EXERCISE 30.3

Differentiate the following with respect to x:

1. x^4 – 2sin x + 3 cos x

Solution:

Given:

f (x) = x^4 – 2sin x + 3 cos x

Differentiate on both the sides with respect to x, we get

$$\frac{d}{dx}\{f(x)\} = \frac{d}{dx}(x^4 - 2\sin x + 3\cos x)$$

By using algebra of derivatives,

$$f' = \frac{d}{dx}(x^4) - 2\frac{d}{dx}(\sin x) + 3\frac{d}{dx}(\cos x)$$

We know that,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

So,

= $4x^{4-1}$ – 2 cos x + 3 (– sin x)

= $4x^3$ – 2 cos x – 3 sin x

∴ Derivative of f (x) is $4x^3$ – 2 cos x – 3 sin x

2. $3^x + x^3 + 3^3$

Solution:

Given:

f (x) = $3^x + x^3 + 3^3$

Differentiate on both the sides with respect to x, we get

$$\frac{d}{dx}\{f(x)\} = \frac{d}{dx}(3^x + x^3 + 3^3)$$

By using algebra of derivatives,

$$f' = \frac{d}{dx}(3^x) + \frac{d}{dx}(x^3) + \frac{d}{dx}(3^3)$$

We know that,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$\frac{d}{dx}(a^x) = a^x \log a$

$\frac{d}{dx}(\text{constant}) = 0$

$f' = 3^x \log_e 3 + 3x^{3-1} + 0$

$= 3^x \log_e 3 + 3x^2$

∴ Derivative of f (x) is $3^x \log_e 3 + 3x^2$

3. $\frac{x^3}{3} - 2\sqrt{x} + \frac{5}{x^2}$

Solution:

Given:

$f(x) = \frac{x^3}{3} - 2\sqrt{x} + \frac{5}{x^2}$

Differentiate on both the sides with respect to x, we get

$\frac{d}{dx}\{f(x)\} = \frac{d}{dx}\left(\frac{x^3}{3} - 2\sqrt{x} + \frac{5}{x^2}\right)$

By using algebra of derivatives,

$f' = \frac{d}{dx}\left(\frac{x^3}{3}\right) - 2\frac{d}{dx}(\sqrt{x}) + 5\frac{d}{dx}\left(\frac{1}{x^2}\right)$

$= \frac{1}{3}\frac{d}{dx}(x^3) - 2\frac{d}{dx}(x^{\frac{1}{2}}) + 5\frac{d}{dx}(x^{-2})$

We know that,

$\frac{d}{dx}(x^n) = nx^{n-1}$

$f' = \frac{1}{3}(3x^{3-1}) - 2 \times \frac{1}{2} x^{\frac{1}{2}-1} + 5(-2)x^{-2-1}$

$= 3 \times \frac{1}{3} x^2 - x^{-\frac{1}{2}} - 10x^{-3}$

$= x^2 - x^{(-1/2)} - 10x^{-3}$

∴ Derivative of f (x) is $x^2 - x^{(-1/2)} - 10x^{-3}$

4. $e^{x \log a} + e^{a \log x} + e^{a \log a}$

Solution:

Given:

$f(x) = e^{x \log a} + e^{a \log x} + e^{a \log a}$

We know that,

$e^{\log f(x)} = f(x)$

So,

$f(x) = a^x + x^a + a^a$

Differentiate on both the sides with respect to x, we get

$\frac{d}{dx}\{f(x)\} = \frac{d}{dx}(a^x + x^a + a^a)$

By using algebra of derivatives,

$f' = \frac{d}{dx}(a^x) + \frac{d}{dx}(x^a) + \frac{d}{dx}(a^a)$

We know that,

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(a^x) = a^x \log a$

$\frac{d}{dx}(\text{constant}) = 0$

$f' = a^x \log_e a - ax^{a-1} + 0$

$= a^x \log a - ax^{a-1}$

∴ Derivative of f (x) is $a^x \log a - ax^{a-1}$

5. $(2x^2 + 1)(3x + 2)$

Solution:

Given:

$f(x) = (2x^2 + 1)(3x + 2)$

$= 6x^3 + 4x^2 + 3x + 2$

Differentiate on both the sides with respect to x, we get

$\frac{d}{dx}\{f(x)\} = \frac{d}{dx}(6x^3 + 4x^2 + 3x + 2)$

By using algebra of derivatives,

$f' = 6\frac{d}{dx}(x^3) + 4\frac{d}{dx}(x^2) + 3\frac{d}{dx}(x) + \frac{d}{dx}(2)$

We know that,

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(\text{constant}) = 0$

$f' = 6(3x^{3-1}) + 4(2x^{2-1}) + 3(x^{1-1}) + 0$

$= 18x^2 + 8x + 3 + 0$

$= 18x^2 + 8x + 3$

∴ Derivative of f (x) is $18x^2 + 8x + 3$