$$= \frac{9!}{(9-r)! \times r!} \; 2^{9-r} \, (3)^r (x)^r$$

$$\therefore t_r = \binom{n}{r-1} \, a^{n-r+1} \, b^{r-1}$$

$$= \binom{9}{r-1} \, 2^{9-r+1} \, (3x)^{r-1}$$

$$= \frac{9!}{(9-r+1)! \times (r-1)!} \; 2^{10-r} \, (3)^{r-1} (x)^{r-1}$$

$$= \frac{9!}{(10-r)! \times (r-1)!} \; 2^{10-r} \, (3)^{r-1} (x)^{r-1}$$

$$\therefore \frac{t_{r+1}}{t_r} \geq 1$$

$$\therefore \frac{\frac{9!}{(9-r)! \times r!} \; 2^{9-r} \, (3)^r (x)^r}{\frac{9!}{(10-r)! \times (r-1)!} \; 2^{10-r} \, (3)^{r-1} (x)^{r-1}} \geq 1$$

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$$\therefore \frac{9!}{(9-r)! \times r!} \; 2^{9-r} \, (3)^r (x)^r \geq \frac{9!}{(10-r)! \times (r-1)!} \; 2^{10-r} \, (3)^{r-1} (x)^{r-1}$$

$$\therefore \frac{9!}{(9-r)! \times r(r-1)!} \; 2^{9-r} \, (3)(3)^{r-1} (x)(x)^{r-1}$$
$$\geq \frac{9!}{(10-r)(9-r)! \times (r-1)!} \; (2) 2^{9-r} \, (3)^{r-1} (x)^{r-1}$$

$$\therefore \frac{1}{r} \, (3)(x) \geq \frac{1}{(10-r)} \, (2)$$

At x = 3/2

$$\therefore \frac{1}{r} \, (3) \frac{3}{2} \geq \frac{1}{(10-r)} \, (2)$$

$\therefore \frac{9}{4} \geq \frac{r}{(10-r)}$

$\therefore 9(10-r) \geq 4r$

$\therefore 90 - 9r \geq 4r$

• $90 \geq 13r$

• $r \leq 6.923$

Therefore, r=6 and hence the 7th term is numerically greater.

By using formula,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

$t_7 = \binom{9}{7} 2^{9-7} (3x)^7$

$= \binom{9}{2} 2^2 (3)^7 (x)^7$

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Conclusion : the 7th term is numerically greater with value $\binom{9}{2} 2^2 (3)^7 (x)^7$

Q. 43. If the coefficients of 2nd, 3rd and 4th terms in the expansion of $(1 + x)^{2n}$ are in AP, show that $2n^2 - 9n + 7 = 0$.

Answer : For $(1 + x)^{2n}$

a=1, b=x and N=2n

We have, $t_{r+1} = \binom{N}{r} a^{N-r} b^r$

For the 2nd term, r=1

$\therefore t_2 = t_{1+1}$

$= \binom{2n}{1} (1)^{2n-1} (x)^1$

$= (2n)\ x$ .... $[\because \binom{n}{1} = n]$

Therefore, the coefficient of 2nd term = (2n)

For the 3rd term, r=2

$\therefore t_3 = t_{2+1}$

$= \binom{2n}{2} (1)^{2n-2} (x)^2$

$= \frac{(2n)!}{(2n-2)! \times 2!} x^2$

$= \frac{(2n)(2n-1)(2n-2)!}{(2n-2)! \times 2} x^2$ ....(n! = n. (n-1)!)

$= (n)(2n-1)\ x^2$

Therefore, the coefficient of 3rd term = (n)(2n-1)

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For the 4th term, r=3

$\therefore t_4 = t_{3+1}$

$= \binom{2n}{3} (1)^{2n-3} (x)^3$

$= \frac{(2n)!}{(2n-3)! \times 3!} x^3$

$= \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)! \times 6} x^3$ ....(n! = n. (n-1)!)

$= \frac{(n)(2n-1).2(n-1)}{3} x^3$

$= \frac{2(n)(2n-1).(n-1)}{3} x^3$

Therefore, the coefficient of 3rd term $= \frac{2(n)(2n-1).(n-1)}{3}$

As the coefficients of 2nd, 3rd and 4th terms are in A.P.

