## Exercise 8(C)

### Solution 1:

Given that $\log_{10} 8 = 0.90$

$\Rightarrow \log_{10} 2 \times 2 \times 2 = 0.90$

$\Rightarrow \log_{10} 2^3 = 0.90$

$\Rightarrow 3\log_{10} 2 = 0.90$

$\Rightarrow \log_{10} 2 = \frac{0.90}{3}$

$\Rightarrow \log_{10} 2 = 0.30 \ldots (1)$

(i)

$\log 4 = \log_{10}(2 \times 2)$

$\Rightarrow \quad = \log_{10}(2^2)$

$\Rightarrow \quad = 2\log_{10} 2$

$\Rightarrow \quad = 2(0.30) \quad$ [from (1)]

$\Rightarrow \quad = 0.60$



(ii)

$\log \sqrt{32} = \log_{10}(32)^{\frac{1}{2}}$

$\Rightarrow \quad = \frac{1}{2}\log_{10}(32)$

$\Rightarrow \quad = \frac{1}{2}\log_{10}(2 \times 2 \times 2 \times 2 \times 2)$

$\Rightarrow \quad = \frac{1}{2}\log_{10}(2^5)$

$\Rightarrow \quad = \frac{1}{2} \times 5\log_{10} 2$

$\Rightarrow \quad = \frac{1}{2} \times 5(0.30) \quad$ [from (1)]

$\Rightarrow \quad = 5 \times 0.15$

$\Rightarrow \quad = 0.75$

(iii)

$$\log 0.125 = \log_{10}\frac{125}{1000}$$
$$= \log_{10}\frac{1}{8}$$
$$= \log_{10}\frac{1}{2\times2\times2}$$
$$= \log_{10}\left(\frac{1}{2^3}\right)$$
$$= \log_{10} 2^{-3}$$
$$= -3\times(0.30) \quad [\text{from } (1)]$$
$$= -0.9$$

**Solution 2:**

$$\log 27 = 1.431$$
$$\Rightarrow \log 3\times3\times3 = 1.431$$
$$\Rightarrow \log 3^3 = 1.431$$
$$\Rightarrow 3\log 3 = 1.431$$
$$\Rightarrow \log 3 = \frac{1.431}{3}$$
$$\Rightarrow \log 3 = 0.477 \ldots\ldots(1)$$



(i)

$$\log 9 = \log(3\times3)$$
$$= \log 3^2$$
$$= 2\log 3$$
$$= 2\times0.477 \quad [\text{from } (1)]$$
$$= 0.954$$

(ii)

$$\log 300 = \log(3\times100)$$
$$= \log 3 + \log 100$$
$$= \log 3 + 2 \quad [\because \log_{10} 100 = 2]$$
$$= 0.477 + 2 \quad [\text{from } (1)]$$
$$= 2.477$$

**Solution 3:**

$\log_{10} a = b$

$\Rightarrow 10^b = a$

$\Rightarrow (10^b)^3 = (a)^3$ [cubing both sides]

$\Rightarrow \frac{10^{3b}}{10^2} = \frac{a^3}{10^2}$ [dividing both sides by $10^2$]

$\Rightarrow 10^{3b-2} = \frac{a^3}{100}$

**Solution 4:**

$\log_5 x = y$ [given]

$\Rightarrow 5^y = x$

$\Rightarrow (5^y)^2 = x^2$

$\Rightarrow 5^{2y} = x^2$

$\Rightarrow 5^{2y} \times 5^3 = x^2 \times 5^3$

$\Rightarrow 5^{2y+3} = 125x^2$

**Solution 5:**

Given that $\log_3 m = x$ and $\log_3 n = y$

$\Rightarrow 3^x = m$ and $3^y = n$

(i)

Consider the given expression:

$3^{2x-3} = 3^{2x} \cdot 3^{-3}$

$= 3^{2x} \cdot \dfrac{1}{3^3}$

$= \dfrac{3^{2x}}{3^3}$

$= \dfrac{(3^x)^2}{3^3}$

$= \dfrac{m^2}{27}$

Therefore, $3^{2x-3} = \dfrac{m^2}{27}$

(ii)

Consider the given expression:

$3^{1-2y+3x} = 3^1 \cdot 3^{-2y} \cdot 3^{3x}$

$= 3 \cdot \dfrac{1}{3^{2y}} \cdot 3^{3x}$

$= \dfrac{3}{(3^y)^2} \cdot (3^x)^3$

$= \dfrac{3}{(n)^2} \cdot (m)^3$

$= \dfrac{3m^3}{n^2}$

Therefore, $3^{1-2y+3x} = \dfrac{3m^3}{n^2}$

(iii)

Consider the given expression:

