EXERCISE 6.3

In Fig. 6.9, OD is the bisector of ∠AOC, OE is the bisector of ∠BOC and OD ⊥ OE. Show that points A, O and B are collinear.

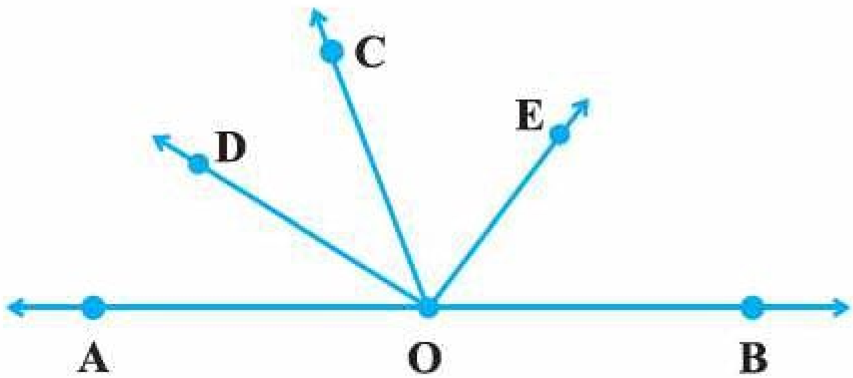


Fig. 6.9

Solution:

According to the question,

In figure,

OD ⊥ OE,

OD and OE are the bisectors of ∠AOC and ∠BOC.

To prove: Points A, O and B are collinear

i.e., AOB is a straight line.

Proof:

Since OD and OE bisect angles ∠AOC and ∠BOC, respectively.

∠AOC = 2∠DOC …(eq.1)

And ∠COB = 2∠COE …(eq.2)

Adding (eq.1) and (eq.2), we get

∠AOC = ∠COB = 2∠DOC + 2∠COE

∠AOC +∠COB = 2(∠DOC +∠COE)

∠AOC + ∠COB = 2∠DOE

Since, OD⊥OE

We get,

∠AOC +∠COB = $2\times 90^\circ$

∠AOC +∠COB =180°

$\angle AOB = 180^{\circ}$

So, $\angle AOC + \angle COB$ form linear pair.

Therefore, AOB is a straight line.

Hence, points A, O and B are collinear.

1. In Fig. 6.10, $\angle 1 = 60°$ and $\angle 6 = 120°$. Show that the lines m and n are parallel.

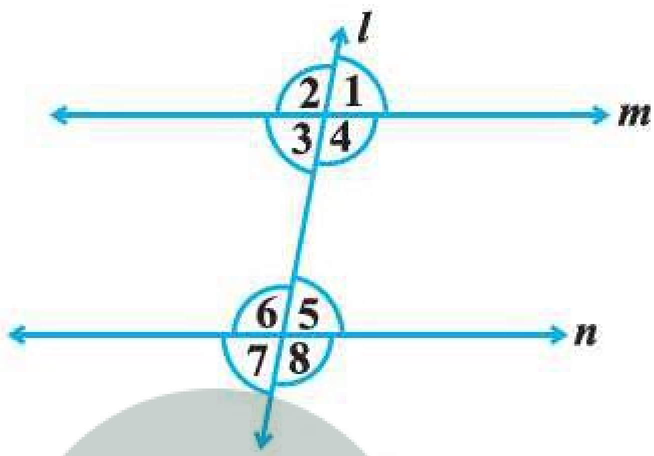


Fig. 6.10

Solution:

According to the question,

We have from the figure $\angle 1 = 60°$ and $\angle 6 = 120°$

Since, $\angle 1 = 60°$ and $\angle 6 = 120°$

Here, $\angle 1 = \angle 3$ [since they are vertically opposite angles]

$\angle 3 = \angle 1 = 60°$Now, $\angle 3 + \angle 6 = 60° + 120°$

$\Rightarrow \angle 3 + \angle 6 = 180°$

We know that,

If the sum of two interior angles on the same side of *l* is 180°, then the lines are parallel.

Therefore, m || n

2. AP and BQ are the bisectors of the two alternate interior angles formed by the intersection of a transversal t with parallel lines l and m (Fig. 6.11). Show that AP || BQ.

