NCERT Solutions for Class-XII Maths

Chapter-9.3

NCERT Math Class 12

1. $\frac{x}{a}+\frac{y}{b}=1$

1. $\frac{x}{a}+\frac{y}{b}=1$

Differentiating both sides of the given equation with respect to x, we get:

$$\frac{1}{a}+\frac{1}{b}\frac{dy}{dx}=0$$

$$\Rightarrow \frac{1}{a}+\frac{1}{b}y'=0$$

Again, differentiating both sides with respect to x, we get:

$$0+\frac{1}{b}y''=0$$

$$\Rightarrow \frac{1}{b}y''=0$$

$$\Rightarrow y''=0$$

Hence, the required differential equation of the given curve is $y''=0$.

2. $y^2=a\left(b^2-x^2\right)$

2. The given equation is $y^2 = a(b^2 - x^2)$

Now, differentiating both sides w.r.t x, we get,

$$2y\frac{dy}{dx} = a(-2x)$$

$\Rightarrow$ 2yy’ = -2ax

$\Rightarrow$ yy’ = -ax ----------------(1)

Now, again differentiating both sides, we get,

y’.y’ +yy’’ = -a

$\Rightarrow (y')^2 + yy'' = -a$--------------(2)

Now, dividing equation (2) by (1), we get,

$$\frac{(y')^2+yy''}{yy'} = \frac{-a}{ax}$$

$\Rightarrow xyy'' + x(y')^2 - yy'' = 0$

Therefore, the required differential equation is $xyy'' + x(y')^2 - yy'' = 0$.

3. $y = ae^{3x} + be^{-2x}$

3. $y = ae^{3x} + be^{-2x}$ ...(1)

Differentiating both sides with respect to x, we get:

$y' = 3ae^{3x} - 2be^{-2x}$ ...(2)

Again, differentiating both sides with respect to x, we get:

$y'' = 9ae^{3x} + 4be^{-2x}$ ...(3)

Multiplying equation (1) with (2) and then adding it to equation (2), we get:

$$\left(2ae^{3x} + 2be^{-2x}\right) + \left(3ae^{3x} - 2be^{-2x}\right) = 2y + y'$$

$$\Rightarrow 5ae^{3x} = 2y + y'$$

$$\Rightarrow ae^{3x} = \frac{2y + y'}{5}$$

Now, multiplying equation (1) with equation (3) and subtracting equation (2) from it, we get:

$$\left(3ae^{3x} + 3be^{-2x}\right) - \left(3ae^{3x} - 2be^{-2x}\right) = 3y - y'$$

$$\Rightarrow 5be^{-2x} = 3y - y'$$

$$\Rightarrow be^{-2x} = \frac{3y - y'}{5}$$

Substituting the values of ae^{3x} and be^{-2x} in equation (3), we get:

$$y'' = 9 \cdot \frac{(2y + y')}{5} + 4\frac{(3y - y')}{5}$$

$$\Rightarrow y'' = \frac{18y + 9y'}{5} + \frac{12y - 4y'}{5}$$

$$\Rightarrow y'' = \frac{30y + 5y'}{5}$$

$$\Rightarrow y'' = 6y + y'$$

$$\Rightarrow y'' - y' - 6y = 0$$

4. $y = e^{2x}\left(a + bx\right)$

4. It is given $y = e^{2x}(a + bx)$ -----------------(1)

Now, differentiating both side w.r.t. x, we get,

$y' = 2e^{2x}(a + bx) + e^{2x}.b$---------------(2)

Now, let us multiply equation (1) with 2 and then subtracting it to equation (2), we get,

$y' - 2y = e^{2x}(2a + 2bx + b) - e^{2x}(2a + 2bx)$

$\Rightarrow y' - 2y = be^{2x}$--------------------(3)

Now, again differentiating both sides w.r.t. x, we get,

$y'' - 2y' = 2be^{2x}$--------------------(4)

Dividing equation (4) by equation (3), we get,

$\frac{y''-2y'}{y'-2y} = 2$

$\Rightarrow$ y" – 2y' = 2y' – 4y

$\Rightarrow$ y" – 4y' – 4y = 0

Therefore, the required differential equation is y" – 4y' - 4y = 0.