$2\ \log_3 A = 5x - 3y$

$\Rightarrow 2\ \log_3 A = 5\ \log_3 m - 3\log_3 n$

$\Rightarrow \log_3 A^2 = \log_3 m^5 - \log_3 n^3$

$\Rightarrow \log_3 A^2 = \log_3\left(\frac{m^5}{n^3}\right)$

$\Rightarrow A^2 = \left(\frac{m^5}{n^3}\right)$

$\Rightarrow A = \sqrt{\left(\frac{m^5}{n^3}\right)}$

**Solution 6:**

(i)

$\log(a)^3 - \log a = 3\log a - \log a$

$= 2\log a$

(ii)

$\log(a)^3 \div \log a = 3\log a \div \log a$

$= \frac{3\log a}{\log a}$

$= 3$



**Solution 7:**

$\log(a + b) = \log a + \log b$

$\Rightarrow \log(a + b) = \log ab$

$\Rightarrow a + b = ab$

$\Rightarrow a - ab = -b$

$\Rightarrow -ab + a = -b$

$\Rightarrow -a(b - 1) = -b$

$\Rightarrow a(b - 1) = b$

$\Rightarrow a = \frac{b}{b - 1}$

## Solution 8:

(i)

$L.H.S = (\log a)^2 - (\log b)^2$

$\Rightarrow L.H.S = (\log a + \log b)(\log a - \log b)$

$\Rightarrow L.H.S = \log(ab)\log\left(\frac{a}{b}\right)$

$\Rightarrow L.H.S = \log\left(\frac{a}{b}\right) \times \log(ab)$

$\Rightarrow L.H.S = R.H.S$

Hence proved.

(ii)

Given that

$a\log b + b\log a - 1 = 0$

$\Rightarrow a\log b + b\log a = 1$

$\Rightarrow \log b^a + \log a^b = 1$

$\Rightarrow \log b^a + \log a^b = \log 10$

$\Rightarrow \log(b^a \cdot a^b) = \log 10$

$\Rightarrow b^a \cdot a^b = 10$



## Solution 9:

(i)

Given that

$\log(a+1) = \log(4a-3) - \log 3$

$\Rightarrow \log(a+1) = \log\left(\frac{4a-3}{3}\right)$

$\Rightarrow a+1 = \frac{4a-3}{3}$

$\Rightarrow 3a + 3 = 4a - 3$

$\Rightarrow 4a - 3a = 3 + 3$

$\Rightarrow a = 6$

(ii)

$2\log y - \log x - 3 = 0$

$\Rightarrow 2\log y - \log x = 3$

$\Rightarrow \log y^2 - \log x = 3$

$\Rightarrow \log y^2 - \log x = \log 1000$

$\Rightarrow \log\frac{y^2}{x} = \log 1000$

$\Rightarrow \frac{y^2}{x} = 1000$

$\Rightarrow x = \frac{y^2}{1000}$

(iii)

$\log_{10}125 = 3(1-\log_{10}2)$

$L.H.S. = \log_{10}125$

$= \log_{10}5\times5\times5$

$= \log_{10}5^3$

$= 3\log_{10}5.....(1)$

$R.H.S = 3(1-\log_{10}2)$

$= 3(\log_{10}10-\log_{10}2)$

$= 3\log_{10}\left(\frac{10}{2}\right)$

$= 3\log_{10}5.....(2)$

From (1) and (2), we have

L.H.S.=R.H.S.

Hence proved.

**Solution 10:**

Given $\log x = 2m - n$, $\log y = n - 2m$ and $\log z = 3m - 2n$

$\log\frac{x^2y^3}{z^4} = \log x^2y^3 - \log z^4$

$= \log x^2 + \log y^3 - \log z^4$

$= 2\log x + 3\log y - 4\log z$

$= 2(2m-n) + 3(n-2m) - 4(3m-2n)$

$= 4m - 2n + 3n - 6m - 12m - 8n$

$= -14m - 7n$

**Solution 11:**

$\log_x 25 - \log_x 5 = 2 - \log_x\frac{1}{125}$

$\Rightarrow \log_x 5^2 - \log_x 5 = 2 - \log_x\left(\frac{1}{5}\right)^3$

$\Rightarrow \log_x 5^2 - \log_x 5 = 2 - \log_x 5^{-3}$

$\Rightarrow 2\log_x 5 - \log_x 5 = 2 + 3\log_x 5$

$\Rightarrow 2\log_x 5 - \log_x 5 - 3\log_x 5 = 2$

$\Rightarrow -2\log_x 5 = 2$

$\Rightarrow \log_x 5 = -1$

$\Rightarrow x^{-1} = 5$

$\Rightarrow \frac{1}{x} = 5$

$\Rightarrow x = \frac{1}{5}$