

Exercise 1.5

1. Show that the following numbers are irrational.

(i) $1/\sqrt{2}$

Solution:

Consider $1/\sqrt{2}$ is a rational number

Let us assume $1/\sqrt{2} = r$ where r is a rational number

On further calculation we get

$$1/r = \sqrt{2}$$

Since r is a rational number, $1/r = \sqrt{2}$ is also a rational number

But we know that $\sqrt{2}$ is an irrational number

So our supposition is wrong.

Hence, $1/\sqrt{2}$ is an irrational number.

(ii) $7\sqrt{5}$

Solution:

Let's assume on the contrary that $7\sqrt{5}$ is a rational number. Then, there exist positive integers a and b such that

$$7\sqrt{5} = a/b \text{ where, } a \text{ and } b, \text{ are co-primes}$$

$$\Rightarrow \sqrt{5} = a/7b$$

$$\Rightarrow \sqrt{5} \text{ is rational} \quad [\because 7, a \text{ and } b \text{ are integers } \therefore a/7b \text{ is a rational number}]$$

This contradicts the fact that $\sqrt{5}$ is irrational. So, our assumption is incorrect.

Hence, $7\sqrt{5}$ is an irrational number.

(iii) $6 + \sqrt{2}$

Solution:

Let's assume on the contrary that $6 + \sqrt{2}$ is a rational number. Then, there exist co prime positive integers a and b such that

$$6 + \sqrt{2} = a/b$$

$$\Rightarrow \sqrt{2} = a/b - 6$$

$$\Rightarrow \sqrt{2} = (a - 6b)/b$$

$$\Rightarrow \sqrt{2} \text{ is rational} \quad [\because a \text{ and } b \text{ are integers } \therefore (a-6b)/b \text{ is a rational number}]$$

This contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption is incorrect.

Hence, $6 + \sqrt{2}$ is an irrational number.

(iv) $3 - \sqrt{5}$

Solution:

Let's assume on the contrary that $3 - \sqrt{5}$ is a rational number. Then, there exist co prime positive integers a and b such that

$$3 - \sqrt{5} = a/b$$

$$\Rightarrow \sqrt{5} = a/b + 3$$

$$\Rightarrow \sqrt{5} = (a + 3b)/b$$

$$\Rightarrow \sqrt{5} \text{ is rational} \quad [\because a \text{ and } b \text{ are integers } \therefore (a+3b)/b \text{ is a rational number}]$$

This contradicts the fact that $\sqrt{5}$ is irrational. So, our assumption is incorrect.

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Hence, $3-\sqrt{5}$ is an irrational number.

2. Prove that the following numbers are irrationals.

(i) $2/\sqrt{7}$

Solution:

Let's assume on the contrary that $2/\sqrt{7}$ is a rational number. Then, there exist co-prime positive integers a and b such that

$$\begin{aligned}2/\sqrt{7} &= a/b \\ \Rightarrow \sqrt{7} &= 2b/a \\ \Rightarrow \sqrt{7} &\text{ is rational} \quad [\because 2, a \text{ and } b \text{ are integers } \therefore 2b/a \text{ is a rational number}]\end{aligned}$$

This contradicts the fact that $\sqrt{7}$ is irrational. So, our assumption is incorrect.
Hence, $2/\sqrt{7}$ is an irrational number.

(ii) $3/(2\sqrt{5})$

Solution:

Let's assume on the contrary that $3/(2\sqrt{5})$ is a rational number. Then, there exist co – prime positive integers a and b such that

$$\begin{aligned}3/(2\sqrt{5}) &= a/b \\ \Rightarrow \sqrt{5} &= 3b/2a \\ \Rightarrow \sqrt{5} &\text{ is rational} \quad [\because 3, 2, a \text{ and } b \text{ are integers } \therefore 3b/2a \text{ is a rational number}]\end{aligned}$$

This contradicts the fact that $\sqrt{5}$ is irrational. So, our assumption is incorrect.
Hence, $3/(2\sqrt{5})$ is an irrational number.

(iii) $4 + \sqrt{2}$

Solution:

Let's assume on the contrary that $4 + \sqrt{2}$ is a rational number. Then, there exist co prime positive integers a and b such that

$$\begin{aligned}4 + \sqrt{2} &= a/b \\ \Rightarrow \sqrt{2} &= a/b - 4 \\ \Rightarrow \sqrt{2} &= (a - 4b)/b \\ \Rightarrow \sqrt{2} &\text{ is rational} \quad [\because a \text{ and } b \text{ are integers } \therefore (a - 4b)/b \text{ is a rational number}]\end{aligned}$$

This contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption is incorrect.
Hence, $4 + \sqrt{2}$ is an irrational number.

