Therefore ,

$\Rightarrow\int \frac{1+\sin x(1+\sin x)}{1-\sin x(1+\sin x)}dx$

$\Rightarrow\int \frac{(1+\sin x)^2}{1-sin^2 x}dx = \int \frac{1+sin^2 x+2\sin x}{cos^2 x}dx$

$\Rightarrow\int \frac{1}{cos^2 x}dx + 2\int \frac{\sin x}{cos^2 x}dx + \int \frac{sin^2 x}{cos^2 x}dx$

$\Rightarrow\int sec^2 x\,dx + 2\int \frac{\sin x}{cos^2 x}dx + \int tan^2 x\,dx$

$\Rightarrow\int sec^2 x\,dx + 2\int \frac{\sin x}{cos^2 x}dx + \int (-1 + sec^2 x)\,dx$

$\Rightarrow 2\int sec^2 x\,dx + 2\int \frac{\sin x}{cos^2 x}dx - \int 1\,dx$

Put cos x = t

Therefore -> sin x dx = - dt

$\Rightarrow 2\tan x - 2\int \frac{dt}{t^2} - x + c$

$\Rightarrow 2\tan x + 2\frac{1}{t} - x + c$

$\Rightarrow 2\tan x + 2\sec x - x + c$

54. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{x^4}{(1+x^2)}dx = ?$

A. $\frac{x^3}{3} + x + \tan^{-1} x + C$

B. $\frac{-x^3}{3} + x - \tan^{-1} x + C$

C. $\frac{x^3}{3} - x + \tan^{-1} x + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int sec^2 x dx = tan\, x$; $\int \frac{1}{1+x^2}dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow\int \frac{x^4+1-1}{1+x^2}dx$

$\Rightarrow\int \frac{x^4-1}{1+x^2}dx + \int \frac{1}{1+x^2}dx = \int \frac{(1+x^2)(x^2-1)}{1+x^2}dx + \int \frac{1}{1+x^2}dx$

$\Rightarrow\int x^2 - 1\,dx + \int \frac{1}{1+x^2}dx$

$\Rightarrow\frac{x^3}{3} - x + \tan^{-1} x + c$

55. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin(x-\alpha)}{\sin(x+\alpha)} dx = ?$$

A. x cos 2α - sin 2α . log |sin (x + α)| + C

B. x cos 2α + sin 2α . log|sin (x + α)| + C

C. x cos 2α + sin α . log |sin (x + α)| + C

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \frac{\sin(x+\propto-2\propto)}{\sin(x+\propto)} dx$

$\Rightarrow \int \frac{\sin(x+\propto)\cos(-2\propto)+\cos(x+\propto)\sin(-2\propto)}{\sin(x+\propto)} dx$

$\Rightarrow \int \cos(2\propto) dx - \sin 2\propto \int \cot(x+\propto) dx$

$\Rightarrow \cos(2\propto) x - \sin 2\propto \log|\sin(x+\propto)| + c$

56. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{\left(\sqrt{x+3}-\sqrt{x+2}\right)} dx = ?$$

A. $\frac{2}{3}(x+3)^{3/2} - \frac{2}{3}(x+3)^{3/2} + C$

B. $\frac{2}{3}(x+3)^{3/2} + \frac{2}{3}(x+3)^{3/2} + C$

C. $\frac{3}{2}(x+3)^{3/2} - \frac{3}{2}(x+3)^{3/2} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \frac{(\sqrt{x+3}+\sqrt{x+2})}{(\sqrt{x+3}-\sqrt{x+2})(\sqrt{x+3}+\sqrt{x+2})} dx$ (Rationalizing the denominator)

$\Rightarrow \int (\sqrt{x+3} + \sqrt{x+2}) dx$

$\Rightarrow \int \sqrt{x+3}\,dx + \int \sqrt{x+2}\,dx$

$\Rightarrow \frac{2(x+3)^{\frac{3}{2}}}{3} + \frac{2(x+2)^{\frac{3}{2}}}{3} + c$

57. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{(1+\tan x)}{(1-\tan x)} dx = ?$$

A. - log |cos x – sin x| + C

B. log |cos x – sin x| + C

C. log |cos x + sin x | + C

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}} dx$ (Rationalizing the denominator)

$\Rightarrow \int \frac{\cos x + \sin x}{\cos x - \sin x} dx$

Put cos x - sin x = t

(- sin x - cos x) dx = dt

(sin x + cos x) dx = -dt

$\Rightarrow \int \frac{-dt}{t} = -\log t + c$

$\Rightarrow -\log|\cos x - \sin x| + c$

59. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{3x^2}{(1+x^6)} dx = ?$$

A. $\sin^{-1} x^3 + C$

B. $\cos^{-1} x^3 + C$

C. $\tan^{-1} x^3 + C$

D. $\cot^{-1} x^3 + C$

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

Put $x^3 = t\ 3x^2dx = dt$

$\Rightarrow \int \frac{dt}{1+t^2}$

$\Rightarrow \tan^{-1} t + c$

$\Rightarrow \tan^{-1} x^3 + c$

59. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{x\sqrt{x^6 - 1}} = ?$$

A. $\frac{1}{3}\sec^{-1} x^3 + C$

B. $\frac{1}{3}\text{cosec}^{-1} x^3 + C$

C. $\frac{1}{3}\cot^{-1} x^3 + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$

Therefore ,

Put $x^3 = t$, $3x^2dx = dt$

$\Rightarrow \int \frac{dt}{x \times 3x^2\sqrt{t^2-1}} = \int \frac{dt}{3t\sqrt{t^2-1}}$

$\Rightarrow \frac{1}{3}\int \frac{dt}{t\sqrt{t^2-1}}$

$\Rightarrow \frac{1}{3}\sec^{-1} t + c$

$\Rightarrow \frac{1}{3}\sec^{-1} x^3 + c$

60. Question

Mark (√) against the correct answer in each of the following:

$$\int \left\{(2x+1)\sqrt{x^2+x+1}\right\} dx = ?$$

A. $\frac{3}{2}(x^2+x+1)^{3/2} + C$

B. $\frac{2}{3}(x^2+x+1)^{3/2} + C$

C. $\frac{3}{2}(2x+1)^{3/2} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$

Therefore ,

Put $x^2 + x + 1 = t$, $(2x+1)dx = dt$

$\Rightarrow \int \sqrt{t} dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$

