

### EXERCISE 5.3

Find the values of the following trigonometric ratios:

(i)  $\sin 5\pi/3$

(ii)  $\sin 17\pi$

(iii)  $\tan 11\pi/6$

(iv)  $\cos (-25\pi/4)$

(v)  $\tan 7\pi/4$

(vi)  $\sin 17\pi/6$

(vii)  $\cos 19\pi/6$

(viii)  $\sin (-11\pi/6)$

(ix)  $\operatorname{cosec} (-20\pi/3)$

(x)  $\tan (-13\pi/4)$

(xi)  $\cos 19\pi/4$

(xii)  $\sin 41\pi/4$

(xiii)  $\cos 39\pi/4$

(xiv)  $\sin 151\pi/6$

**Solution:**

(i)  $\sin 5\pi/3$

$$\begin{aligned} 5\pi/3 &= (5/3 \times 180)^\circ \\ &= 300^\circ \\ &= (90 \times 3 + 30)^\circ \end{aligned}$$

Since,  $300^\circ$  lies in IV quadrant in which sine function is negative.

$$\begin{aligned} \sin 5\pi/3 &= \sin (300)^\circ \\ &= \sin (90 \times 3 + 30)^\circ \\ &= -\cos 30^\circ \\ &= -\sqrt{3}/2 \end{aligned}$$

(ii)  $\sin 17\pi$

$$\begin{aligned} \sin 17\pi &= \sin 3060^\circ \\ &= \sin (90 \times 34 + 0)^\circ \end{aligned}$$

Since,  $3060^\circ$  lies in the negative direction of x-axis i.e., on boundary line of II and III quadrants.

$$\begin{aligned} \sin 17\pi &= \sin (90 \times 34 + 0)^\circ \\ &= -\sin 0^\circ \\ &= 0 \end{aligned}$$

(iii)  $\tan 11\pi/6$

$$\tan 11\pi/6 = (11/6 \times 180)^\circ$$

$$= 330^\circ$$

Since,  $330^\circ$  lies in the IV quadrant in which tangent function is negative.

$$\begin{aligned}\tan 11\pi/6 &= \tan (300)^\circ \\ &= \tan (90 \times 3 + 60)^\circ \\ &= -\cot 60^\circ \\ &= -1/\sqrt{3}\end{aligned}$$

**(iv)**  $\cos (-25\pi/4)$

$$\begin{aligned}\cos (-25\pi/4) &= \cos (-1125)^\circ \\ &= \cos (1125)^\circ\end{aligned}$$

Since,  $1125^\circ$  lies in the I quadrant in which cosine function is positive.

$$\begin{aligned}\cos (1125)^\circ &= \cos (90 \times 12 + 45)^\circ \\ &= \cos 45^\circ \\ &= 1/\sqrt{2}\end{aligned}$$

**(v)**  $\tan 7\pi/4$

$$\begin{aligned}\tan 7\pi/4 &= \tan 315^\circ \\ &= \tan (90 \times 3 + 45)^\circ\end{aligned}$$

Since,  $315^\circ$  lies in the IV quadrant in which tangent function is negative.

$$\begin{aligned}\tan 315^\circ &= \tan (90 \times 3 + 45)^\circ \\ &= -\cot 45^\circ \\ &= -1\end{aligned}$$

**(vi)**  $\sin 17\pi/6$

$$\begin{aligned}\sin 17\pi/6 &= \sin 510^\circ \\ &= \sin (90 \times 5 + 60)^\circ\end{aligned}$$

Since,  $510^\circ$  lies in the II quadrant in which sine function is positive.

$$\begin{aligned}\sin 510^\circ &= \sin (90 \times 5 + 60)^\circ \\ &= \cos 60^\circ \\ &= 1/2\end{aligned}$$

**(vii)**  $\cos 19\pi/6$

$$\begin{aligned}\cos 19\pi/6 &= \cos 570^\circ \\ &= \cos (90 \times 6 + 30)^\circ\end{aligned}$$

Since,  $570^\circ$  lies in III quadrant in which cosine function is negative.

$$\begin{aligned}\cos 570^\circ &= \cos (90 \times 6 + 30)^\circ \\ &= -\cos 30^\circ \\ &= -\sqrt{3}/2\end{aligned}$$

