EXERCISE 20.1

Question. 1
Solution:

As we know that the formula $\int_a^b (x)^n \, dx = \left[\frac{(x)^{n+1}}{n+1}\right]_a^b$

$\int_4^9 \frac{1}{\sqrt{x}} dx$ this can be written as $\int_4^9 (x)^{-\frac{1}{2}}$

Then, $\int_4^9 (x)^{-\frac{1}{2}} = \left[\frac{(x)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]_4^9$

On simplifying we get,

$$= \left[\frac{(x)^{\frac{1}{2}}}{\frac{1}{2}}\right]_4^9$$

Then, applying limits

$$= \left[\frac{(9)^{\frac{1}{2}}}{\frac{1}{2}}\right] - \left[\frac{(4)^{\frac{1}{2}}}{\frac{1}{2}}\right]$$

$$= \left[\frac{3}{\frac{1}{2}}\right] - \left[\frac{2}{\frac{1}{2}}\right]$$

= (3 × 2) – (2 × 2)
= 6 – 4
= 2

Therefore, $\int_4^9 (x)^{-\frac{1}{2}} = 2$

Question. 2
Solution:

As we know that the formula ∫dx/x = log x + c

Then, $\int_{-2}^{3} \frac{1}{x+7} dx = [\log(x+7)]_{-2}^{3}$

Now applying limits, we get,

= [log (3 + 7) – log (-2 + 7)]

= [log 10 – log 5]

We know that, log a – log b = log (a/b)

= log 10/5

= log 2

Therefore, $\int_{-2}^{3} \frac{1}{x+7} dx = \log 2$

Question. 3

Solution:

Let us assume that x = sin θ,

dx = cos θ dθ

Then, substitute x = 0

θ = 0

Again substitute x = ½

θ = π/6

$\int_{0}^{\frac{\pi}{6}} \frac{1}{\sqrt{1-\sin^2\theta}} \cos\theta \, d\theta$

We know that, 1 – sin θ = cos θ

So, $\int_{0}^{\frac{\pi}{6}} \frac{\cos\theta \, d\theta}{\cos\theta}$

Then, $\int_{0}^{\frac{\pi}{6}} d\theta$

On integrating we get, $[\theta]_{0}^{\frac{\pi}{6}}$

Now applying limits,

= [π/6 - 0]

= π/6

Therefore, $\int_{0}^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} = \frac{\pi}{6}$

Question. 4
Solution:

As we know that the formula $\int_a^b \frac{1}{(1+x^2)} dx = [\tan^{-1} x]_a^b$

Then, $\int_0^1 \frac{1}{1+x^2} dx = \left[\tan^{-1} x\right]_0^1$

Now applying limits,
$= [\tan^{-1} 1 - \tan^{-1} 0]$
$= [\pi/4 - 0]$
$= \pi/4$
Therefore, $\int_0^1 \frac{1}{1+x^2} dx = \frac{\pi}{4}$

Question. 5
Solution:
Let us assume that $x^2 + 1 = t$
Then, 2x dx = dt
x dx = dt/2
Now substitute x = 2
t = 5
Again substitute x = 3
t = 10
Then,
$\int_2^3 \frac{x}{x^2+1} dx = \frac{1}{2} \int_5^{10} \frac{dt}{t}$
As we know that the formula ∫dx/x = log x + c

$= \frac{1}{2} [\log t]_5^{10}$
Now applying limits, we get,
= ½ [log 10 – log 5]
We know that, log a – log b = log (a/b)
= ½ [log 10/5]
= ½ [log 2]
= log √2

Therefore, $\int_2^3 \frac{x}{x^2+1} = \log\sqrt{2}$