Therefore,

2×coefficient of 3rd term = coefficient of 2nd term + coefficient of the 4th term

$$\therefore 2 \times (n)(2n-1) = (2n) + \frac{2(n)(2n-1).(n-1)}{3}$$

Dividing throughout by (2n),

$$\therefore 2n-1 = 1 + \frac{(2n-1)(n-1)}{3}$$

$$\therefore 2n-1 = \frac{3+(2n-1)(n-1)}{3}$$

• 3 (2n-1) = 3 + (2n-1)(n-1)

• $6n - 3 = 3 + (2n^2 - 2n - n + 1)$

• $6n - 3 = 3 + 2n^2 - 3n + 1$

• $3 + 2n^2 - 3n + 1 - 6n + 3 = 0$

• $2n^2 - 9n + 7 = 0$

<u>Conclusion</u> : If the coefficients of 2nd, 3rd and 4th terms of $(1 + x)^{2n}$ are in A.P. then $2n^2 - 9n + 7 = 0$

Q. 44. Find the 6th term of the expansion $(y^{1/2} + x^{1/3})^n$, if the binomial coefficient of the 3rd term from the end is 45.

Answer : <u>Given</u> : 3rd term from the end =45

<u>To Find</u> : 6th term

For $(y^{1/2} + x^{1/3})^n$,

$a= y^{1/2}$, $b= x^{1/3}$

We have, $t_{r+1} = \binom{n}{r} a^{n-r} b^r$

As n=n, therefore there will be total (n+1) terms in the expansion.

3rd term from the end = (n+1-3+1)th i.e. (n-1)th term from the starting

For (n-1)th term, r = (n-1-1) = (n-2)

$t_{(n-1)} = t_{(n-2)+1}$

$= \binom{n}{n-2} \left(y^{\frac{1}{2}}\right)^{n-(n-2)} \left(x^{\frac{1}{3}}\right)^{(n-2)}$

$= \binom{n}{2} \left(y^{\frac{1}{2}}\right)^{2} (x)^{\frac{n-2}{3}}$ ……….. $\because \binom{n}{n-r} = \binom{n}{r}$

$= \frac{n(n-1)}{2} (y) (x)^{\frac{n-2}{3}}$

Therefore 3rd term from the end $= \frac{n(n-1)}{2} (y) (x)^{\frac{n-2}{3}}$

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Therefore coefficient 3rd term from the end $= \frac{n(n-1)}{2}$

$\therefore 45 = \frac{n(n-1)}{2}$

- 90 = n (n-1)
- 10 (9) = n (n-1)

Comparing both sides, n=10

For 6th term, r=5

$t_6 = t_{5+1}$

$= \binom{10}{5} \left(y^{\frac{1}{2}}\right)^{10-5} \left(x^{\frac{1}{3}}\right)^{5}$

$= \binom{10}{5} (y)^{\frac{5}{2}} (x)^{\frac{5}{3}}$

$= \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} (y)^{\frac{5}{2}} (x)^{\frac{5}{3}}$

$= 252 (y)^{\frac{5}{2}} (x)^{\frac{5}{3}}$

Conclusion : 6th term $= 252 (y)^{\frac{5}{2}} (x)^{\frac{5}{3}}$

Q. 45. If the 17th and 18th terms in the expansion of $(2 + a)^{50}$ are equal, find the value of a.

Answer : Given : $t_{17} = t_{18}$

To Find : value of a

For $(2 + a)^{50}$

A=2, b=a and n=50

We have, $t_{r+1} = \binom{n}{r} A^{n-r} b^r$

For the 17th term, r=16

$\therefore t_{17} = t_{16+1}$

$= \binom{50}{16} (2)^{50-16} (a)^{16}$

$= \binom{50}{16} (2)^{34} (a)^{16}$

For the 18th term, r=17

$\therefore t_{18} = t_{17+1}$

$= \binom{50}{17} (2)^{50-17} (a)^{17}$

$= \binom{50}{17} (2)^{33} (a)^{17}$

As 17^{th} and 18^{th} terms are equal

$\therefore t_{18} = t_{17}$

$\therefore \binom{50}{17} (2)^{33} (a)^{17} = \binom{50}{16} (2)^{34} (a)^{16}$

$\therefore \binom{50}{17} (2)^{33} (a)^{17} = \binom{50}{16} (2)^{34} (a)^{16}$

$$\therefore \frac{50!}{(50-17)! \times (17)!} (2)^{33} (a)^{17} = \frac{50!}{(50-16)! \times (16)!} (2)^{34} (a)^{16}$$

.... $\left[\because \binom{n}{r} = \frac{n!}{(n-r)! \times (r)!}\right]$

$$\therefore \frac{(a)^{17}}{(a)^{16}} = \frac{50!}{(50-16)! \times (16)!} \cdot \frac{(50-17)! \times (17)!}{50!} \cdot \frac{(2)^{34}}{(2)^{33}}$$

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$$\therefore a = \frac{(50-17) \times (50-16)! \times 17 \times (16)!}{(50-16)! \times (16)!} . (2)$$

.... $[\because n! = n(n-1)!]$

$\therefore a = (50-17) \times 17 . (2)$

• a = 1122

<u>Conclusion</u> : value of a = 1122

Q. 46. Find the coefficient of x^4 in the expansion of $(1 + x)^n (1 - x)^n$. Deduce that $C_2 = C_0C_4 - C_1C_3 + C_2C_2 - C_3C_1 + C_4C_0$, where C_r stands for ^nC_r.

