - \cos 2A}{2}}$ , find the value of  $\sin 30^\circ$ , it being given that  $\cos 60^\circ = \frac{1}{2}$ .

**Sol:**

$$A = 30^\circ$$

$$\Rightarrow 2A = 2 \times 30^\circ = 60^\circ$$

By substituting the value of the given T-ratio, we get:

$$\sin A = \sqrt{\frac{1 - \cos 2A}{2}}$$

$$\sin 30^\circ = \sqrt{\frac{1 - \cos 60^\circ}{2}} = \sqrt{\frac{1 - \frac{1}{2}}{2}} = \sqrt{\frac{\frac{1}{2}}{2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$\therefore \sin 30^\circ = \frac{1}{2}$$

20. In the adjoining figure,  $\triangle ABC$  is a right-angled triangle in which  $\angle B = 90^\circ$ ,  $\angle 30^\circ$  and  $AC = 20\text{cm}$ .

Find (i)  $BC$ , (ii)  $AB$ .

**Sol:**

From the given right-angled triangle, we have:

$$\frac{BC}{AC} = \sin 30^\circ$$

$$\Rightarrow \frac{BC}{20} = \frac{1}{2}$$

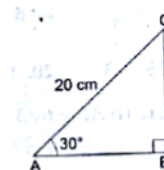
$$\Rightarrow BC = \frac{20}{2} = 10\text{cm}$$

$$\text{Also, } \frac{AB}{AC} = \cos 30^\circ$$

$$\Rightarrow \frac{AB}{20} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow AB = \left(20 \times \frac{\sqrt{3}}{2}\right) = 10\sqrt{3}\text{ cm}$$

$$\therefore BC = 10\text{cm and } AB = 10\sqrt{3}\text{ cm}$$



21. In the adjoining figure,  $\triangle ABC$  is right-angled at  $B$  and  $\angle A = 30^\circ$ . If  $BC = 6\text{cm}$ , find (i)  $AB$ , (ii)  $AC$ .

**Sol:**

From the given right-angled triangle, we have:

$$\frac{BC}{AB} = \tan 30^\circ$$

$$\Rightarrow \frac{6}{AB} = \frac{1}{\sqrt{3}}$$

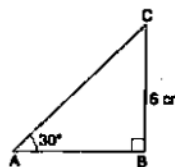
$$\Rightarrow AB = 6\sqrt{3}\text{cm}$$

$$\text{Also, } \frac{BC}{AC} = \sin 30^\circ$$

$$\Rightarrow \frac{6}{AC} = \frac{1}{2}$$

$$\Rightarrow AC = (2 \times 6) = 12\text{ cm}$$

$$\therefore AB = 6\sqrt{3}\text{ cm and } AC = 12\text{ cm}$$



22. In the adjoining figure,  $\triangle ABC$  is right-angled at B and  $\angle A = 45^\circ$ . If  $AC = 3\sqrt{2}$  cm, find (i) BC, (ii) AB.

**Sol:**

From the right-angled  $\triangle ABC$ , we have:

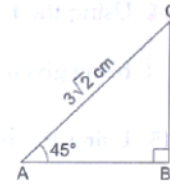
$$\frac{BC}{AC} = \sin 45^\circ$$

$$\Rightarrow \frac{BC}{3\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow BC = 3 \text{ cm}$$

$$\text{Also, } \frac{AB}{AC} = \cos 45^\circ$$

$$\Rightarrow \frac{AB}{3\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow AB = 3 \text{ cm}$$

$$\therefore BC = 3 \text{ cm and } AB = 3 \text{ cm}$$



23. If  $\sin(A + B) = 1$  and  $\cos(A - B) = 1$ ,  $0^\circ \leq (A + B) \leq 90^\circ$  and  $A > B$ , then find A and B.

**Sol:**

$$\text{Here, } \sin(A + B) = 1$$

$$\Rightarrow \sin(A + B) = 90^\circ \quad [\because \sin 90^\circ = 1]$$

$$\Rightarrow (A + B) = 90^\circ \quad \dots\dots(i)$$

$$\text{Also, } \cos(A - B) = 1$$

$$\Rightarrow \cos(A - B) = 0^\circ \quad [\because \cos 0^\circ = 1]$$

$$\Rightarrow A - B = 0^\circ \quad \dots\dots(ii)$$

Solving (i) and (ii), we get:

$$A = 45^\circ \text{ and } B = 45^\circ$$

24. If  $\sin(A - B) = \frac{1}{2}$  and  $\cos(A + B) = \frac{1}{2}$ ,  $0^\circ \leq (A + B) \leq 90^\circ$  and  $A > B$ , then find A and B.

