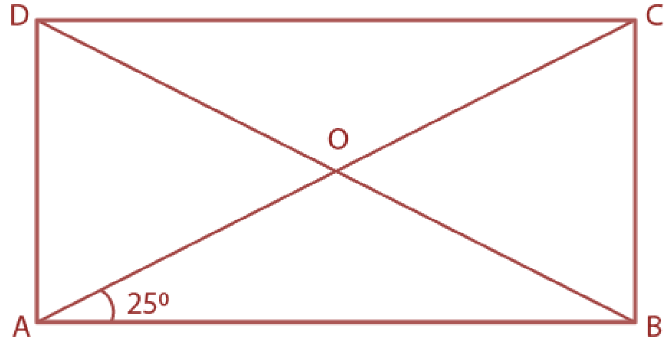
EXERCISE 8.1

Write the correct answer in each of the following:
1. Three angles of a quadrilateral are 75º, 90º and 75º. The fourth angle is
(A) 90º
(B) 95º
(C) 105º
(D) 120º
Solution:

(D) 120º
Explanation:
According to the question,
Three angles of quadrilateral are 75°, 90° and 75°
Consider the fourth angle to be x.
We know that,
Sum of all angles of a quadrilateral = 360°
$\Rightarrow 75° + 90° + 75° + x = 360°$
$\Rightarrow 240° + x = 360°$
$\Rightarrow x = 360° – 240°$
$\Rightarrow x = 120°$
Hence, the fourth angle is 120°.
Therefore, option (D) is the correct answer.

2. A diagonal of a rectangle is inclined to one side of the rectangle at 25º. The acute angle between the diagonals is
(A) 55º
(B) 50º
(C) 40º
(D) 25º
Solution:

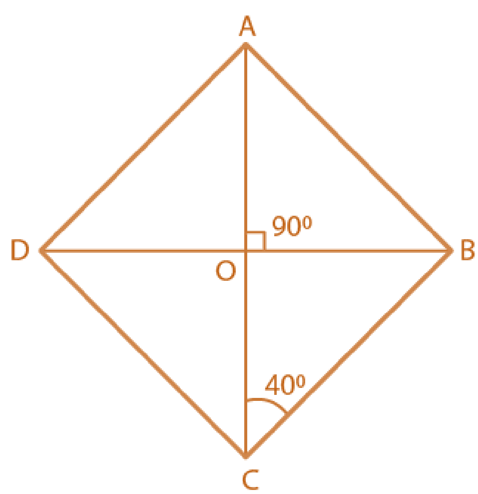
(B) 50º
Explanation:
According to the question,
A diagonal of a rectangle is inclined to one side of the rectangle at 25º
i.e., Angle between a side of rectangle and its diagonal = 25°
Consider the acute angle between diagonals to be = x



We know that diagonals of a rectangle are equal in length i.e.,
AC = BD
Dividing RHS and LHS by 2,
⇒ ½ AC = ½ BD
Since, O is mid-point of AC and BD
⇒ OD = OC
Since, angles opposite to equal sides are equal
⇒ ∠y = 25°
We also know that,
Exterior angle is equal to the sum of two opposite interior angles.
So, ∠BOC = ∠ODC + ∠OCD
⇒ ∠x = ∠y + 25°
⇒ ∠x = 25° + 25°
⇒ ∠x = 50°
Hence, the acute angle between diagonals is 50°.
Therefore, option (B) is the correct answer.

3. ABCD is a rhombus such that ∠ACB = 40º. Then ∠ADB is
(A) 40º
(B) 45º
(C) 50º
(D) 60º
Solution:

(C) 50º
Explanation:
According to the question,
ABCD is a rhombus
∠ACB = 40°



∵ ∠ACB = 40°
⇒ ∠OCB = 40°
∵ AD || BC
⇒ ∠DAC = ∠BCA = 40° [Alternate interior angles]
⇒ ∠DAO = 40°

Since, diagonals of a rhombus are perpendicular to each other
We have,
∠AOD = 90°
We know that,
Sum of all angles of a triangle = 180°
⇒ ∠AOD + ∠ADO + ∠DAO = 180°
⇒ 90° + ∠ADO + 40° = 180°
⇒ 130° + ∠ADO = 180°
⇒ ∠ADO = 180° – 130°
⇒ ∠ADO = 50°
⇒ ∠ADB = 50°
Hence, ∠ADB = 50°
Therefore, option (C) is the correct answer.

4. The quadrilateral formed by joining the mid-points of the sides of a quadrilateral PQRS, taken in order, is a rectangle, if
(A) PQRS is a rectangle
(B) PQRS is a parallelogram
(C) diagonals of PQRS are perpendicular
(D) diagonals of PQRS are equal.
Solution:

(C) diagonals of PQRS are perpendicular
Explanation:
Let the rectangle be ABCD,
We know that,
Diagonals of rectangle are equal
∴ AC = BD
⇒ PQ = QR
∴ PQRS is a rhombus
Diagonals of a rhombus are perpendicular.
Hence, diagonals of PQRS are perpendicular
Therefore, option (C) is the correct answer.