$\Rightarrow \frac{2}{3} t^{\frac{3}{2}} + c$

$\Rightarrow \frac{2}{3}(x^2 + x + 1)^{\frac{3}{2}} + c$

61. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left\{\sqrt{2x+3} + \sqrt{2x+3}\right\}} = ?$$

A. $\frac{1}{18}(2x+3)^{3/2} + \frac{1}{18}(2x-3)^{3/2} + C$

B. $\frac{1}{18}(2x+3)^{3/2} - \frac{1}{18}(2x-3)^{3/2} + C$

C. $\frac{1}{12}(2x+3)^{3/2} - \frac{1}{12}(2x-3)^{3/2} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \frac{(\sqrt{2x+3} - \sqrt{2x-3})}{(\sqrt{2x+3} + \sqrt{2x-3})(\sqrt{2x+3} - \sqrt{2x-3})} dx$ (Rationalizing the denominator)

$\Rightarrow \int \frac{\sqrt{2x+3} - \sqrt{2x-3}}{6} dx$

$\Rightarrow \frac{1}{6} \int \sqrt{2x+3}\, dx - \frac{1}{6} \int \sqrt{2x-3}\, dx$

$\Rightarrow \frac{2(2x+3)^{\frac{3}{2}}}{3 \times 6 \times 2} - \frac{2(2x-3)^{\frac{3}{2}}}{3 \times 6 \times 2} + c$

$\Rightarrow \frac{(2x+3)^{\frac{3}{2}}}{18} - \frac{(2x-3)^{\frac{3}{2}}}{18} + c$

62. Question

Mark (√) against the correct answer in each of the following:

$$\int \tan x \, dx = ?$$

A. log |cos x| + C

B. - log |cos x| + C

C. log |sin x| + C

D. - log |sin x| + C

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \frac{\sin x}{\cos x} dx$

Put cos x = t -sin x dx = dt

$\Rightarrow \int \frac{-dt}{t}$

$\Rightarrow -\log t + c$

$\Rightarrow -\log|\cos x| + c$

63. Question

Mark (√) against the correct answer in each of the following:

$\int \sec x \, dx = ?$

A. log |sec x - tan x| + C

B. - log |sec x + tan x| + C

C. log |sec x + tan x| + C

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \sec x \frac{\sec x + \tan x}{\sec x + \tan x} dx$

$\Rightarrow \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx$

Put $\sec x + \tan x = t$, $(\sec^2 x + \sec x \tan x)$dx = dt

$\Rightarrow \int \frac{dt}{t}$

$\Rightarrow \log t + c$

$\Rightarrow \log|\sec x + \tan x| + c$

64. Question

Mark (√) against the correct answer in each of the following:

$\int \operatorname{cosec} x \, dx = ?$

A. log |cosec x - cot x| + C

B. - log |cosec x - cot x| + C

C. log |cosec x + cot x| + C

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \operatorname{cosec} x \frac{\operatorname{cosec} x - \cot x}{\operatorname{cosec} x - \cot x} dx$

$\Rightarrow \int \frac{cosec^2 x - \operatorname{cosec} x \cot x}{\operatorname{cosec} x - \cot x} dx$

Put $\operatorname{cosec} x - \cot x = t$, $(cosec^2 x - \operatorname{cosec} x \cot x)$dx = dt

$\Rightarrow \int \frac{dt}{t}$

$\Rightarrow \log t + c$

$\Rightarrow \log|\operatorname{cosec} x - \cot x| + c$

65. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{(1+\sin x)}{(1+\cos x)} dx = ?$

A. $\tan\frac{x}{2} + 2\log\left|\cos\frac{x}{2}\right| + C$

B. $-\tan\frac{x}{2} + 2\log\left|\cos\frac{x}{2}\right| + C$

C. $\tan\frac{x}{2} - 2\log\left|\cos\frac{x}{2}\right| + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int sec^2 x dx = \tan x$

Therefore ,

$\Rightarrow \int \frac{1+\sin x}{2cos^2\frac{x}{2}} dx$

$$\Rightarrow \int \frac{1}{2cos^2\frac{x}{2}} + \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2cos^2\frac{x}{2}}dx = \frac{1}{2}\int sec^2\frac{x}{2}dx + \int \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}dx$$

$$\Rightarrow \frac{1}{2}\tan\frac{x}{2} \times 2 + \int tan\ \frac{x}{2}dx$$

$$\Rightarrow \tan\frac{x}{2} + 2\left(-\log\cos\frac{x}{2}\right) + c$$

$$\Rightarrow \tan\frac{x}{2} - 2\log|\cos\frac{x}{2}| + c$$

66. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\tan x}{(\sec x + \cos x)}dx = ?$$

A. $\tan^{-1}(\cos x) + C$

B. $-\tan^{-1}(\cos x) + C$

C. $\cot^{-1}(\cos x) + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ; \int sec^2 x dx = tan\ x$

Therefore ,

$$\Rightarrow \int \frac{\sec x \tan x}{sec^2 x + 1}dx$$

Put sec x = t (sec x tan x) dx = dt

$$\Rightarrow \int \frac{dt}{1+t^2} = \tan^{-1} t + c$$

$$\Rightarrow \tan^{-1}\sec x + c$$

$$\Rightarrow -\tan^{-1}(\cos x) + c$$

67. Question

Mark (√) against the correct answer in each of the following:

$$\int \sqrt{\frac{1+x}{1-x}}dx = ?$$

A. $\sin^{-1}x + \sqrt{1-x^2} + C$

B. $\sin^{-1}x + \left(1+x^2\right) + C$

C. $\sin^{-1}x - \sqrt{1-x^2} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ; \int sec^2 x dx = tan\ x$

Therefore ,

$\Rightarrow \int \sqrt{\frac{(1+x)^2}{(1+x)(1-x)}}dx$

$\Rightarrow \int \frac{1+x}{\sqrt{1-x^2}}dx = \int \frac{1}{\sqrt{1-x^2}}dx + \int \frac{x}{\sqrt{1-x^2}}dx$

Put $1 - x^2 = t$ -2x dx = dt

$\Rightarrow \sin^{-1} x - \frac{1}{2}\int \frac{1}{\sqrt{t}}dt + c$

$\Rightarrow \sin^{-1} x - \frac{1}{2}\frac{\sqrt{t}}{\frac{1}{2}} + c$

$\Rightarrow \sin^{-1} x - \sqrt{t} + c \;=\; \sin^{-1} x - \sqrt{1-x^2} + c$

68. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{1}{x^2} e^{-1/x} dx = ?$