**(viii)**  $\sin(-11\pi/6)$

$$\begin{aligned}\sin(-11\pi/6) &= \sin(-330^\circ) \\ &= -\sin(90 \times 3 + 60)^\circ\end{aligned}$$

Since,  $330^\circ$  lies in the IV quadrant in which the sine function is negative.

$$\begin{aligned}\sin(-330^\circ) &= -\sin(90 \times 3 + 60)^\circ \\ &= -(-\cos 60^\circ) \\ &= -(-1/2) \\ &= 1/2\end{aligned}$$

**(ix)**  $\operatorname{cosec}(-20\pi/3)$

$$\begin{aligned}\operatorname{cosec}(-20\pi/3) &= \operatorname{cosec}(-1200)^\circ \\ &= -\operatorname{cosec}(1200)^\circ \\ &= -\operatorname{cosec}(90 \times 13 + 30)^\circ\end{aligned}$$

Since,  $1200^\circ$  lies in the II quadrant in which cosec function is positive.

$$\begin{aligned}\operatorname{cosec}(-1200)^\circ &= -\operatorname{cosec}(90 \times 13 + 30)^\circ \\ &= -\sec 30^\circ \\ &= -2/\sqrt{3}\end{aligned}$$

**(x)**  $\tan(-13\pi/4)$

$$\begin{aligned}\tan(-13\pi/4) &= \tan(-585)^\circ \\ &= -\tan(90 \times 6 + 45)^\circ\end{aligned}$$

Since,  $585^\circ$  lies in the III quadrant in which the tangent function is positive.

$$\begin{aligned}\tan(-585)^\circ &= -\tan(90 \times 6 + 45)^\circ \\ &= -\tan 45^\circ \\ &= -1\end{aligned}$$

**(xi)**  $\cos 19\pi/4$

$$\begin{aligned}\cos 19\pi/4 &= \cos 855^\circ \\ &= \cos(90 \times 9 + 45)^\circ\end{aligned}$$

Since,  $855^\circ$  lies in the II quadrant in which the cosine function is negative.

$$\begin{aligned}\cos 855^\circ &= \cos(90 \times 9 + 45)^\circ \\ &= -\sin 45^\circ \\ &= -1/\sqrt{2}\end{aligned}$$

**(xii)**  $\sin 41\pi/4$

$$\begin{aligned}\sin 41\pi/4 &= \sin 1845^\circ \\ &= \sin(90 \times 20 + 45)^\circ\end{aligned}$$

Since,  $1845^\circ$  lies in the I quadrant in which the sine function is positive.

$$\sin 1845^\circ = \sin(90 \times 20 + 45)^\circ$$

$$= \sin 45^\circ$$

$$= 1/\sqrt{2}$$

(xiii)  $\cos 39\pi/4$

$$\cos 39\pi/4 = \cos 1755^\circ$$

$$= \cos (90 \times 19 + 45)^\circ$$

Since,  $1755^\circ$  lies in the IV quadrant in which the cosine function is positive.

$$\cos 1755^\circ = \cos (90 \times 19 + 45)^\circ$$

$$= \sin 45^\circ$$

$$= 1/\sqrt{2}$$

(xiv)  $\sin 151\pi/6$

$$\sin 151\pi/6 = \sin 4530^\circ$$

$$= \sin (90 \times 50 + 30)^\circ$$

Since,  $4530^\circ$  lies in the III quadrant in which the sine function is negative.

$$\sin 4530^\circ = \sin (90 \times 50 + 30)^\circ$$

$$= -\sin 30^\circ$$

$$= -1/2$$

2. prove that:

(i)  $\tan 225^\circ \cot 405^\circ + \tan 765^\circ \cot 675^\circ = 0$

(ii)  $\sin 8\pi/3 \cos 23\pi/6 + \cos 13\pi/3 \sin 35\pi/6 = 1/2$

(iii)  $\cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ + \cos 300^\circ = 1/2$

(iv)  $\tan (-125^\circ) \cot (-405^\circ) - \tan (-765^\circ) \cot (675^\circ) = 0$