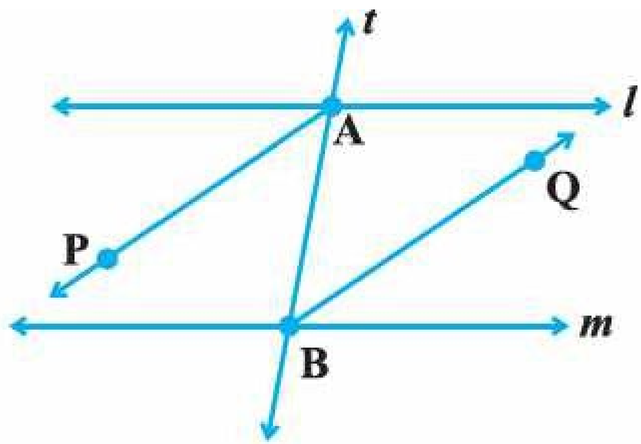
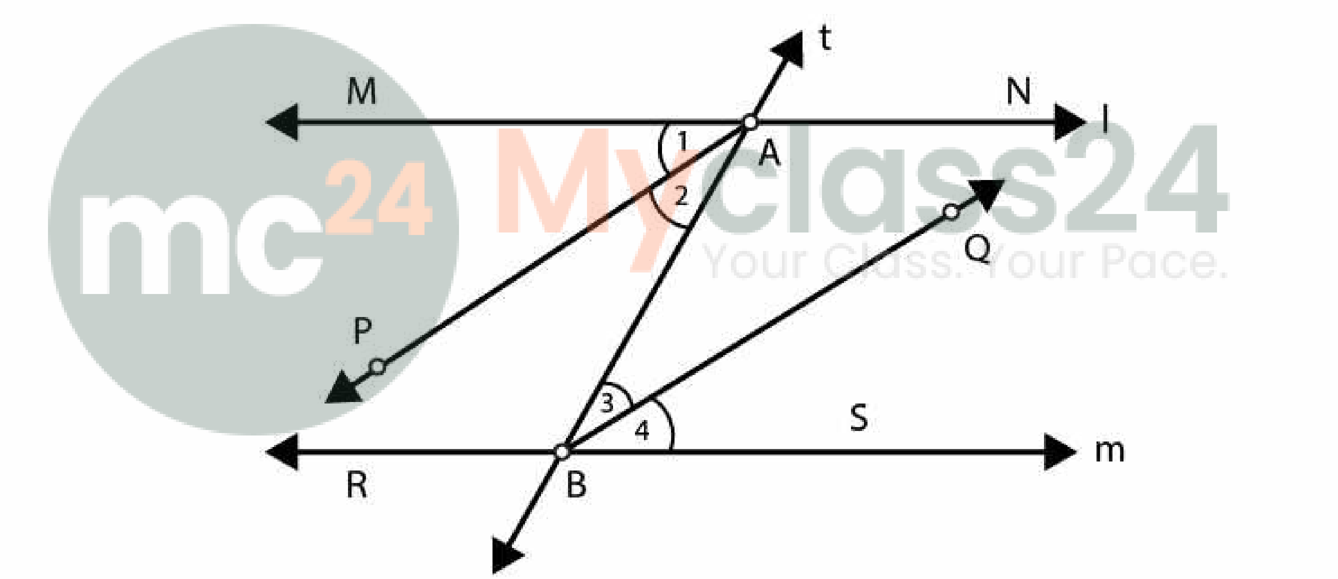


Fig. 6.11

Solution:



l || m and t is the transversal

∠MAB = ∠SBA [alternate angles]

⇒ ½ ∠MAB = ½ ∠SBA

⇒ ∠PAB = ∠QBA

⇒ ∠2 = ∠3

But, ∠2 and ∠3 are alternate angles.

Hence, AP||BQ.

3. If in Fig. 6.11, bisectors AP and BQ of the alternate interior angles are parallel, then show that *l* || m.

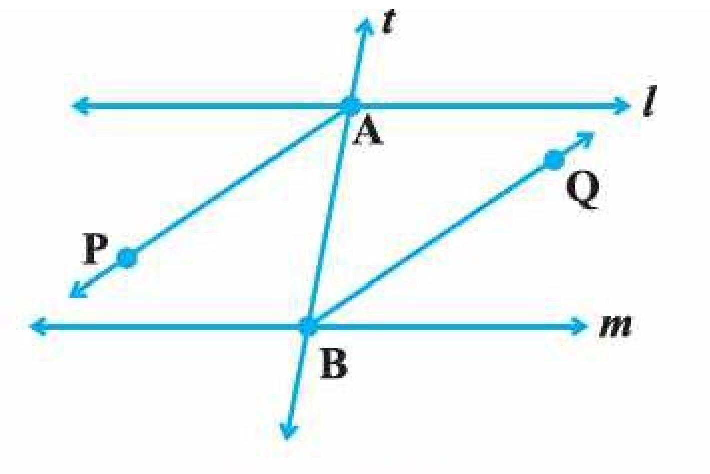


Fig. 6.11

Solution:

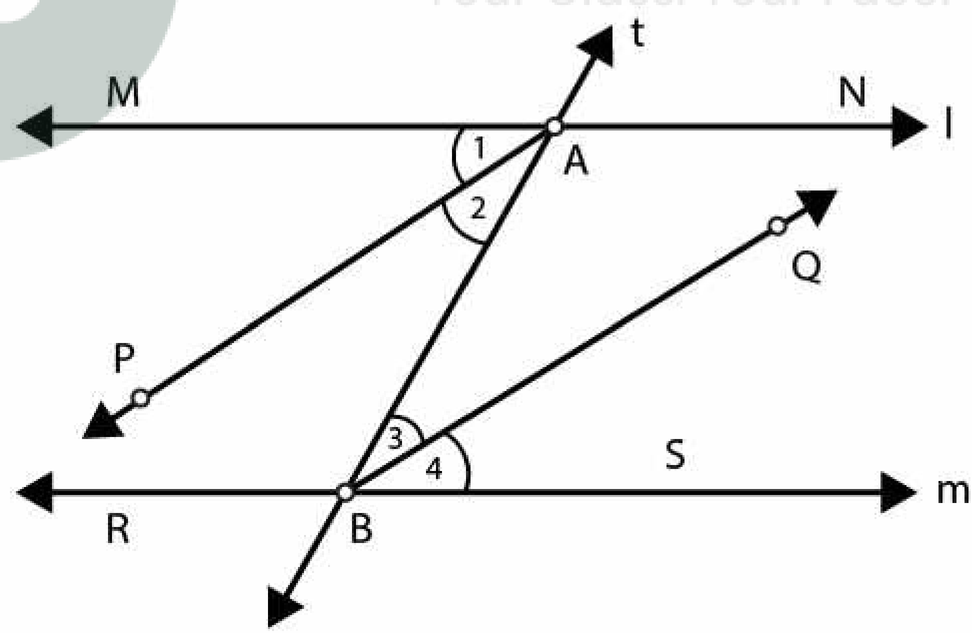
AP is the bisector of ∠MAB

BQ is the bisector of ∠SBA.

Given: AP||BQ.

As AP||BQ,

We have,



So ∠2 = ∠3 [Alternate angles]

2∠2 = 2∠3

⇒ ∠2 + ∠2 = ∠3 +∠3

From figure, we have ∠1= ∠2and ∠3 = ∠4

⇒ ∠1+ ∠2 = ∠3 +∠4

⇒ ∠MAB = ∠SBA

But, we know that these are alternate angles.

Hence, the lines *l* and m are parallel, i.e., *l* ||m.

4. In Fig. 6.12, BA || ED and BC || EF. Show that ∠ABC = ∠DEF [Hint: Produce DE to intersect BC at P (say)].

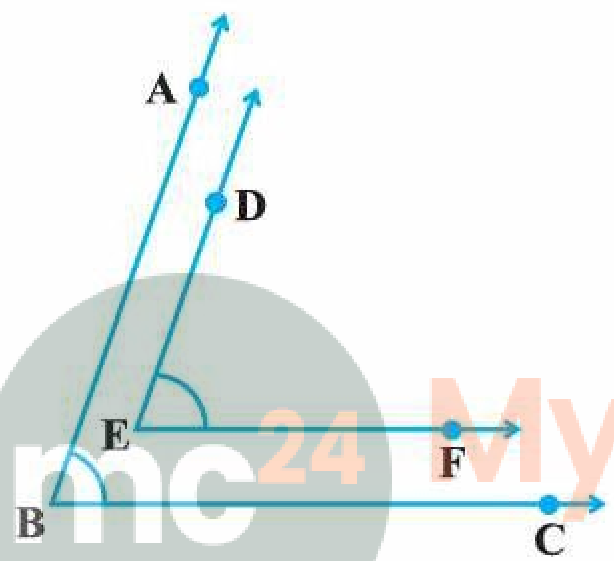


Fig. 6.12

Solution:

Construction:

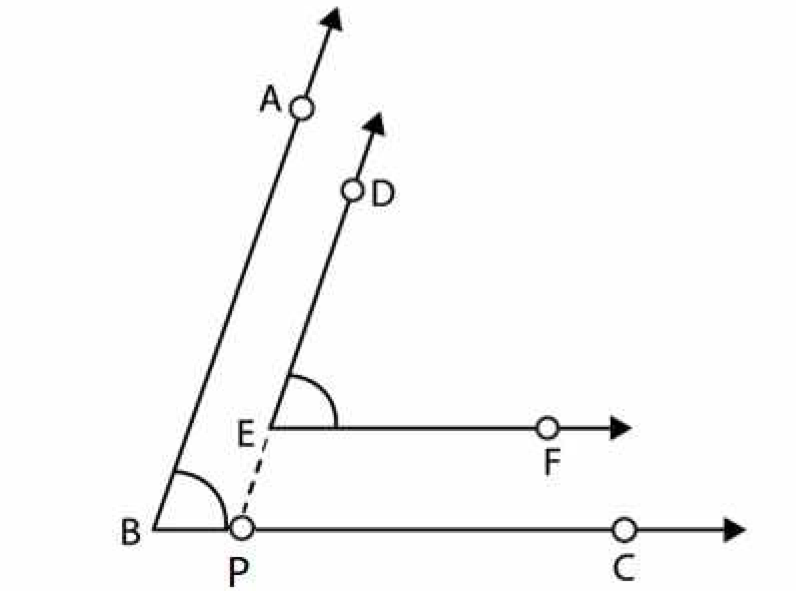
Extend DE to intersect BC at point, P.

Given, EF||BC and DP are the transversal,

$\angle DEF = \angle DPC$ …(eq.1) [Corresponding angles]

Also given, AB||DP and BC is the transversal,

$\angle DPC = \angle ABC$ …(eq.2) [Corresponding angles]



From (eq.1) and (eq.2), we get

∠ABC = ∠DEF

Hence, Proved.