**Sol:**

$$\text{Here, } \sin(A - B) = \frac{1}{2}$$

$$\Rightarrow \sin(A - B) = 30^\circ \quad [\because \sin 30^\circ = \frac{1}{2}]$$

$$\Rightarrow (A - B) = 30^\circ \quad \dots\dots(i)$$

$$\text{Also, } \cos(A + B) = \frac{1}{2}$$

$$\Rightarrow \cos(A + B) = \cos 60^\circ \quad [\because \cos 60^\circ = \frac{1}{2}]$$

$$\Rightarrow A + B = 60^\circ \quad \dots\dots(ii)$$

Solving (i) and (ii), we get:

$$A = 45^\circ \text{ and } B = 15^\circ$$

25. If  $\tan(A - B) = \frac{1}{\sqrt{3}}$  and  $\tan(A + B) = \sqrt{3}$ ,  $0^\circ \leq (A + B) \leq 90^\circ$  and  $A > B$ , then find A and B.

**Sol:**

$$\text{Here, } \tan(A - B) = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan(A - B) = \tan 30^\circ \quad [\because \tan 30^\circ = \frac{1}{\sqrt{3}}]$$

$$\Rightarrow (A - B) = 30^\circ \quad \dots\dots(i)$$

$$\text{Also, } \tan(A + B) = \sqrt{3}$$

$$\Rightarrow \tan (A + B) = \tan 60^{\circ} \quad [ \because \tan 60^{\circ} = \sqrt{3} ]$$

$$\Rightarrow A + B = 60^{\circ} \quad \dots\dots(ii)$$

Solving (i) and (ii), we get:

$$A = 45^{\circ} \text{ and } B = 15^{\circ}$$

26. If  $3x = \operatorname{cosec} \theta$  and  $\frac{3}{x} = \cot \theta$  find the value of  $3\left(x^2 - \frac{1}{x^2}\right)$

**Sol:**

$$\begin{aligned} & 3\left(x^2 - \frac{1}{x^2}\right) \\ &= \frac{9}{3}\left(x^2 - \frac{1}{x^2}\right) \\ &= \frac{1}{3}\left(9x^2 - \frac{9}{x^2}\right) \\ &= \frac{1}{3}\left[(3x^2) - \left(\frac{3}{x}\right)^2\right] \\ &= \frac{1}{3}[(\operatorname{cosec} \theta)^2 - (\cot \theta)^2] \\ &= \frac{1}{3}(\operatorname{cosec}^2 \theta - \cot^2 \theta) \\ &= \frac{1}{3}(1) = \frac{1}{3} \end{aligned}$$

27. If  $\sin (A+B) = \sin A \cos B + \cos A \sin B$  and  $\cos (A-B) = \cos A \cos B + \sin A \sin B$

(i)  $\sin (75^{\circ})$

(ii)  $\cos (15^{\circ})$

**Sol:**

Let  $A = 45^{\circ}$  and  $B = 30^{\circ}$

(i) As,  $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$$\Rightarrow \sin (45^{\circ} + 30^{\circ}) = \sin 45^{\circ} \cos 30^{\circ} + \cos 45^{\circ} \sin 30^{\circ}$$

$$\Rightarrow \sin (75^{\circ}) = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$\Rightarrow \sin (75^{\circ}) = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$\therefore \sin (75^{\circ}) = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

(ii) As,  $\cos (A - B) = \cos A \cos B + \sin A \sin B$

$$\Rightarrow \cos (45^{\circ} - 30^{\circ}) = \cos 45^{\circ} \cos 30^{\circ} + \sin 45^{\circ} \sin 30^{\circ}$$

$$\Rightarrow \cos (15^{\circ}) = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$\Rightarrow \cos (15^{\circ}) = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$\therefore \cos (15^{\circ}) = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Disclaimer:  $\cos 15^{\circ}$  can also be written by taking  $A = 60^{\circ}$  and  $B = 45^{\circ}$ .