5. $y = e^x(a\cos x + b\sin x)$

5. $y = e^x(a\cos x + b\sin x)$ ...(1)

Differentiating both sides with respect to x, we get:

$y' = e^x(a\cos x + b\sin x) + e^x(-a\sin x + b\cos x)$

$\Rightarrow y' = e^x[(a+b)\cos x - (a-b)\sin x]$ ...(2)

Again, differentiating with respect to x, we get:

$y'' = e^x[(a+b)\cos x - (a-b)\sin x] + e^x[-(a+b)\sin x - (a-b)\cos x]$

$y'' = e^x[2b\cos x - 2a\sin x]$

$y'' = 2e^x(b\cos x - a\sin x)$

$\Rightarrow \frac{y''}{2} = e^x(b\cos x - a\sin x)$ ...(3)

Adding equations (1) and (3), we get:

$y + \frac{y''}{2} = e^x[(a+b)\cos x - (a-b)\sin x]$

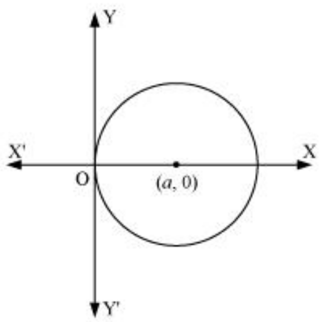
$\Rightarrow y + \frac{y''}{2} = y'$

$\Rightarrow 2y + y'' = 2y'$

$\Rightarrow y'' - 2y' + 2y = 0$

This is the required differential equation of the given curve.

6. Form the differential equation of the family of circles touching the y-axis at the origin.



6.

The center of the circle touching the y- axis at orgin lies on the x – axis.

Let (a,0) be the centre of the circle.

Thus, it touches the y – axis at orgin, its radius is a.

Now, the equation of the circle with centre (a,0) and radius (a) is

$(x - a)^2 - y^2 = a^2$

$\Rightarrow x^2 + y^2 = 2ax$

Now, differentiating both sides w.r.t. x , we get,

$2x + 2yy' = 2a$

$\Rightarrow x + yy' = a$

Now, on substituting the value of a in the equation, we get,

$x^2 + y^2 = 2(x + yy')x$

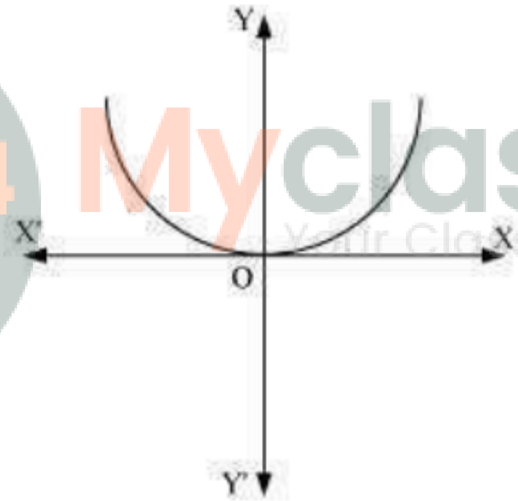
$\Rightarrow 2xyy' + x^2 = y^2$

Therefore, the required differential equation is $2xyy' + x^2 = y^2$.

7. Form the differential equation of the family of parabolas having vertex at origin and axis along positive y-axis.

7. The equation of the parabola having the vertex at origin and the axis along the positive y-axis is:

$x^2 = 4ay$ …(1)



Differentiating equation (1) with respect to x, we get:

$2x = 4ay'$ …(2)

Dividing equation (2) by equation (1), we get:

$$\frac{2x}{x^2} = \frac{4ay'}{4ay}$$

$$\Rightarrow \frac{2}{x} = \frac{y'}{y}$$

$\Rightarrow xy' = 2y$

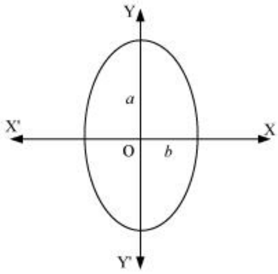
$\Rightarrow xy' - 2y = 0$

This is the required differential equation.

8. Form the differential equation of the family of ellipses having foci on y-axis and centre at origin.

8. We know that the equation of the family of ellipses having foci on y – axis and the centre at origin is

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$--------------------(1)



Now, differentiating equation (1) w.r.t. x, we get,

$\frac{2x}{b^2} + \frac{2yy'}{a^2} = 0$

$\Rightarrow \frac{x}{b^2} + \frac{yy'}{a^2} = 0$--------------(2)