5. The quadrilateral formed by joining the mid-points of the sides of a quadrilateral PQRS, taken in order, is a rhombus, if
(A) PQRS is a rhombus
(B) PQRS is a parallelogram
(C) diagonals of PQRS are perpendicular
(D) diagonals of PQRS are equal.
Solution:

(D) diagonals of PQRS are equal.
Explanation:
Since, ABCD is a rhombus
We have,
AB = BC = CD = DA
Now,

Since, D and C are midpoints of PQ and PS
By midpoint theorem,
We have,
DC = ½ QS
Also,
Since, B and C are midpoints of SR and PS
By midpoint theorem
We have,
BC = ½ PR
Now, again, ABCD is a rhombus
∴ BC = CD
⇒ ½ QS = ½ PR
⇒ QS = PR
Hence, diagonals of PQRS are equal
Therefore, option (D) is the correct answer.

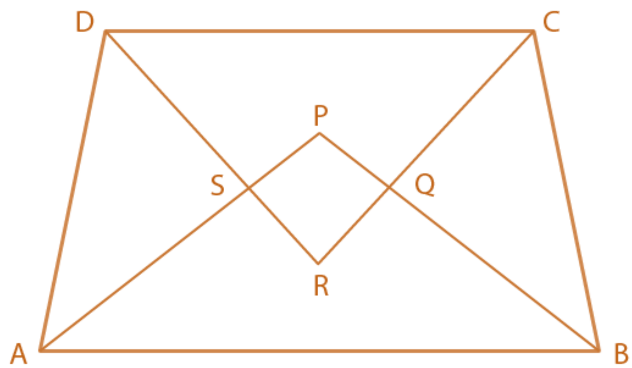
6. If angles A, B, C and D of the quadrilateral ABCD, taken in order, are in the ratio 3:7:6:4, then ABCD is a
(A) rhombus
(B) parallelogram
(C) trapezium
(D) kite
Solution:
(C) trapezium
Explanation:
As angle A, B, C and D of the quadrilateral ABCD, taken in order, are in the ratio 3: 7: 6: 4,
We have the angles A, B, C and D = 3x, 7x, 6x and 4x.
Now, sum of the angle of a quadrilateral = 360°.
$3x + 7x + 6x + 4x = 360°$
$\Rightarrow 20x = 360°$
$\Rightarrow x = 360 \div 20 = 18°$
So, the angles A, B, C and D of quadrilateral ABCD are,
$\angle A = 3\times18° = 54°$,
$\angle B = 7\times18° = 126°$
$\angle C = 6\times18° = 108°$
$\angle D = 4\times18° = 72°$
AD and BC are two lines cut by a transversal CD
Now, sum of angles ∠C and ∠D on the same side of transversal,
$\angle C + \angle D = 108° + 72° = 180$
Hence, AD|| BC
So, ABCD is a quadrilateral in which one pair of opposite sides are parallel.
Hence, ABCD is a trapezium.
Therefore, option (C) is the correct answer.

7. If bisectors of ∠A and ∠B of a quadrilateral ABCD intersect each other at P, of ∠B and ∠C at Q, of ∠C and ∠D at R and of ∠D and ∠A at S, then PQRS is a

(A) rectangle
(B) rhombus
(C) parallelogram
(D) quadrilateral whose opposite angles are supplementary
Solution:

(D) quadrilateral whose opposite angles are supplementary

Explanation:



We know that,
Sum of all angles of a quadrilateral = 360°
⇒ ∠A + ∠B + ∠C + ∠D = 360°
Dividing LHS and RHS by 2,
⇒ ½ (∠A + ∠B + ∠C + ∠D) = ½ × 360° = 180°
Since, AP, PB, RC and RD are bisectors of ∠A, ∠B, ∠C and ∠D
⇒ ∠PAB + ∠ABP + ∠RCD + ∠RDC = 180° … (1)
We also know that,
Sum of all angles of a triangle = 180°
∠PAB + ∠APB + ∠ABP = 180°
⇒ ∠PAB + ∠ABP = 180° – ∠APB …(2)
Similarly,
∴ ∠RDC + ∠RCD + ∠CRD = 180°
⇒ ∠RDC + ∠RCD = 180° – ∠CRD …(3)
Substituting the value of equations (2) and (3) in equation (1),
180° – ∠APB + 180° – ∠CRD = 180°
⇒ 360° – ∠APB – ∠CRD = 180°
⇒ ∠APB + ∠CRD = 360° – 180°
⇒ ∠APB + ∠CRD = 180° …(4)
Now,
∠SPQ = ∠APB [vertically opposite angles]
∠SRQ = ∠DRC [vertically opposite angles]
Substituting in equation (4),
⇒ ∠SPQ + ∠SRQ = 180°
Hence, PQRS is a quadrilateral whose opposite angles are supplementary.
Therefore, option (D) is the correct answer.