Answer : <u>To Find</u> : Coefficients of x^4

For $(1+x)^n$

a=1, b=x

We have a formula,

$$(1+x)^n = \sum_{r=0}^{n} \binom{n}{r} (1)^{n-r} x^r$$

$$= \binom{n}{0} (1)^n x^0 + \binom{n}{1} (1)^{n-1} x^1 + \binom{n}{2} (1)^{n-2} x^2 + \cdots + \binom{n}{n} (1)^{n-n} x^n$$

$$= \binom{n}{0} x^0 + \binom{n}{1} x + \binom{n}{2} x^2 + \cdots + \binom{n}{n} x^n$$

For $(1-x)^n$

a=1, b=-x and n=n

We have formula,

$$(1-x)^n = \sum_{r=0}^{n} \binom{n}{r} (1)^{n-r} (-x)^r$$

$$= \binom{n}{0} (1)^n (-x)^0 + \binom{n}{1} (1)^{n-1} (-x)^1 + \binom{n}{2} (1)^{n-2} (-x)^2 + \cdots + \binom{n}{n} (1)^{n-n} (-x)^n$$

$$= \binom{n}{0} (-x)^0 - \binom{n}{1} (x)^1 + \binom{n}{2} (x)^2 + \cdots + \binom{n}{n} (-x)^n$$

$$\therefore (1+x)^3 (1-x)^6$$

$$= \left\{\binom{n}{0} x^0 + \binom{n}{1} x + \binom{n}{2} x^2 + \cdots + \binom{n}{n} x^n\right\}\left\{\binom{n}{0} (-x)^0 - \binom{n}{1} (x)^1 + \binom{n}{2} (x)^2 + \cdots + \binom{n}{n} (-x)^n\right\}$$

Coefficients of x^4 are

$x^0.x^4 = \binom{n}{0} \times \binom{n}{4} = C_0C_4$

$x^1.x^3 = \binom{n}{1} \times (-1)\binom{n}{3} = -\binom{n}{1}\binom{n}{3} = -C_1C_3$

$x^2.x^2 = \binom{n}{2} \times \binom{n}{2}$ = C_2C_2

$x^3.x^1 = \binom{n}{3} \times (-1)\binom{n}{1} = -\binom{n}{3}\binom{n}{1}$ = - C_3C_1

$x^4.x^0 = \binom{n}{4} \times \binom{n}{0}$ = C_4C_0

Therefore, Coefficient of x^4

= C_4C_0 - C_1C_3 + C_2C_2 - C_3C_1 + C_4C_0

Let us assume, n=4, it becomes

${}^4C_4\ {}^4C_0$ - ${}^4C_1\ {}^4C_3$ + ${}^4C_2\ {}^4C_2$ - ${}^4C_3\ {}^4C_1$ + ${}^4C_4\ {}^4C_0$

We know that,

$$\binom{n}{r} = \frac{n!}{(n-r)!\ \times r!}$$

By using above formula, we get,

${}^4C_4\ {}^4C_0$ - ${}^4C_1\ {}^4C_3$ + ${}^4C_2\ {}^4C_2$ - ${}^4C_3\ {}^4C_1$ + ${}^4C_4\ {}^4C_0$

= (1)(1) – (4)(4) + (6)(6) – (4)(4) + (1)(1)

= 1 – 16 + 36 – 16 + 1

= 6

= ${}^4C_2$

Therefore, in general,

C_4C_0 - C_1C_3 + C_2C_2 - C_3C_1 + C_4C_0 = C_2

Therefore, Coefficient of x^4 = C_2

<u>Conclusion</u> :

• Coefficient of x^4 = C_2

• C_4C_0 - C_1C_3 + C_2C_2 - C_3C_1 + C_4C_0 = C_2

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Q. 47. Prove that the coefficient of xn in the binomial expansion of $(1 + x)^{2n}$ is twice the coefficient of x^n in the binomial expansion of $(1 + x)^{2n-1}$.

