EXERCISE 20.1

Show that each one of the following progressions is a G.P. Also, find the common ratio in each case:
(i) 4, -2, 1, -1/2, ….
(ii) -2/3, -6, -54, ….
(iii) a, $3a^2/4$, $9a^3/16$, ….
(iv) ½, 1/3, 2/9, 4/27, …
Solution:
(i) 4, -2, 1, -1/2, ….
Let a = 4, b = -2, c = 1
In GP,
$b^2 = ac$
$(-2)^2 = 4(1)$
4 = 4
So, the Common ratio = r = -2/4 = -1/2

(ii) -2/3, -6, -54, ….
Let a = -2/3, b = -6, c = -54
In GP,
$b^2 = ac$
$(-6)^2 = -2/3 \times (-54)$
36 = 36
So, the Common ratio = r = -6/(-2/3) = -6 × 3/-2 = 9

(iii) a, $3a^2/4$, $9a^3/16$, ….
Let a = a, b = $3a^2/4$, c = $9a^3/16$
In GP,
$b^2 = ac$
$(3a^2/4)^2 = 9a^3/16 \times a$
$9a^4/16 = 9a^4/16$
So, the Common ratio = r = $(3a^2/4)/a = 3a^2/4a = 3a/4$

(iv) ½, 1/3, 2/9, 4/27, …
Let a = 1/2, b = 1/3, c = 2/9
In GP,
$b^2 = ac$
$(1/3)^2 = 1/2 \times (2/9)$
1/9 = 1/9

So, the Common ratio = r = (1/3)/(1/2) = (1/3) × 2 = 2/3

2. Show that the sequence defined by $a_n = 2/3^n$, n ∈ N is a G.P.
Solution:
Given:
$a_n = 2/3^n$
Let us consider n = 1, 2, 3, 4, … since n is a natural number.
So,
$a_1 = 2/3$
$a_2 = 2/3^2 = 2/9$
$a_3 = 2/3^3 = 2/27$
$a_4 = 2/3^4 = 2/81$
In GP,
a_3/a_2 = (2/27) / (2/9)
= 2/27 × 9/2
= 1/3

a_2/a_1 = (2/9) / (2/3)
= 2/9 × 3/2
= 1/3
∴ Common ratio of consecutive term is 1/3. Hence n ∈ N is a G.P.

3. Find:
(i) the ninth term of the G.P. 1, 4, 16, 64, ….
(ii) the 10th term of the G.P. -3/4, ½, -1/3, 2/9, ….
(iii) the 8th term of the G.P. 0.3, 0.06, 0.012, ….
(iv) the 12th term of the G.P. $1/a^3x^3$, ax, a^5x^5, ….
(v) nth term of the G.P. √3, 1/√3, 1/3√3, …
(vi) the 10th term of the G.P. √2, 1/√2, 1/2√2, ….
Solution:
(i) the ninth term of the G.P. 1, 4, 16, 64, ….
We know that,
$t_1 = a = 1$, $r = t_2/t_1 = 4/1 = 4$
By using the formula,
$T_n = ar^{n-1}$
$T_9 = 1\ (4)^{9-1}$
$= 1\ (4)^8$
$= 4^8$

(ii) the 10^{th} term of the G.P. -3/4, ½, -1/3, 2/9, ….
We know that,
$t_1 = a = -3/4$, $r = t_2/t_1 = (1/2) / (-3/4) = ½ \times -4/3 = -2/3$
By using the formula,
$T_n = ar^{n-1}$
$T_{10} = -3/4\ (-2/3)^{10-1}$
$= -3/4\ (-2/3)^9$
$= ½\ (2/3)^8$

(iii) the 8^{th} term of the G.P., 0.3, 0.06, 0.012, ….
We know that,
$t_1 = a = 0.3$, $r = t_2/t_1 = 0.06/0.3 = 0.2$
By using the formula,
$T_n = ar^{n-1}$
$T_8 = 0.3\ (0.2)^{8-1}$
$= 0.3\ (0.2)^7$

(iv) the 12^{th} term of the G.P. $1/a^3x^3$, ax, a^5x^5, ….
We know that,
$t_1 = a = 1/a^3x^3$, $r = t_2/t_1 = ax/(1/a^3x^3) = ax\ (a^3x^3) = a^4x^4$
By using the formula,
$T_n = ar^{n-1}$
$T_{12} = 1/a^3x^3\ (a^4x^4)^{12-1}$
$= 1/a^3x^3\ (a^4x^4)^{11}$
$= (ax)^{41}$

(v) nth term of the G.P. √3, 1/√3, 1/3√3, …
We know that,
$t_1 = a = \sqrt{3}$, $r = t_2/t_1 = (1/\sqrt{3})/\sqrt{3} = 1/(\sqrt{3}\times\sqrt{3}) = 1/3$
By using the formula,
$T_n = ar^{n-1}$
$T_n = \sqrt{3}\ (1/3)^{n-1}$