A. $e^{-1/x} + C$

B. $-e^{-1/x} + C$

C. $\frac{e^{-1/x}}{x} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int sec^2 x dx = tan\, x$

Therefore ,

Put $-\frac{1}{x} = t \; \frac{1}{x^2}dx = dt$

$\Rightarrow \int e^t dt$

$\Rightarrow e^t + c$

$\Rightarrow e^{-\frac{1}{x}} + c$

69. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{x^3}{(1+x^8)} dx = ?$

A. $\tan^{-1} x^4 + C$

B. $4 \tan^{-1} x^4 + C$

C. $\frac{1}{4}\tan^{-1} x^4 + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

Put $x^4 = t$ $4x^3 dx = dt$

$\Rightarrow \frac{1}{4} \int \frac{1}{1+t^2} dt$

$\Rightarrow \frac{1}{4} \tan^{-1} t + c$

$\Rightarrow \frac{1}{4} \tan^{-1} x^4 + c$

70. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{(x+1)(x+\log x)^2}{x} dx = ?$$

A. $\frac{1}{3}(x+\log x)^3 + C$

B. $\frac{x^2}{2} + x + C$

C. $\frac{x^3}{3} + \frac{x^2}{2} + x + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

Put $x^1 + \log x = t$ $(1 + \frac{1}{x})dx = dt \Rightarrow (\frac{x+1}{x})dx = dt$

$\Rightarrow \int t^2 dt$

$\Rightarrow \frac{t^3}{3} + c$

$\Rightarrow \frac{(x+\log x)^3}{3} + c$

71. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{2x \tan^{-1} x^2}{(1+x^4)} dx = ?$$

A. $(\tan^{-1}x^2)^2 + C$

B. $2 \tan^{-1} x^2 + C$

C. $\frac{1}{2}(\tan^{-1} x^2)^2 + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\ \int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

Put $\tan^{-1} x^2 = t\ (\frac{1}{1+(x^2)^2} \times 2x)dx = dt \Rightarrow (\frac{2x}{1+x^4})dx = dt$

$\Rightarrow \int t^1 dt$

$\Rightarrow \frac{t^2}{2} + c$

$\Rightarrow \frac{(\tan^{-1} x^2)^2}{2} + c$

72. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{dx}{(2-3x)} = ?$

A. $-3 \log |2 - 3x| + C$

B. $-\frac{1}{3}\log|2-3x|+C$

C. $-\log |2 - 3x| + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\ \int \frac{1}{x^1} dx = \log x + c$

Therefore ,

Put $2 - 3x = t\ -3dx = dt$

$\Rightarrow -\frac{1}{3}\int \frac{1}{t} dt$

$\Rightarrow -\frac{1}{3}\log t + c$

$\Rightarrow -\frac{1}{3}\log|2-3x| + c$

73. Question

Mark (√) against the correct answer in each of the following:

$\int x\sqrt{x^2-1}\,dx = ?$

A. $\frac{2}{3}(x^2-1)^{3/2}+C$

B. $\frac{1}{3}(x^2-1)^{3/2}+C$

C. $\frac{1}{\sqrt{x^2-1}}+C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\ \int \frac{1}{x^1} dx = \log x + c$

Therefore ,

Put $x^2 - 1 = t\ 2xdx = dt$

$\Rightarrow \int \sqrt{t}dt$

$\Rightarrow \frac{1}{2}\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c \Rightarrow \frac{t^{\frac{3}{2}}}{3} + c$

$\Rightarrow \frac{(x^2-1)^{\frac{3}{2}}}{3} + c$

74. Question

Mark (√) against the correct answer in each of the following:

$\int e^{(5-3x)} dx = ?$

A. $\frac{3^{(5-3x)}}{3(\log 3)} + C$

B. $\frac{3^{(4-3x)}}{(\log 3)} + C$

C. $-3^{(5-3x)} \log 3 + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\ \int a^x dx = \frac{a^x}{\log a} + c$

Therefore ,

Put $5 - 3x = t\ -3dx = dt$

$\Rightarrow -\frac{1}{3}\int 3^t dt$

$\Rightarrow -\frac{1}{3} \times \frac{3^t}{\log 3} + c \Rightarrow -\frac{1}{3} \times \frac{3^{(5-3x)}}{\log 3} + c$

$\Rightarrow -\frac{3^{(5-3x)}}{3\log 3} + c$

75. Question

Mark (√) against the correct answer in each of the following:

$\int e^{\tan x} \sec^2 x\, dx = ?$

A. $e^{\tan x} + \tan x + C$

B. $e^{\tan x} . \tan x + C$

C. $e^{\tan x} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $\tan x = t$ $sec^2 x dx = dt$

$\Rightarrow \int e^t dt$

$\Rightarrow e^t + c \Rightarrow e^{\tan x} + c$

76. Question

Mark (√) against the correct answer in each of the following:

$\int e^{\cos^2 x} \sin 2x \, dx = ?$

A. $e^{\cos^2 x} + C$

B. $-e^{\cos^2 x} + C$

C. $e^{\sin^2 x} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $cos^2 x = t \Rightarrow 2\cos x\,(-\sin x) dx = dt \Rightarrow -\sin 2x \; dx = dt$

$\Rightarrow -\int e^t dt$

$\Rightarrow -e^t + c \Rightarrow -e^{cos^2 x} + c$

77. Question

Mark (√) against the correct answer in each of the following:

$\int x \sin^3 x^2 \cos x^2 dx = ?$

A. $\frac{1}{4}\sin^4 x^2 + C$

B. $\frac{1}{8}\sin^4 x^2 + C$

C. $\frac{1}{2}\sin^4 x^2 + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $\sin x^2 = t \Rightarrow 2x \cos x^2 \, dx = dt$

$\Rightarrow \frac{1}{2}\int t^3 dt$

$\Rightarrow \frac{1}{2}\frac{t^4}{4} + c \Rightarrow \frac{t^4}{8} + c$

$\Rightarrow \frac{(\sin x^2)^4}{8} + c$

78. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{e^{\sqrt{x}} \cos\left(e^{\sqrt{x}}\right)}{\sqrt{x}} dx = ?$$