Answer : <u>To Prove</u> : coefficient of x^n in $(1+x)^{2n}$ = 2 × coefficient of x^n in $(1+x)^{2n-1}$

For $(1+x)^{2n}$,

a=1, b=x and m=2n

We have a formula,

$$t_{r+1} = \binom{m}{r} a^{m-r} b^r$$

$$= \binom{2n}{r} (1)^{2n-r} (x)^r$$

$$= \binom{2n}{r} (x)^r$$

To get the coefficient of x^n, we must have,

$x^n = x^r$

• r = n

Therefore, the coefficient of x^n $= \binom{2n}{n}$

$= \frac{(2n)!}{n! \times (2n-n)!}$ ……… $\left(\because \binom{n}{r} = \frac{n!}{r! \times (n-r)!}\right)$

$$= \frac{(2n)!}{n! \times n!}$$

$= \frac{2n \times (2n-1)!}{n! \times n(n-1)!}$ ……….. $(\because n! = n(n-1)!)$

$= \frac{2 \times (2n-1)!}{n! \times (n-1)!}$ ………cancelling n

Therefore, the coefficient of x^n in $(1+x)^{2n}$ $= \frac{2 \times (2n-1)!}{n! \times (n-1)!}$ ………eq(1)

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Now for $(1+x)^{2n-1}$,

a=1, b=x and m=2n-1

We have formula,

$$t_{r+1} = \binom{m}{r} a^{m-r} b^r$$

$$= \binom{2n-1}{r} (1)^{2n-1-r} (x)^r$$

$$= \binom{2n-1}{r} (x)^r$$

To get the coefficient of x^n, we must have,

$x^n = x^r$

• r = n

Therefore, the coefficient of x^n in $(1+x)^{2n-1} = \binom{2n-1}{n}$

$$= \frac{(2n-1)!}{n! \times (2n-1-n)!}$$

$$= \frac{1}{2} \times \frac{2 \times (2n-1)!}{n! \times (n-1)!}$$

…..multiplying and dividing by 2

Therefore,

Coefficient of x^n in $(1+x)^{2n-1}$ = � × coefficient of x^n in $(1+x)^{2n}$

Or coefficient of x^n in $(1+x)^{2n}$ = 2 × coefficient of x^n in $(1+x)^{2n-1}$

Hence proved.

Q. 48. Find the middle term in the expansion of $\left(\frac{p}{2}+2\right)^8$

Answer : Given : $a = \frac{p}{2}$, b=2 and n=8

To find : middle term

Formula :

- The middle term $= \left(\frac{n+2}{2}\right)$
- $t_{r+1} = \binom{n}{r} a^{n-r} b^r$

Here, n is even.

Hence,

$$\left(\frac{n+2}{2}\right) = \left(\frac{8+2}{2}\right) = 5$$

Therefore, 5^{th} the term is the middle term.

For t_5, r=4

We have, $t_{r+1} = \binom{n}{r} a^{n-r} b^r$

$$\therefore\ t_5 = \binom{8}{4}\left(\frac{p}{2}\right)^{8-4} 2^4$$

$$\therefore\ t_5 = \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1}.\left(\frac{p}{2}\right)^4.(16)$$

$$\therefore\ t_5 = 70.\left(\frac{p^4}{16}\right).(16)$$

$$\therefore\ t_5 = 70\ p^4$$

Conclusion : The middle term is $70\ p^4$.

Exercise 10B

Q. 1. Show that the term independent of x in the expansion of $\left(x - \frac{1}{x}\right)^{10}$ is -252.

Answer : To show: the term independent of x in the expansion of $\left(x - \frac{1}{x}\right)^{10}$ is -252.

Formula Used:

General term, T_{r+1} of binomial expansion $(x + y)^n$ is given by,

$T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$ where

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

Now, finding the general term of the expression, $\left(x - \frac{1}{x}\right)^{10}$, we get

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$$T_{r+1} = {}^{10}C_r \times x^{10-r} \times \left(\frac{-1}{x}\right)^r$$

For finding the term which is independent of x,

10 – 2r=5

r=5

Thus, the term which would be independent of x is T_6

$T_6 = {}^{10}C_5 \times x^{10-5} \times \left(\frac{-1}{x}\right)^5$

$T_6 = {}^{10}C_5 \times x^{10-5} \times \left(\frac{-1}{x}\right)^5$

$T_6 = -\ {}^{10}C_5$

$T_6 = -\ \frac{10!}{5!(10-5)!}$

$T_6 = -\ \frac{10!}{5! \times 5!}$

$T_6 = -\ \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5! \times 5 \times 4 \times 3 \times 2}$

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$T_6 = -\ \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5! \times 5 \times 4 \times 3 \times 2}$

$T_6 = 252$

Thus, the term independent of x in the expansion of $\left(x - \frac{1}{x}\right)^{10}$ is -252.