Now, again differentiating w.r.t. x, we get,

$\frac{1}{b^2} + \frac{y'y'+yy"}{a^2} = 0$

$\Rightarrow \frac{1}{b^2} + \frac{1}{a^2}(y'^2 + yy") = 0$

$\Rightarrow \frac{1}{b^2} = -\frac{1}{a^2}(y'^2 + yy")$

Let us substitute the value in eq. (2), we get,

$x\left[-\frac{1}{a^2}(y'^2 + yy")\right] + \frac{yy'}{a^2} = 0$

$\Rightarrow$ $-x(y')^2 - xyy'' + yy' = 0$

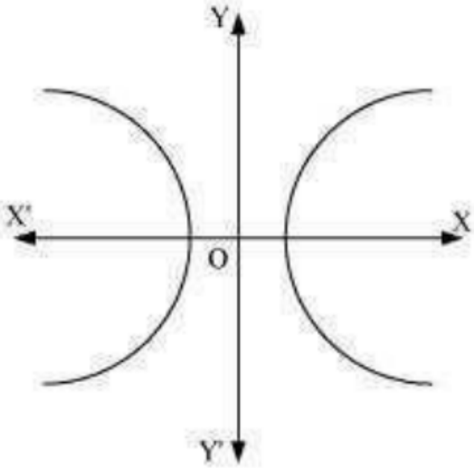
$\Rightarrow xyy'' + x(y')^2 - yy' = 0$

Therefore, the required differential equation is $xyy'' + x(y')^2 - yy' = 0$.

9. Form the differential equation of the family of hyperbolas having foci on x-axis and centre at origin.

9. The equation of the family of hyperbolas with the centre at origin and foci along the xaxis is:

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ …(1)



Differentiating both sides of equation (1) with respect to x, we get:

$$\frac{2x}{a^2} - \frac{2yy'}{b^2} = 0$$

$$\Rightarrow \frac{x}{a^2} - \frac{yy'}{b^2} = 0 \qquad \ldots(2)$$

Again, differentiating both sides with respect to x, we get:

$$\frac{1}{a^2} - \frac{y' \cdot y' + yy''}{b^2} = 0$$

$$\Rightarrow \frac{1}{a^2} = \frac{1}{b^2}\left((y')^2 + yy''\right)$$

Substituting the value of $\frac{1}{a^2}$ in equation (2)

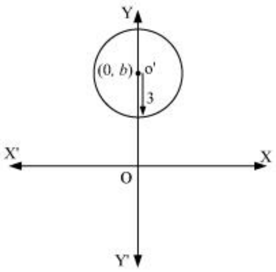
$$\frac{x}{b^2}\left((y')^2 + yy''\right) - \frac{yy'}{b^2} = 0$$

$$\Rightarrow x(y')^2 + xyy'' - yy' = 0$$

$$\Rightarrow xyy'' + x(y')^2 - yy' = 0$$

This is the required differential equation.

10. Form the differential equation of the family of circles having centre on y-axis and radius 3 units.



10.

Let the centre of the circle on y – axis be (0,b).

We know that the differential equation of the family of circles with centre at (0, b) and radius 3 is:

$x^2 + (y- b)^2 = 3^2$

$\Rightarrow x^2 + (y- b)^2 = 9$--------------------(1)

Now, differentiating both sides w.r.t. x, we get,

$2x + 2(y – b).y' = 0$

$\Rightarrow (y – b). y' = x$

$\Rightarrow y – b = \frac{-x}{y'}$

Thus, substituting the value of (y – b) in equation (1), we get,

$x^2 + \left(\frac{-x}{y'}\right)^2 = 9$

$\Rightarrow x^2 \left[1 + \frac{1}{(y')^2}\right] = 9$

$\Rightarrow x^2((y')^2 + 1) = 9(y')^2$

$\Rightarrow (x^2 – 9)(y')^2 + x^2 = 0$

Therefore, the required differential equation is $(x^2 – 9)(y')^2 + x^2 = 0$

11. Which of the following differential equations has $y = c_1e^x + c_2e^{-x}$ as the general solution?

(a) $\frac{d^2y}{dx^2} + y = 0$ (b) $\frac{d^2y}{dx^2} - y = 0$

(c) $\frac{d^2y}{dx^2} + 1 = 0$ (d) $\frac{d^2y}{dx^2} - 1 = 0$

11. The given equation is:

$y = c_1e^x + c_2e^{-x}$ …(1)

Differentiating with respect to x, we get:

$\frac{dy}{dx} = c_1e^x + c_2e^{-x}$

Again, differentiating with respect to x, we get:

$\frac{d^2y}{dx^2} = c_1e^x + c_2e^{-x}$

$\Rightarrow \frac{d^2y}{dx^2} = y$

$\Rightarrow \frac{d^2y}{dx^2} - y = 0$

This is the required differential equation of the given equation of curve. Hence, the correct answer is B.

12. Which of the following differential equation has $y = x$ as one of its particular solution?

(a) $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = x$

(b) $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = x$

(c) $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0$

(d) $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = 0$

12. Explanation: It is given that that y = x

Now, differentiating both sides w.r.t. x, we get,

$\frac{dy}{dx} = 1$

Again, differentiating both sides w.r.t. x, we get,

$\frac{d^2y}{dx^2} = 0$

Now, substitute the value of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in the given options, we will see that the differential equation in option (C) is correct.

$\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0 - x^2.1 + x.x$

$= -x^2 + x^2$

$= 0$