A. $\sin\left(e^{\sqrt{x}}\right) + C$

B. $\frac{1}{2}\sin\left(e^{\sqrt{x}}\right) + C$

C. $2\sin\left(e^{\sqrt{x}}\right) + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $\sin e^{\sqrt{x}} = t \Rightarrow (\cos e^{\sqrt{x}}) \times (e^{\sqrt{x}}) \times (\frac{1}{2\sqrt{x}})dx = dt$

$\Rightarrow \int 2dt$

$\Rightarrow 2t + c \Rightarrow 2\sin e^{\sqrt{x}} + c$

79. Question

Mark (√) against the correct answer in each of the following:

$$\int x^2 \sin x^3 dx = ?$$

A. $\cos x^3 + C$

B. $-\cos x^3 + C$

C. $-\frac{1}{3}\cos x^3 + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $x^3 = t \Rightarrow 3x^2 dx = dt$

$\Rightarrow \frac{1}{3}\int \sin t\, dt$

$\Rightarrow -\frac{1}{3}\cos t + c \Rightarrow -\frac{1}{3}\cos x^3 + c$

80. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{(x+1)e^x}{\cos^2(xe^x)}dx = ?$$

A. tan (xe^x) + C

B. - tan (xe^x) + C

C. cot (xe^x) + C

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $xe^x = t \Rightarrow (e^x + xe^x)dx = dt \Rightarrow e^x(1+x)dx = dt$

$\Rightarrow \int \frac{dt}{\cos^2 t} \Rightarrow \int \sec^2 t\, dt = \tan t + c$

$\Rightarrow \tan(xe^x) + c$

81. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{x\sqrt{x^4 - 1}}dx = ?$$

A. $\sec^{-1} x^2 + C$

B. $\frac{1}{2}\sec^{-1} x^2 + C$

C. $\operatorname{cosec}^{-1} x^2 + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{t\sqrt{t^2-1}} dt = \sec^{-1} t + c$

Therefore ,

Put $x^2 = t \Rightarrow 2x dx = dt$

$\Rightarrow \int \frac{1}{x\sqrt{t^2-1}} \times \frac{dt}{2x} \Rightarrow \frac{1}{2}\int \frac{1}{t\sqrt{t^2-1}} dt$

$\Rightarrow \frac{1}{2}\sec^{-1} t + c \Rightarrow \frac{1}{2}\sec^{-1} x^2 + c$

82. Question

Mark (√) against the correct answer in each of the following:

$$\int x\sqrt{x-1}\,dx = ?$$

A. $\frac{2}{3}(x-1)^{3/2}+C$

B. $\frac{2}{5}(x-1)^{5/2}+C$

C. $\frac{2}{5}(x-1)^{5/2}+\frac{3}{2}(x-1)^{3/2}+C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1}+c$; $\int \frac{1}{t\sqrt{t^2-1}}dt = \sec^{-1}t + c$

Therefore ,

Put $x-1=t \Rightarrow x = t+1 \Rightarrow dx = dt$

$\Rightarrow \int (t+1)\times \sqrt{t}dt \Rightarrow \int t^{\frac{3}{2}}dt + \int t^{\frac{1}{2}}dt$

$\Rightarrow \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c \Rightarrow \frac{2t^{\frac{5}{2}}}{5} + \frac{2t^{\frac{3}{2}}}{3} + c$

$\Rightarrow \frac{2(x-1)^{\frac{5}{2}}}{5} + \frac{2(x-1)^{\frac{3}{2}}}{3} + c$

83. Question

Mark (√) against the correct answer in each of the following:

$\int x\sqrt{x^2-x}\,dx = ?$

A. $\frac{1}{3}(x^2-1)^{3/2}+C$

B. $\frac{2}{3}(x^2-1)^{3/2}+C$

C. $\frac{1}{\sqrt{x^2-1}}+C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1}+c$; $\int \frac{1}{t\sqrt{t^2-1}}dt = \sec^{-1}t + c$

Therefore ,

$\Rightarrow \int x\sqrt{x^2-1}dx$

Put $x^2-1=t \Rightarrow 2xdx = dt$

$\Rightarrow \int \sqrt{t}\frac{dt}{2} \Rightarrow \frac{1}{2}\int \frac{t^{\frac{3}{2}}}{\frac{3}{2}}dt$

$\Rightarrow \frac{t^{\frac{3}{2}}}{3} + c \Rightarrow \frac{(x^2-1)^{\frac{3}{2}}}{3} + c$

$\Rightarrow \frac{1}{3}(x^2-1)^{\frac{3}{2}}+c$

84. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(1+\sqrt{x}\right)}=?$$

A. $\sqrt{x}-\log\left|1+\sqrt{x}\right|+C$

B. $\sqrt{x}+\log\left|1+\sqrt{x}\right|+C$

C. $2\sqrt{x}-2\log\left|1+\sqrt{x}\right|+C$

D. none of these

Answer

Formula :- $\int x^n dx=\frac{x^{n+1}}{n+1}+c$; $\int \frac{1}{t\sqrt{t^2-1}}dt=\sec^{-1}t+c$

Therefore ,

$\Rightarrow \int \frac{1}{1+\sqrt{x}}dx$

Put $x=t^2 \Rightarrow dx=2tdt$

$\Rightarrow \int \frac{2t}{1+t}dt \Rightarrow 2\int \frac{t}{1+t}dt \Rightarrow 2\int \frac{t+1-1}{1+t}dt \Rightarrow 2\int dt-2\int \frac{1}{1+t}dt$

$\Rightarrow 2t-2\log(1+t)+c \Rightarrow 2\sqrt{x}-2\log(1+\sqrt{x})+c$

85. Question

Mark (√) against the correct answer in each of the following:

$$\int \sqrt{e^x-1}\,dx$$

A. $\frac{3}{2}\left(e^x-1\right)^{3/2}+C$

B. $\frac{1}{2}\left(e^x-1\right)^{1/2}+C$

C. $\frac{2}{3}\left(e^x-1\right)^{3/2}+C$

D. none of these

Answer

Formula :- $\int x^n dx=\frac{x^{n+1}}{n+1}+c$

Therefore ,

$\Rightarrow \int \sqrt{e^x-1}dx$

Put $e^x-1=t \Rightarrow e^x dx=dt$

$\Rightarrow \int \sqrt{t}\frac{dt}{1+t} \Rightarrow \int \frac{\sqrt{t}}{1+t}dt$