Q. 2. If the coefficients of x^2 and x^3 in the expansion of $(3 + px)^9$ are the same then prove that $P = \frac{9}{7}$.

Answer : To prove: that. If the coefficients of x^2 and x^3 in the expansion of $(3 + px)^9$ are the same then $P = \frac{9}{7}$.

Formula Used:

General term, T_{r+1} of binomial expansion $(x + y)^n$ is given by,

$T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$ where

${}^nC_r = \frac{n!}{r!(n-r)!}$

Now, finding the general term of the expression, $(3 + px)^9$, we get

$T_{r+1} = {}^9C_r \times 3^{9-r} \times (px)^r$

For finding the term which has x^2 in it, is given by

r=2

Thus, the coefficients of x^2 are given by,

$T_3 = {}^9C_2 \times 3^{9-2} \times (px)^2$

$T_3 = {}^9C_2 \times 3^7 \times p^2 \times x^2$

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For finding the term which has x^2 in it, is given by

r=3

Thus, the coefficients of x^3 are given by,

$T_3 = {}^9C_3 \times 3^{9-3} \times (px)^3$

$T_3 = {}^9C_3 \times 3^6 \times p^3 \times x^3$

As the coefficients of x^2 and x^3 in the expansion of $(3 + px)^9$ are the same.

${}^9C_3 \times 3^6 \times p^3 = {}^9C_2 \times 3^7 \times p^2$

${}^9C_3 \times p = {}^9C_2 \times 3$

$$\frac{9!}{3! \times 6!} \times p = \frac{9!}{2! \times 7!} \times 3$$

$$\frac{9!}{3 \times 2! \times 6!} \times p = \frac{9!}{2! \times 7 \times 6!} \times 3$$

$$p = \frac{9}{7}$$

Thus, the value of p for which coefficients of x^2 and x^3 in the expansion of $(3 + px)^9$ are the same is $\frac{9}{7}$

Q. 3. Show that the coefficient of x^{-3} in the expansion of $\left(x - \frac{1}{x}\right)^{11}$ is -330.

Answer : To show: that the coefficient of x^{-3} in the expansion of $\left(x - \frac{1}{x}\right)^{11}$ is -330.

Formula Used:

General term, T_{r+1} of binomial expansion $(x + y)^n$ is given by,

$T_{r+1} = {}^nC_r\, x^{n-r} y^r$ where

${}^nC_r = \frac{n!}{r!(n-r)!}$

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Now, finding the general term of the expression, $\left(x - \frac{1}{x}\right)^{11}$, we get

$T_{r+1} = {}^{11}C_r \times x^{11-r} \times \left(\frac{-1}{x}\right)^r$

For finding the term which has x^{-3} in it , is given by

11 – 2r=3

2r = 14

R = 7

Thus, the term which the term which has x^{-3} in it isT_8

$T_8 = {}^{11}C_7 \times x^{11-7} \times \left(\frac{-1}{x}\right)^7$

$T_8 = -{}^{11}C_7 \times x^{-3}$

$T_8 = -\frac{11!}{7!(11-7)!}$

$T_6 = -\frac{11\times10\times9\times8\times7!}{7!\times4\times3\times2}$

$T_6 = -330$

Thus, the coefficient of x^{-3} in the expansion of $\left(x - \frac{1}{x}\right)^{11}$ is -330.

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Q. 4. Show that the middle term in the expansion of $\left(\frac{2x^2}{3} + \frac{3}{2x^2}\right)^{10}$ is 252.

Answer : To show: that the middle term in the expansion of $\left(\frac{2x^2}{3} + \frac{3}{2x^2}\right)^{10}$ is 252.

Formula Used:

General term, T_{r+1} of binomial expansion $(x + y)^n$ is given by,

$T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$ where

${}^nC_r = \frac{n!}{r!(n-r)!}$

Total number of terms in the expansion is 11

Thus, the middle term of the expansion is T_6 and is given by,

$T_6 = {}^{10}C_5 \times \left(\frac{2x^2}{3}\right)^5 \times \left(\frac{3}{2x^2}\right)^5$

$T_6 = {}^{10}C_5$

$T_6 = \frac{10\times9\times8\times7\times6\times5!}{5!\times5\times4\times3\times2}$

$T_6 = 252$

Thus, the middle term in the expansion of $\left(\frac{2x^2}{3} + \frac{3}{2x^2}\right)^{10}$ is 252.

Q. 5. Show that the coefficient of x^4 in the expansion of $\left(\frac{x}{2} - \frac{3}{x^2}\right)^{10}$ is $\frac{405}{256}$.