(iv) $5\sqrt{2}$

Solution:

Let's assume on the contrary that $5\sqrt{2}$ is a rational number. Then, there exist positive integers a and b such that

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$$\begin{aligned} &5\sqrt{2} = a/b \text{ where, } a \text{ and } b, \text{ are co-primes} \\ \Rightarrow &\sqrt{2} = a/5b \\ \Rightarrow &\sqrt{2} \text{ is rational} \quad [\because a \text{ and } b \text{ are integers } \therefore a/5b \text{ is a rational number}] \end{aligned}$$

This contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption is incorrect.
Hence, $5\sqrt{2}$ is an irrational number.

3. Show that $2 - \sqrt{3}$ is an irrational number.

Solution:

Let's assume on the contrary that $2 - \sqrt{3}$ is a rational number. Then, there exist co prime positive integers a and b such that

$$\begin{aligned} &2 - \sqrt{3} = a/b \\ \Rightarrow &\sqrt{3} = 2 - a/b \\ \Rightarrow &\sqrt{3} = (2b - a)/b \\ \Rightarrow &\sqrt{3} \text{ is rational} \quad [\because a \text{ and } b \text{ are integers } \therefore (2b - a)/b \text{ is a rational number}] \end{aligned}$$

This contradicts the fact that $\sqrt{3}$ is irrational. So, our assumption is incorrect.
Hence, $2 - \sqrt{3}$ is an irrational number.

4. Show that $3 + \sqrt{2}$ is an irrational number.

Solution:

Let's assume on the contrary that $3 + \sqrt{2}$ is a rational number. Then, there exist co prime positive integers a and b such that

$$\begin{aligned} &3 + \sqrt{2} = a/b \\ \Rightarrow &\sqrt{2} = a/b - 3 \\ \Rightarrow &\sqrt{2} = (a - 3b)/b \\ \Rightarrow &\sqrt{2} \text{ is rational} \quad [\because a \text{ and } b \text{ are integers } \therefore (a - 3b)/b \text{ is a rational number}] \end{aligned}$$

This contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption is incorrect.
Hence, $3 + \sqrt{2}$ is an irrational number.

5. Prove that $4 - 5\sqrt{2}$ is an irrational number.

Solution:

Let's assume on the contrary that $4 - 5\sqrt{2}$ is a rational number. Then, there exist co prime positive integers a and b such that

$$\begin{aligned} &4 - 5\sqrt{2} = a/b \\ \Rightarrow &5\sqrt{2} = 4 - a/b \\ \Rightarrow &\sqrt{2} = (4b - a)/(5b) \\ \Rightarrow &\sqrt{2} \text{ is rational} \quad [\because 5, a \text{ and } b \text{ are integers } \therefore (4b - a)/5b \text{ is a rational number}] \end{aligned}$$

This contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption is incorrect.
Hence, $4 - 5\sqrt{2}$ is an irrational number.

6. Show that $5 - 2\sqrt{3}$ is an irrational number.

Solution:

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Let's assume on the contrary that $5 - 2\sqrt{3}$ is a rational number. Then, there exist co prime positive integers a and b such that

$$5 - 2\sqrt{3} = a/b$$

$$\Rightarrow 2\sqrt{3} = 5 - a/b$$

$$\Rightarrow \sqrt{3} = (5b - a)/(2b)$$

$$\Rightarrow \sqrt{3} \text{ is rational} \quad [\because 2, a \text{ and } b \text{ are integers } \therefore (5b - a)/2b \text{ is a rational number}]$$

This contradicts the fact that $\sqrt{3}$ is irrational. So, our assumption is incorrect.

Hence, $5 - 2\sqrt{3}$ is an irrational number.

7. Prove that $2\sqrt{3} - 1$ is an irrational number.

Solution:

Let's assume on the contrary that $2\sqrt{3} - 1$ is a rational number. Then, there exist co prime positive integers a and b such that

$$2\sqrt{3} - 1 = a/b$$

$$\Rightarrow 2\sqrt{3} = a/b + 1$$

$$\Rightarrow \sqrt{3} = (a + b)/(2b)$$

$$\Rightarrow \sqrt{3} \text{ is rational} \quad [\because 2, a \text{ and } b \text{ are integers } \therefore (a + b)/2b \text{ is a rational number}]$$

This contradicts the fact that $\sqrt{3}$ is irrational. So, our assumption is incorrect.

Hence, $2\sqrt{3} - 1$ is an irrational number.

8. Prove that $2 - 3\sqrt{5}$ is an irrational number.

Solution:

Let's assume on the contrary that $2 - 3\sqrt{5}$ is a rational number. Then, there exist co prime positive integers a and b such that

$$2 - 3\sqrt{5} = a/b$$

$$\Rightarrow 3\sqrt{5} = 2 - a/b$$

$$\Rightarrow \sqrt{5} = (2b - a)/(3b)$$

$$\Rightarrow \sqrt{5} \text{ is rational} \quad [\because 3, a \text{ and } b \text{ are integers } \therefore (2b - a)/3b \text{ is a rational number}]$$

This contradicts the fact that $\sqrt{5}$ is irrational. So, our assumption is incorrect.