Put $t = z^2$ dt = 2z dz

$\Rightarrow \int \frac{2z^2}{1+z^2}dz \Rightarrow \int \frac{2+2z^2-2}{1+z^2}dz \Rightarrow 2\int \frac{1+z^2}{1+z^2}dz - 2\int \frac{1}{1+z^2}dz$

$\Rightarrow 2\int dz - 2\int \frac{1}{1+z^2}dz \Rightarrow 2z - 2\tan^{-1} z + c$

$\Rightarrow 2\sqrt{t} - 2\tan^{-1}\sqrt{t} + c \Rightarrow 2\sqrt{e^x - 1} - 2\tan^{-1}\sqrt{e^x-1} + c$

86. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin x}{(\sin x - \cos x)}dx = ?$$

A. $\frac{1}{2}x - \frac{1}{2}\log|\sin x - \cos x| + C$

B. $\frac{1}{2}x + \frac{1}{2}\log|\sin x - \cos x| + C$

C. log |sin x – cos x| + C

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\,;\, \int e^x dx = e^x + c$

Therefore ,

We can write $\sin x = \frac{1}{2}[(\sin x - \cos x) + (\sin x + \cos x)]$

$\Rightarrow \int \frac{\frac{1}{2}[(\sin x - \cos x) + (\sin x + \cos x)]}{(\sin x - \cos x)}dx$

$\Rightarrow \frac{1}{2}\int \frac{(\sin x - \cos x)}{(\sin x - \cos x)}dx + \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)}dx$

$\Rightarrow \frac{1}{2}\int dx + \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)}dx \Rightarrow \frac{x}{2} + \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)}dx$

Put $(\sin x - \cos x) = t$ $(\sin x + \cos x)\, dx = dt$

$\Rightarrow \frac{x}{2} + \frac{1}{2}\int \frac{1}{t}dt \Rightarrow \frac{x}{2} + \frac{1}{2}\log t + c \Rightarrow \frac{1}{2}x + \frac{1}{2}\log|\sin x - \cos x| + c$

87. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{(1 - \tan x)} = ?$$

A. $\frac{1}{2}\log|\sin x - \cos x| + C$

B. $\frac{1}{2}x + \frac{1}{2}\log|\sin x - \cos x| + C$

C. $\dfrac{1}{2}x - \dfrac{1}{2}\log|\sin x - \cos x| + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

$\Rightarrow \int \frac{1}{1-\frac{\sin x}{\cos x}} dx \Rightarrow \int \frac{\cos x}{\cos x - \sin x} dx$

We can write $\cos x = \frac{1}{2}[(\cos x - \sin x) + (\sin x + \cos x)]$

$\Rightarrow \int \frac{\frac{1}{2}[(\cos x - \sin x) + (\sin x + \cos x)]}{(\cos x - \sin x)} dx$

$\Rightarrow \frac{1}{2}\int \frac{(\cos x - \sin x)}{\cos x - \sin x} dx + \frac{1}{2}\int \frac{(\sin x + \cos x)}{\cos x - \sin x} dx$

$\Rightarrow \frac{1}{2}\int dx + \frac{1}{2}\int \frac{(\sin x + \cos x)}{\cos x - \sin x} dx \Rightarrow \frac{x}{2} + \frac{1}{2}\int \frac{(\sin x + \cos x)}{\cos x - \sin x} dx$

Put $(\cos x - \sin x) = t$ $(\sin x + \cos x)\ dx = -dt$

$\Rightarrow \frac{x}{2} - \frac{1}{2}\int \frac{1}{t} dt \Rightarrow \frac{x}{2} - \frac{1}{2}\log t + c \Rightarrow \frac{1}{2}x - \frac{1}{2}\log|\cos x - \sin x| + c$

88. Question

Mark (√) against the correct answer in each of the following:

$\int \dfrac{dx}{(1-\cot x)} = ?$

A. log |sin x - cos x| + C

B. $\dfrac{1}{2}\log|\sin x - \cos x| + C$

C. $\dfrac{1}{2}x - \dfrac{1}{2}\log|\sin x - \cos x| + C$

D. $\dfrac{1}{2}x + \dfrac{1}{2}\log|\sin x - \cos x| + C$

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

$\Rightarrow \int \frac{1}{1-\frac{\cos x}{\sin x}} dx \Rightarrow \int \frac{\sin x}{\sin x - \cos x} dx$

We can write $\sin x = \frac{1}{2}[(\sin x - \cos x) + (\sin x + \cos x)]$

$\Rightarrow \int \frac{\frac{1}{2}[(\sin x - \cos x) + (\sin x + \cos x)]}{(\sin x - \cos x)} dx$

$\Rightarrow \frac{1}{2}\int \frac{(\sin x - \cos x)}{(\sin x - \cos x)} dx + \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)} dx$

$\Rightarrow \frac{1}{2}\int dx + \frac{1}{2}\int \frac{(\sin x+\cos x)}{(\sin x-\cos x)}dx \Rightarrow \frac{x}{2} + \frac{1}{2}\int \frac{(\sin x+\cos x)}{(\sin x-\cos x)}dx$

Put $(\sin x - \cos x) = t \; (\sin x + \cos x)\; dx = dt$

$\Rightarrow \frac{x}{2} + \frac{1}{2}\int \frac{1}{t}dt \Rightarrow \frac{x}{2} + \frac{1}{2}\log t + c \Rightarrow \frac{1}{2}x + \frac{1}{2}\log|\sin x - \cos x| + c$

89. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}dx = ?$$

A. $\sin^{-1}(\tan x) + C$

B. $\cos^{-1}(\sin x) + C$

C. tan-1 (cos x) + C

D. $\tan^{-1}(\sin x) + C$

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c \; ; \int \frac{1}{1+x^2}dx = \tan^{-1}x + c$

Therefore ,

Put $\tan x = t \Rightarrow sec^2 x\; dx = dt$

$\Rightarrow \int \frac{1}{\sqrt{1-t^2}}dt \Rightarrow \sin^{-1}t + c$

$\Rightarrow \sin^{-1}(\tan x) + c$

90. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{(x^2+1)}{(x^4+1)}dx = ?$$

A. $\frac{1}{\sqrt{2}}\tan^{-1}\left(x - \frac{1}{x}\right) + C$

B. $\frac{1}{\sqrt{2}}\cot^{-1}\left\{\left(x - \frac{1}{x}\right)\right\} + C$

C. $\frac{1}{\sqrt{2}}\tan^{-1}\left\{\frac{1}{\sqrt{2}}\left(x - \frac{1}{x}\right)\right\} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c \; ; \int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