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Answer : To show: that the coefficient of x^4 in the expansion of $\left(\frac{x}{2} - \frac{3}{x^2}\right)^{10}$ is -330.

Formula Used:

General term, T_{r+1} of binomial expansion $(x+y)^n$ is given by,

$T_{r+1} = {}^nC_r \, x^{n-r} y^r$ where

${}^nC_r = \frac{n!}{r!(n-r)!}$

Now, finding the general term of the expression, $\left(\frac{x}{2} - \frac{3}{x^2}\right)^{10}$, we get

$T_{r+1} = {}^{10}C_r \times \left(\frac{x}{2}\right)^{10-r} \times \left(\frac{-3}{x^2}\right)^r$

For finding the term which has x^4 in it, is given by

10 – 3r = 4

3r = 6

R = 2

Thus, the term which has x^4 in it is T_3

$$T_3 = {}^{10}C_2 \times \left(\frac{x}{2}\right)^8 \times \left(\frac{-3}{x^2}\right)^2$$

$$T_3 = \frac{10! \times 9}{2! \times 8! \times 2^8}$$

$$T_3 = \frac{10 \times 9 \times 8! \times 9}{2 \times 8! \times 2^8}$$

$$T_3 = \frac{405}{256}$$

Thus, the coefficient of x^4 in the expansion of $\left(\frac{x}{2} - \frac{3}{x^2}\right)^{10}$ is $\frac{405}{256}$

Q. 6. Prove that there is no term involving x^6 in the expansion of $\left(2x^2 - \frac{3}{x}\right)^{11}$.

Answer : To prove: that there is no term involving x^6 in the expansion of $\left(2x^2 - \frac{3}{x}\right)^{11}$

Formula Used:

General term, T_{r+1} of binomial expansion $(x + y)^n$ is given by,

$T_{r+1} = {}^nC_r \, x^{n-r} \, y^r$ where

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

Now, finding the general term of the expression, $\left(2x^2 - \frac{3}{x}\right)^{11}$, we get

$T_{r+1} = {}^{11}C_r \times (2x^2)^{11-r} \times \left(\frac{-3}{x}\right)^r$

For finding the term which has x^6 in it, is given by

22 – 2 r – r = 6

3r = 16

$r = \frac{16}{3}$

Since, $r = \frac{16}{3}$ is not possible as r needs to be a whole number

Thus, there is no term involving x^6 in the expansion of $\left(2x^2 - \frac{3}{x}\right)^{11}$.

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Q. 7. Show that the coefficient of x^4 in the expansion of $(1 + 2x + x^2)^5$ is 212.

Answer : To show: that the coefficient of x^4 in the expansion of $(1 + 2x + x^2)^5$ is 212.

Formula Used:

We have,

$(1 + 2x + x^2)^5 = (1 + x + x + x^2)^5$

$= (1 + x + x(1+x))^5$

$= (1 + x)^5(1 + x)^5$

$= (1 + x)^{10}$

General term, T_{r+1} of binomial expansion $(x + y)^n$ is given by,

$T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$ where s

${}^{n}C_r = \frac{n!}{r!(n-r)!}$

Now, finding the general term,

$T_{r+1} = {}^{10}C_r \times x^{10-r} \times (1)^r$

10-r=4

r=6

Thus, the coefficient of x^4 in the expansion of $(1 + 2x + x^2)^5$ is given by,

${}^{10}C_4 = \frac{10!}{4!6!}$

${}^{10}C_4 = \frac{10 \times 9 \times 8 \times 7 \times 6!}{24 \times 6!}$

${}^{10}C_4 = 210$

Thus, the coefficient of x^4 in the expansion of $(1 + 2x + x^2)^5$ is 210

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Q. 8. Write the number of terms in the expansion of $\left(\sqrt{2}+1\right)^5 + \left(\sqrt{2}-1\right)^5$

Answer : To find: the number of terms in the expansion of $\left(\sqrt{2}+1\right)^5 + \left(\sqrt{2}-1\right)^5$

Formula Used:

Binomial expansion of $(x + y)^n$ is given by,

$$(x + y)^n = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} \times y^r$$

Thus,

$$(\sqrt{2}+1)^5+(\sqrt{2}-1)^5$$
$$=\left((\sqrt{2})^5+(\sqrt{2})^4\binom{5}{1}+\cdots+\binom{5}{5}\right)$$
$$+\left((\sqrt{2})^5-(\sqrt{2})^4\binom{5}{1}+\cdots-\binom{5}{5}\right)$$

So, the no. of terms left would be 6

Thus, the number of terms in the expansion of $(\sqrt{2}+1)^5+(\sqrt{2}-1)^5$ is 6

Q. 9. Which term is independent of x in the expansion of $\left(x-\frac{1}{3x^2}\right)^9$?

Answer : To find: the term independent of x in the expansion of $\left(x-\frac{1}{3x^2}\right)^9$?