Hence, $2 - 3\sqrt{5}$ is an irrational number.

9. Prove that $\sqrt{5} + \sqrt{3}$ is irrational.

Solution:

Let's assume on the contrary that $\sqrt{5} + \sqrt{3}$ is a rational number. Then, there exist co prime positive integers a and b such that

$$\sqrt{5} + \sqrt{3} = a/b$$

$$\Rightarrow \sqrt{5} = (a/b) - \sqrt{3}$$

$$\Rightarrow (\sqrt{5})^2 = ((a/b) - \sqrt{3})^2 \quad [\text{Squaring on both sides}]$$

$$\Rightarrow 5 = (a^2/b^2) + 3 - (2\sqrt{3}a/b)$$

$$\Rightarrow (a^2/b^2) - 2 = (2\sqrt{3}a/b)$$

$$\Rightarrow (a/b) - (2b/a) = 2\sqrt{3}$$

$$\Rightarrow (a^2 - 2b^2)/2ab = \sqrt{3}$$

$$\Rightarrow \sqrt{3} \text{ is rational} \quad [\because a \text{ and } b \text{ are integers } \therefore (a^2 - 2b^2)/2ab \text{ is rational}]$$

This contradicts the fact that $\sqrt{3}$ is irrational. So, our assumption is incorrect.

Hence, $\sqrt{5} + \sqrt{3}$ is an irrational number.

10. Prove that $\sqrt{2} + \sqrt{3}$ is irrational.

Solution:

Let's assume on the contrary that $\sqrt{2} + \sqrt{3}$ is a rational number. Then, there exist co prime positive integers a and b such that

$$\begin{aligned} \sqrt{2} + \sqrt{3} &= a/b \\ \Rightarrow \sqrt{2} &= (a/b) - \sqrt{3} \\ \Rightarrow (\sqrt{2})^2 &= ((a/b) - \sqrt{3})^2 && \text{[Squaring on both sides]} \\ \Rightarrow 2 &= (a^2/b^2) + 3 - (2\sqrt{3}a/b) \\ \Rightarrow (a^2/b^2) + 1 &= (2\sqrt{3}a/b) \\ \Rightarrow (a/b) + (b/a) &= 2\sqrt{3} \\ \Rightarrow (a^2 + b^2)/2ab &= \sqrt{3} \\ \Rightarrow \sqrt{3} &\text{ is rational} && [\because a \text{ and } b \text{ are integers } \therefore (a^2 + 2b^2)/2ab \text{ is rational}] \end{aligned}$$

This contradicts the fact that $\sqrt{3}$ is irrational. So, our assumption is incorrect.

Hence, $\sqrt{2} + \sqrt{3}$ is an irrational number.

11. Prove that for any prime positive integer p , \sqrt{p} is an irrational number.

Solution:

Consider \sqrt{p} as a rational number

Assume $\sqrt{p} = a/b$ where a and b are integers and $b \neq 0$

By squaring on both sides

$$p = a^2/b^2$$

$$pb = a^2/b$$

p and b are integers $pb = a^2/b$ will also be an integer

But we know that a^2/b is a rational number so our supposition is wrong

Therefore, \sqrt{p} is an irrational number.

12. If p, q are prime positive integers, prove that $\sqrt{p} + \sqrt{q}$ is an irrational number.

Solution:

Let's assume on the contrary that $\sqrt{p} + \sqrt{q}$ is a rational number. Then, there exist co prime positive integers a and b such that

$$\begin{aligned} \sqrt{p} + \sqrt{q} &= a/b \\ \Rightarrow \sqrt{p} &= (a/b) - \sqrt{q} \\ \Rightarrow (\sqrt{p})^2 &= ((a/b) - \sqrt{q})^2 && \text{[Squaring on both sides]} \\ \Rightarrow p &= (a^2/b^2) + q - (2\sqrt{q} a/b) \\ \Rightarrow (a^2/b^2) - (p+q) &= (2\sqrt{q} a/b) \\ \Rightarrow (a/b) - ((p+q)b/a) &= 2\sqrt{q} \\ \Rightarrow (a^2 - b^2(p+q))/2ab &= \sqrt{q} \\ \Rightarrow \sqrt{q} &\text{ is rational} && [\because a \text{ and } b \text{ are integers } \therefore (a^2 - b^2(p+q))/2ab \text{ is rational}] \end{aligned}$$

This contradicts the fact that \sqrt{q} is irrational. So, our assumption is incorrect.

Hence, $\sqrt{p} + \sqrt{q}$ is an irrational number.