Therefore ,

$\Rightarrow \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx \Rightarrow \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}-2+2}dx \Rightarrow \int \frac{1+\frac{1}{x^2}}{(x-\frac{1}{x})^2+2}dx$

Put $x - \frac{1}{x} = t \Rightarrow (1 + \frac{1}{x^2})\, dx = dt$

$\Rightarrow \int \frac{1}{t^2+2} dt \Rightarrow \frac{1}{\sqrt{2}} \tan^{-1} \frac{t}{\sqrt{2}} + c$

$\Rightarrow \frac{1}{\sqrt{2}} \tan^{-1}[\frac{1}{\sqrt{2}}(x - \frac{1}{x})] + c$

91. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{\sin^6 x}{\cos^8} dx = ?$

A. $\frac{1}{7}\tan^7 x + C$

B. $\frac{1}{7}\sec^7 x + C$

C. $\log|\cos^6 x| + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int \frac{\sin^6 x}{\cos^6 x \cos^2 x} dx \Rightarrow \int \frac{\tan^6 x}{\cos^2 x} dx \Rightarrow \int \tan^6 x \sec^2 x dx$

Put $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$\Rightarrow \int t^6 dt \Rightarrow \frac{t^7}{7} + c$

$\Rightarrow \frac{(\tan x)^7}{7} + c$

92. Question

Mark (√) against the correct answer in each of the following:

$\int \sec^5 x \tan x\, dx = ?$

A. $\frac{1}{5}\tan^5 x + C$

B. $\frac{1}{5}\sec^5 x + C$

C. 5 log |cos x| + C

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int \sec^4 x \sec x \tan x\, dx$

Put $\sec x = t \Rightarrow \sec x \tan x\, dx = dt$

$\Rightarrow \int t^4 dt \Rightarrow \frac{t^5}{5} + c$

$\Rightarrow \frac{(\sec x)^5}{5} + c$

93. Question

Mark (√) against the correct answer in each of the following:

$\int \tan^5 x\, dx = ?$

A. $\frac{1}{6}\tan^6 x + C$

B. $\frac{1}{4}\tan^4 x + \frac{1}{2}\tan^2 x + \log|\sec x| + C$

C. $\frac{1}{4}\tan^4 x - \frac{1}{2}\tan^2 x + \log|\sec x| + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\,;\ \int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int tan^3 x\, tan^2 x dx \Rightarrow \int tan^3 x(sec^2 x - 1)dx$

$\Rightarrow \int tan^3 x sec^2 x dx - \int tan^3 x dx \Rightarrow \int tan^3 x sec^2 x dx - \int tan^1 x\, tan^2 x dx$

$\Rightarrow \int tan^3 x sec^2 x dx - \int \tan x\,(sec^2 x - 1)dx$

$\Rightarrow \int tan^3 x sec^2 x dx - \int \tan x\, sec^2 x dx + \int \tan x\, dx$

Put $\tan x = t \Rightarrow sec^2 x dx = dt$

$\Rightarrow \int t^3 dt - \int t^1 dt + \log|\sec x| \Rightarrow \frac{t^4}{4} - \frac{t^2}{2} + \log|\sec x| + c$

$\Rightarrow \frac{(\tan x)^4}{4} - \frac{(\tan x)^2}{2} + \log|\sec x| + c$

94. Question

Mark (√) against the correct answer in each of the following:

$\int \sin^3 x \cos^3 x\, dx = ?$

A. $-\frac{1}{4}\cos^4 x + \frac{1}{6}\cos^6 x + C$

B. $\frac{1}{4}\cos^4 x - \frac{1}{6}\cos^6 x + C$

C. $\frac{1}{4}\cos^4 x + \frac{1}{6}\cos^6 x + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int \cos x\,(cos^2 x\, sin^3 x)dx \Rightarrow \int \cos x\,((1 - sin^2 x)\, sin^3 x)dx$

$\Rightarrow \int \cos x\,(sin^3 x - sin^5 x)dx \Rightarrow \int sin^3 x cos\, x dx - \int sin^5 x \cos x\, dx$

Put $\sin x = t \Rightarrow \cos x\, dx = dt$

$\Rightarrow \int t^3 dt - \int t^5 dt \Rightarrow \frac{t^4}{4} - \frac{t^6}{6} + c$

$\Rightarrow \frac{(\sin x)^4}{4} - \frac{(\sin x)^6}{6} + c$

95. Question

Mark (√) against the correct answer in each of the following:

$\int \sec^4 x \tan x\, dx = ?$

A. $\frac{1}{2}\sec^2 x + \frac{1}{4}\sec^4 x + C$

B. $\frac{1}{2}\tan^2 x + \frac{1}{4}\tan^4 x + C$

C. $\frac{1}{2}\sec x + \log|\sec x + \tan x| + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int sec^2 x sec^2 x \tan x\, dx \Rightarrow \int (1 + tan^2 x) sec^2 x \tan x\, dx$

$\Rightarrow \int sec^2 x \tan x\, dx + \int tan^3 x sec^2 x dx$

Put $\tan x = t \Rightarrow sec^2 x dx = dt$

$\Rightarrow \int t^1 dt + \int t^3 dt \Rightarrow \frac{t^2}{2} + \frac{t^4}{4} + c$

$\Rightarrow \frac{(\tan x)^2}{2} + \frac{(\tan x)^4}{4} + c$

96. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{\log \tan x}{\sin x \cos x} dx = ?$

A. log {log (tan x)| + C

B. $\frac{1}{2}(\log \tan x)^2 + C$

C. log (sin x cos x) + C

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int sec^2 x sec^2 x \tan x \, dx \Rightarrow \int (1 + tan^2 x) sec^2 x \tan x \, dx$

$\Rightarrow \int sec^2 x \tan x \, dx + \int tan^3 x sec^2 x dx$

Put $\log(\tan x) = t \Rightarrow \frac{1}{\tan x} sec^2 x dx = dt \Rightarrow \frac{1}{\sin x \cos x} dx = dt$

$\Rightarrow \int t^1 dt \Rightarrow \frac{t^2}{2} + c$

$\Rightarrow \frac{(\log|\tan x|)^2}{2} + c$

97. Question

Mark (√) against the correct answer in each of the following:

$\int \sin^3(2x+1)dx = ?$

A. $\frac{1}{8}\sin^4(2x+1) + C$

B. $\frac{1}{2}\cos(2x+1) + \frac{1}{3}\cos^3(2x+1) + C$

C. $-\frac{1}{2}\cos(2x+1) + \frac{1}{6}\cos^3(2x+1) + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int sin^2(2x+1)\sin(2x+1) \, dx \Rightarrow \int (1 - cos^2(2x+1))\sin(2x+1) \, dx$

$\Rightarrow \int \sin(2x+1) \, dx - \int cos^2(2x+1)\sin(2x+1) \, dx$

Put $\cos(2x+1) = t \Rightarrow -2\sin(2x+1) \, dx = dt$

$\Rightarrow -\int \frac{dt}{2} - (-\frac{1}{2}) \int t^2 dt \Rightarrow -\frac{1}{2}\int dt + \frac{1}{2}\int t^2 dt$

$\Rightarrow -\frac{1}{2}t + \frac{1}{2}\frac{t^3}{3} + c \Rightarrow -\frac{1}{2}t + \frac{t^3}{6} + c$

$\Rightarrow -\frac{1}{2}\cos(2x+1) + \frac{[\cos(2x+1)]^3}{6} + c$

98. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{\sqrt{\tan x}}{\sin x + \cos x} dx = ?$

A. $2\sqrt{\tan x}+C$

B. $2\sqrt{\cot x}+C$

C. $2\sqrt{\sec x}+C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int \frac{\sqrt{\tan x}}{\sin x \times \cos x} dx \Rightarrow \int \frac{\sqrt{\tan x}}{\frac{\tan x}{\sec x} \times \frac{1}{\sec x}} dx \Rightarrow \int \frac{\sec^2 x}{\sqrt{\tan x}} dx$

Put $\tan x = t \Rightarrow sec^2 x dx = dt$

$\Rightarrow \int \frac{dt}{\sqrt{t}} \Rightarrow \frac{\sqrt{t}}{\frac{1}{2}} + c \Rightarrow 2\sqrt{t} + c$

$\Rightarrow 2\sqrt{\tan x} + c$

99. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{(\cos + \sin x)}{(1-\sin 2x)} dx = ?$

A. log |sin x – cos x| + C

B. $\frac{1}{(\cos x - \sin x)} + C$

C. log |cos x + sin x| + C

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int \frac{\cos x + \sin x}{cos^2 x + sin^2 x - \sin 2x} dx \Rightarrow \int \frac{\cos x + \sin x}{(\cos x - \sin x)^2} dx$

Put $\cos x - \sin x = t \Rightarrow (\cos x + \sin x) dx = -dt$

$\Rightarrow \int \frac{-dt}{t^2} \Rightarrow \frac{1}{t} + c \Rightarrow \frac{1}{\cos x - \sin x} + c$

100. Question

Mark (√) against the correct answer in each of the following:

$\int \sqrt{e^x - 1} dx = ?$

A. $\frac{2}{3}(e^x - 1)^{3/2} + C$

B. $\frac{1}{2}\cdot\frac{e^x}{\sqrt{e^x-1}}+C$ e

C. $2\sqrt{e^x-1}-2\tan^{-1}\sqrt{e^x-1}+C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1}+c$

Therefore ,

$\Rightarrow \int \sqrt{e^x-1}dx$

Put $e^x-1=t \Rightarrow e^x dx = dt$

$\Rightarrow \int \sqrt{t}\frac{dt}{1+t} \Rightarrow \int \frac{\sqrt{t}}{1+t}dt$

Put $t=z^2$ dt = 2z dz

$\Rightarrow \int \frac{2z^2}{1+z^2}dz \Rightarrow \int \frac{2+2z^2-2}{1+z^2}dz \Rightarrow 2\int \frac{1+z^2}{1+z^2}dz - 2\int \frac{1}{1+z^2}dz$

$\Rightarrow 2\int dz - 2\int \frac{1}{1+z^2}dz \Rightarrow 2z-2\tan^{-1}z+c$

$\Rightarrow 2\sqrt{t}-2\tan^{-1}\sqrt{t}+c \Rightarrow 2\sqrt{e^x-1}-2\tan^{-1}\sqrt{e^x-1}+c$

101. Question

Mark (√) against the correct answer in each of the following:

$\int\frac{dx}{\sqrt{\sin^3 x\cos x}}=?$

A. $2\sqrt{\tan x}+C$

B. $2\sqrt{\cot x}+C$

C. $-2\sqrt{\tan x}+C$

D. $\frac{-2}{\sqrt{\tan x}}+C$

Answer

Let $I=\int\frac{dx}{\sqrt{\sin^3 x\cos x}}$

Now multiplying and dividing by $\cos^2 x$, we get,

$I=\int\frac{dx}{\sqrt{\sin^3 x\times\cos x}}\times\frac{1}{\cos^2 x}\times\cos^2 x$

$I=\int\frac{(\sec^2 x)}{\sqrt{\frac{\sin^3 x}{\cos^3 x}}}dx$

$I=\int\frac{\sec^2 x}{\sqrt{\tan^3 x}}dx$

Let tan x = t

Differentiating both sides, we get,

$\sec^2 x\ dx = dt$

Therefore,

$$I = \int \frac{dt}{t^{3/2}}$$

Integrating, we get,

$$I = \frac{t^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + C$$

$$I = \frac{t^{-\frac{1}{2}}}{-\frac{1}{2}} + C$$

$$I = -\frac{2}{\sqrt{t}} + C$$

$$I = -\frac{2}{\sqrt{\tan x}} + C$$

Exercise 13B

1. Question

Evaluate the following integrals:

(i) $\int \sin^2 x\ dx$

(ii) $\int \cos^2 x\, dx$

Answer

i) $\int sin^2 x dx$

$\Rightarrow \int sin^2 x dx$

Now, we know that 1-cos2x=2sin^{2}x

So, applying this identity in the given integral, we get,

$$\int sin^2 x dx = \int \frac{(1 - cos2x)dx}{2}$$

$\Rightarrow \frac{1}{2}(\int dx - \int cos2xdx)$

$$\Rightarrow \frac{x}{2} - \frac{sin2x}{2 \times 2} + c$$

$$\Rightarrow \frac{x}{2} - \frac{sin2x}{4} + c$$

Ans: $\int sin^2 x dx = \frac{x}{2} - \frac{sin2x}{4} + c$

ii) $\int cos^2 x\, dx$

$\Rightarrow \int cos^2 x\, dx$

Now, we know that 1+cos2x=2cos^{2}x

So, applying this identity in the given integral, we get,

$$\int cos^2x\,dx = \int \frac{(1+cos2x)dx}{2}$$

$\Rightarrow \frac{1}{2}(\int dx + \int cos2xdx)$

$$\Rightarrow \frac{x}{2} + \frac{sin2x}{2\times 2} + c$$

$$\Rightarrow \frac{x}{2} + \frac{sin2x}{4} + c$$

Ans: $\int cos^2x\,dx = \frac{x}{2} + \frac{sin2x}{4} + c$

2. Question

Evaluate the following integrals:

(i) $\int \cos^2(x/2)dx$

(ii) $\int \cot^2(x/2)dx$

Answer

(i) $\int \cos^2(x/2)dx$

$\Rightarrow \int cos^2\left(\frac{x}{2}\right)dx$

Now, we know that 1+cosx=2cos^2 (x/2)

So, applying this identity in the given integral, we get,

$$\int cos^2(\frac{x}{2})\,dx = \int \frac{(1+cosx)dx}{2}$$

$\Rightarrow \frac{1}{2}(\int dx + \int cosxdx)$

$$\Rightarrow \frac{x}{2} + \frac{sin2x}{2} + c$$

$$\Rightarrow \frac{x}{2} + \frac{sin2x}{2} + c$$

Ans: $\frac{x}{2} + \frac{sin2x}{2} + c$

ii) $\int \cot^2(x/2)dx$

$\Rightarrow \int cot^2\left(\frac{x}{2}\right)dx$

Now, we know that cosec^{2}x-cot^2 x=1

So, applying this identity in the given integral we get,

$\Rightarrow \int cot^2\left(\frac{x}{2}\right)dx = \int (cosec^2\left(\frac{x}{2}\right) - 1)\,dx$

$\Rightarrow \int (cosec^2\left(\frac{x}{2}\right) - 1)\,dx = \int cosec^2\left(\frac{x}{2}\right)dx - \int 1dx$

$\Rightarrow \int cosec^2\left(\frac{x}{2}\right)dx - \int 1dx = \frac{-cotx}{\frac{1}{2}} - x + c$

⇒-2cotx-x+c

$\Rightarrow \int cot^2\left(\frac{x}{2}\right)dx$ =-2cotx-x+c

Ans: -2cotx-x+c

3. Question

Evaluate the following integrals:

(i) $\int \sin^2 nx\, dx$

(ii) $\int \sin^5 x\, dx$

Answer

i) $\int sin^2 nxdx$

$\Rightarrow \int sin^2 nxdx$

Now, we know that 1-cos2nx=2sin^{2}nx

So, applying this identity in the given integral, we get,

$\int sin^2 nxdx = \int \frac{(1 - cos2nx)dx}{2}$

$\Rightarrow \frac{1}{2}(\int dx - \int cos2nxdx)$

$\Rightarrow \frac{x}{2} - \frac{sin2nx}{2n \times 2} + c$

$\Rightarrow \frac{x}{2} - \frac{sin2x}{4n} + c$

Ans: $\int sin^2 nxdx = \frac{x}{2} - \frac{sin2nx}{4n} + c$

(ii) $\int \sin^5 x\, dx$

We know that 1-cos^{2}x=sin^{2}x

$\Rightarrow \int sin^5 xdx = \int (1 - cos^2 x)^2 sinxdx$

⇒Put cosx=t

⇒-sinxdx=dt

$\Rightarrow \int (1 - cos^2 x)^2 sinxdx = -\int (1 - t^2)^2 dt$

$\Rightarrow -\int (1 - t^2)^2 dt = -\int (1 + t^4 - 2t^2)dt$

$\Rightarrow -\int dt + \int 2t^2 dt - \int t^4 dt$

$\Rightarrow -t + \frac{2t^3}{3} - \frac{t^5}{5} + c$

Resubstituting the value of t=cosx we get,

$\Rightarrow -cosx + \frac{2cos^3 x}{3} - \frac{cos^5 x}{5} + c$

Ans: $-cosx + \frac{2cos^3 x}{3} - \frac{cos^5 x}{5} + c$

4. Question

Evaluate the following integrals:

$\int \cos^3(3x+5)dx$

Answer

Substitute 3x+5=u

⇒3dx=du

⇒dx=du/3

$\Rightarrow \int cos^3(3x+5)dx = \frac{1}{3}\int cos^3(u)du$

Now We know that 1-$\cos^2$x=$\sin^2$x ,

$\Rightarrow \frac{1}{3}\int cos^3(u)du = \frac{1}{3}\int(1-sin^2(u))cosu\,du$

⇒Substitute sinu=t

⇒cosu du=dt

$\Rightarrow \frac{1}{3}\int(1-sin^2(u))cosu\,du = \frac{1}{3}\int(1-t^2)\,dt$

$\Rightarrow \frac{1}{3}\int dt - \frac{1}{3}\int t^2 dt$

$\Rightarrow \frac{t}{3} - \frac{t^3}{3\times 3} + c$

$\Rightarrow \frac{t}{3} - \frac{t^3}{9} + c$

Resubstituting the value of t=sinu and u=3x+5 we get,

$\Rightarrow \frac{\sin(3x+5)}{3} - \frac{sin^3(3x+5)}{9} + c$

Ans: $\frac{\sin(3x+5)}{3} - \frac{sin^3(3x+5)}{9} + c$

5. Question

Evaluate the following integrals:

$\int \sin^7(3-2x)dx$

Answer

$\Rightarrow -\int sin^7(2x-3)dx$

Substitute 2x-3=u

⇒ 2dx=du

⇒dx=du/2

$\Rightarrow -\left(\frac{1}{2}\right)\int sin^7(u)du$

⇒ We know that 1-$\cos^2$x=$\sin^2$x

$\Rightarrow -\left(\frac{1}{2}\right)\int\left(1-cos^2(u)\right)^3 sinu\,du$

⇒Put cosu=t

⇒-sinxdu=dt

$\Rightarrow \left(\frac{1}{2}\right)\int(1-t^2)^3 dt$

$\Rightarrow \left(\frac{1}{2}\right)\int(1-t^6-3t^2+3t^4)dt$