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Formula Used:

A general term, T_{r+1} of binomial expansion $(x+y)^n$ is given by,

$T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$ where

${}^nC_r = \frac{n!}{r!(n-r)!}$

Now, finding the general term of the expression, $\left(x-\frac{1}{3x^2}\right)^9$, we get

$T_{r+1} = {}^{9}C_r \times x^{9-r} \times \left(\frac{-1}{3x^2}\right)^r$

$T_{r+1} = {}^{9}C_r \times x^{9-r} \times (-1) \times 3x^{-2r}$

$T_{r+1} = {}^{9}C_r \times (-1) \times 3x^{9-3r}$

For finding the term which is independent of x,

9-3r=0

r=3

Thus, the term which would be independent of x is T_4

Thus, the term independent of x in the expansion of $\left(x - \frac{1}{x}\right)^{10}$ is T_4 i.e 4th term

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Q. 10. Write the coefficient of the middle term in the expansion of $(1 + x)^{2n}$.

Answer : To find: that the middle term in the expansion of $\left(\frac{2x^2}{3} + \frac{3}{2x^2}\right)^{10}$ is 252.

Formula Used:

A general term, T_{r+1} of binomial expansion $(x + y)^n$ is given by,

$T_{r+1} = {}^{n}C_r\, x^{n-r} y^r$ where

${}^{n}C_r = \frac{n!}{r!(n-r)!}$

Total number of terms in the expansion is 11

Thus, the middle term of the expansion is T_6 and is given by,

$T_6 = {}^{10}C_5 \times \left(\frac{2x^2}{3}\right)^5 \times \left(\frac{3}{2x^2}\right)^5$

$T_6 = {}^{10}C_5$

$T_6 = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5! \times 5 \times 4 \times 3 \times 2}$

$T_6 = 252$

Thus, the middle term in the expansion of $\left(\frac{2x^2}{3} + \frac{3}{2x^2}\right)^{10}$ is 252.

Q. 11. Write the coefficient of x^7y^2 in the expansion of $(x + 2y)^9$

Answer : To find: the coefficient of x^7y^2 in the expansion of $(x + 2y)^9$

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Formula Used:

A general term, T_{r+1} of binomial expansion $(x + y)^n$ is given by,

$T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$ where

${}^nC_r = \frac{n!}{r!(n-r)!}$

Now, finding the general term of the expression, $(x + 2y)^9$, we get

$T_{r+1} = {}^9C_r \times x^{9-r} \times (2y)^r$

The value of r for which coefficient of x^7y^2 is defined

R = 2

Hence, the coefficient of x^7y^2 in the expansion of $(x + 2y)^9$ is given by:

$T_3 = {}^9C_3 \times x^{9-2} \times (2y)^2$

$T_3 = {}^9C_3 \times 4 \times x^7 \times (y)^2$

$T_3 = \frac{9!}{3! \times 6!} \times 4 \times x^7 \times (y)^2$

$T_3 = \frac{9 \times 8 \times 7 \times 6!}{6 \times 6!} \times 4 \times x^7 \times (y)^2$

$T_3 = 336$

Thus, the coefficient of x^7y^2 in the expansion of $(x + 2y)^9$ is 336.

Q. 12. If the coefficients of (r – 5)th and (2r – 1)th terms in the expansion of $(1 + x)^{34}$ are equal, find the value of r.

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Answer : To find: the value of r with respect to the binomial expansion of $(1 + x)^{34}$ where the coefficients of the (r – 5)th and (2r – 1)th terms are equal to each other

Formula Used:

The general term, T_{r+1} of binomial expansion $(x + y)^n$ is given by,

$T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$ where

${}^nC_r = \frac{n!}{r!(n-r)!}$

Now, finding the (r – 5)th term, we get

$T_{r-5} = {}^{34}C_{r-6} \times x^{r-6}$

Thus, the coefficient of (r – 5)th term is ${}^{34}C_{r-6}$

Now, finding the (2r – 1)th term, we get

$$T_{2r-1} = {}^{34}C_{2r-2} \times (x)^{2r-2}$$

Thus, coefficient of (2r – 1)th term is ${}^{34}C_{2r-2}$

As the coefficients are equal, we get

$${}^{34}C_{2r-2} = {}^{34}C_{r-6}$$

2r – 2 = r – 6

R = – 4

Value of r=-4 is not possible

2r – 2 + r – 6 = 34

3r = 42

R = 14

Thus, value of r is 14

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Q. 13. Write the 4th term from the end in the expansion of $\left(\frac{3}{x^2} - \frac{x^3}{6}\right)^7$