(v)  $\cos 570^\circ \sin 510^\circ + \sin (-330^\circ) \cos (-390^\circ) = 0$

(vi)  $\tan 11\pi/3 - 2 \sin 4\pi/6 - 3/4 \operatorname{cosec}^2 \pi/4 + 4 \cos^2 17\pi/6 = (3 - 4\sqrt{3})/2$

(vii)  $3 \sin \pi/6 \sec \pi/3 - 4 \sin 5\pi/6 \cot \pi/4 = 1$

**Solution:**

(i)  $\tan 225^\circ \cot 405^\circ + \tan 765^\circ \cot 675^\circ = 0$

Let us consider LHS:

$$\tan 225^\circ \cot 405^\circ + \tan 765^\circ \cot 675^\circ$$

$$\tan (90^\circ \times 2 + 45^\circ) \cot (90^\circ \times 4 + 45^\circ) + \tan (90^\circ \times 8 + 45^\circ) \cot (90^\circ \times 7 + 45^\circ)$$

We know that when n is odd,  $\cot \rightarrow \tan$ .

$$\tan 45^\circ \cot 45^\circ + \tan 45^\circ [-\tan 45^\circ]$$

$$\tan 45^\circ \cot 45^\circ - \tan 45^\circ \tan 45^\circ$$

$$1 \times 1 - 1 \times 1$$

$$1 - 1$$

$$0 = \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

$$(ii) \sin 8\pi/3 \cos 23\pi/6 + \cos 13\pi/3 \sin 35\pi/6 = 1/2$$

Let us consider LHS:

$$\sin 8\pi/3 \cos 23\pi/6 + \cos 13\pi/3 \sin 35\pi/6$$

$$\sin 480^\circ \cos 690^\circ + \cos 780^\circ \sin 1050^\circ$$

$$\sin (90^\circ \times 5 + 30^\circ) \cos (90^\circ \times 7 + 60^\circ) + \cos (90^\circ \times 8 + 60^\circ) \sin (90^\circ \times 11 + 60^\circ)$$

We know that when n is odd,  $\sin \rightarrow \cos$  and  $\cos \rightarrow \sin$ .

$$\cos 30^\circ \sin 60^\circ + \cos 60^\circ [-\cos 60^\circ]$$

$$\sqrt{3}/2 \times \sqrt{3}/2 - 1/2 \times 1/2$$

$$3/4 - 1/4$$

$$2/4$$

$$1/2$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

$$(iii) \cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ + \cos 300^\circ = 1/2$$

Let us consider LHS:

$$\cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ + \cos 300^\circ$$

$$\cos 24^\circ + \cos (90^\circ \times 1 - 35^\circ) + \cos (90^\circ \times 1 + 35^\circ) + \cos (90^\circ \times 2 + 24^\circ) + \cos (90^\circ \times 3 + 30^\circ)$$

We know that when n is odd,  $\cos \rightarrow \sin$ .

$$\cos 24^\circ + \sin 35^\circ - \sin 35^\circ - \cos 24^\circ + \sin 30^\circ$$

$$0 + 0 + 1/2$$

$$1/2$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

$$(iv) \tan (-125^\circ) \cot (-405^\circ) - \tan (-765^\circ) \cot (675^\circ) = 0$$

Let us consider LHS:

$$\tan (-125^\circ) \cot (-405^\circ) - \tan (-765^\circ) \cot (675^\circ)$$

We know that  $\tan (-x) = -\tan (x)$  and  $\cot (-x) = -\cot (x)$ .

$$[-\tan (225^\circ)] [-\cot (405^\circ)] - [-\tan (765^\circ)] \cot (675^\circ)$$

$$\tan (225^\circ) \cot (405^\circ) + \tan (765^\circ) \cot (675^\circ)$$

$$\tan (90^\circ \times 2 + 45^\circ) \cot (90^\circ \times 4 + 45^\circ) + \tan (90^\circ \times 8 + 45^\circ) \cot (90^\circ \times 7 + 45^\circ)$$

$$\tan 45^\circ \cot 45^\circ + \tan 45^\circ [-\tan 45^\circ]$$

$$1 \times 1 + 1 \times (-1)$$