Answer : To find: 4th term from the end in the expansion of $\left(\frac{3}{x^2} - \frac{x^3}{6}\right)^7$

Formula Used:

A general term, T_{r+1} of binomial expansion $(x + y)^n$ is given by,

$$T_{r+1} = {}^nC_r\, x^{n-r}\, y^r \text{ where}$$

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

Total number of terms in the expansion is 8

Thus, the 4th term of the expansion is T_5 and is given by,

$$T_5 = {}^7C_5 \times \left(\frac{3}{x^2}\right)^3 \times \left(\frac{-x^3}{6}\right)^4$$

$$T_5 = \frac{7\times6\times5!}{2\times5!} \times \frac{3\times3\times3}{6\times6\times6\times6} \times x^{-18}$$

$$T_5 = \frac{7\times6\times5!}{2\times5!} \times \frac{3\times3\times3}{6\times6\times6\times6} \times x^{-18}$$

$$T_5 = \frac{7}{16}x^{-18}$$

Thus, a 4th term from the end in the expansion of $\left(\frac{3}{x^2} - \frac{x^3}{6}\right)^7$ is $T_5 = \frac{7}{16}x^{-18}$

Q. 14. Find the coefficient of x^n in the expansion of $(1 + x)(1 – x)^n$.

Answer : To find: the coefficient of x^n in the expansion of $(1 + x)(1 – x)^n$.

Formula Used:

Binomial expansion of $(x+y)^n$ is given by,

$$(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} \times y^r$$

Thus,

$$(1 + x)(1 - x)^n.$$

$$= (1+x)\left(\binom{n}{0}(-x) + \binom{n}{1}(-x)^1 + \binom{n}{2}(-x)^2 + .. + \binom{n}{n-1}(-x)^{n-1} + \binom{n}{n}(-x)^n\right)$$

Thus, the coefficient of $(x)^n$ is,

${}^nC_n - {}^nC_{n-1}$ (If n is even)

$-{}^nC_n + {}^nC_{n-1}$ (If n is odd)

Thus, the coefficient of $(x)^n$ is, ${}^nC_n - {}^nC_{n-1}$ (If n is even) and $-{}^nC_n + {}^nC_{n-1}$ (If n is odd)

Q. 15. In the binomial expansion of $(a + b)^n$, the coefficients of the 4^{th} and 13^{th}terms are equal to each other. Find the value of n.

Answer : To find: the value of n with respect to the binomial expansion of $(a + b)^n$ where the coefficients of the 4^{th} and 13^{th} terms are equal to each other

Formula Used:

A general term, T_{r+1} of binomial expansion $(x + y)^n$ is given by,

$T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$ where

${}^nC_r = \frac{n!}{r!(n-r)!}$

Now, finding the 4^{th} term, we get

$T_4 = {}^nC_3 \times a^{n-3} \times (b)^3$

Thus, the coefficient of a 4^{th} term is ${}^nC_3$

Now, finding the 13^{th} term, we get

$T_{13} = {}^nC_{12} \times a^{n-12} \times (b)^{12}$

Thus, coefficient of 4^{th} term is ${}^nC_{12}$

As the coefficients are equal, we get

${}^nC_{12} = {}^nC_3$

Also, ${}^nC_r = {}^nC_{n-r}$

${}^nC_{n-12} = {}^nC_3$

n-12=3

n=15

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Thus, value of n is 15

Q. 16. Find the positive value of m for which the coefficient of x^2 in the expansion of $(1 + x)^m$ is 6.

Answer : To find: the positive value of m for which the coefficient of x^2 in the expansion of $(1 + x)^m$ is 6.

Formula Used:

General term, T_{r+1} of binomial expansion $(x + y)^n$ is given by,

$T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$ where

${}^nC_r = \frac{n!}{r!(n-r)!}$

Now, finding the general term of the expression, $(1 + x)^m$, we get

$T_{r+1} = {}^mC_r \times 1^{m-r} \times (x)^r$

$T_{r+1} = {}^mC_r \times (x)^r$

The coefficient of $(x)^2$ is ${}^mC_2$

${}^mC_2 = 6$

$\frac{m!}{2(m-2)!} = 6$

$$\frac{m(m-1)(m-2)!}{2(m-2)!} = 6$$

$$m^2 - m - 6 = 0$$

$$(m-3)(m+2) = 0$$

m=3,-2

Since m cannot be negative. Therefore,

m=3

Thus, positive value of m is 3 for which the coefficient of x2 in the expansion of $(1 + x)^m$ is 6

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