(vi) the 10^{th} term of the G.P. √2, 1/√2, 1/2√2, ….
We know that,
$t_1 = a = \sqrt{2}$, $r = t_2/t_1 = (1/\sqrt{2})/\sqrt{2} = 1/(\sqrt{2}\times\sqrt{2}) = 1/2$
By using the formula,
$T_n = ar^{n-1}$
$T_{10} = \sqrt{2}\ (1/2)^{10-1}$

$= \sqrt{2}\,(1/2)^9$
$= 1/\sqrt{2}\,(1/2)^8$

4. Find the 4^{th} term from the end of the G.P. 2/27, 2/9, 2/3, …., 162.
Solution:
The nth term from the end is given by:
$a_n = l\,(1/r)^{n-1}$ where, l is the last term, r is the common ratio, n is the nth term
Given: last term, l = 162
$r = t_2/t_1 = (2/9) / (2/27)$
$= 2/9 \times 27/2$
$= 3$
n = 4
So, $a_n = l\,(1/r)^{n-1}$
$a_4 = 162\,(1/3)^{4-1}$
$= 162\,(1/3)^3$
$= 162 \times 1/27$
$= 6$
∴ 4^{th} term from last is 6.

5. Which term of the progression 0.004, 0.02, 0.1, …. is 12.5?
Solution:
By using the formula,
$T_n = ar^{n-1}$
Given:
a = 0.004
$r = t_2/t_1 = (0.02/0.004)$
$= 5$
$T_n = 12.5$
n = ?
So, $T_n = ar^{n-1}$
$12.5 = (0.004)\,(5)^{n-1}$
$12.5/0.004 = 5^{n-1}$
$3000 = 5^{n-1}$
$5^5 = 5^{n-1}$
5 = n-1
n = 5 + 1
= 6
∴ 6^{th} term of the progression 0.004, 0.02, 0.1, …. is 12.5.

6. Which term of the G.P.:
(i) √2, 1/√2, 1/2√2, 1/4√2, … is 1/512√2 ?
(ii) 2, 2√2, 4, … is 128 ?
(iii) √3, 3, 3√3, … is 729 ?
(iv) 1/3, 1/9, 1/27… is 1/19683 ?
Solution:
(i) √2, 1/√2, 1/2√2, 1/4√2, … is 1/512√2 ?
By using the formula,
$T_n = ar^{n-1}$
a = √2
$r = t_2/t_1 = (1/\sqrt{2}) / (\sqrt{2})$
$= 1/2$
$T_n = 1/512\sqrt{2}$
n = ?
$T_n = ar^{n-1}$
$1/512\sqrt{2} = (\sqrt{2}) (1/2)^{n-1}$
$1/512\sqrt{2}\times\sqrt{2} = (1/2)^{n-1}$
$1/512\times2 = (1/2)^{n-1}$
$1/1024 = (1/2)^{n-1}$
$(1/2)^{10} = (1/2)^{n-1}$
10 = n – 1
n = 10 + 1
= 11
∴ 11^{th} term of the G.P is 1/512√2

(ii) 2, 2√2, 4, … is 128 ?
By using the formula,
$T_n = ar^{n-1}$
a = 2
$r = t_2/t_1 = (2\sqrt{2}/2)$
$= \sqrt{2}$
$T_n = 128$
n = ?
$T_n = ar^{n-1}$
$128 = 2 (\sqrt{2})^{n-1}$
$128/2 = (\sqrt{2})^{n-1}$
$64 = (\sqrt{2})^{n-1}$
$2^6 = (\sqrt{2})^{n-1}$
12 = n – 1

n = 12 + 1
= 13
∴ 13^{th} term of the G.P is 128

(iii) √3, 3, 3√3, … is 729 ?
By using the formula,
$T_n = ar^{n-1}$
a = √3
$r = t_2/t_1 = (3/\sqrt{3})$
$= \sqrt{3}$
$T_n = 729$
n = ?
$T_n = ar^{n-1}$
$729 = \sqrt{3} (\sqrt{3})^{n-1}$
$729 = (\sqrt{3})^n$
$3^6 = (\sqrt{3})^n$
$(\sqrt{3})^{12} = (\sqrt{3})^n$
n = 12
∴ 12^{th} term of the G.P is 729

(iv) 1/3, 1/9, 1/27… is 1/19683 ?
By using the formula,
$T_n = ar^{n-1}$
a = 1/3
$r = t_2/t_1 = (1/9) / (1/3)$
$= 1/9 \times 3/1$
$= 1/3$
$T_n = 1/19683$
n = ?
$T_n = ar^{n-1}$
$1/19683 = (1/3) (1/3)^{n-1}$
$1/19683 = (1/3)^n$
$(1/3)^9 = (1/3)^n$
n = 9
∴ 9^{th} term of the G.P is 1/19683

7. Which term of the progression 18, -12, 8, … is 512/729 ?
Solution:
By using the formula,

$T_n = ar^{n-1}$
a = 18
$r = t_2/t_1 = (-12/18)$
$= -2/3$
$T_n = 512/729$
n = ?
$T_n = ar^{n-1}$
$512/729 = 18\,(-2/3)^{n-1}$
$2^9/(729 \times 18) = (-2/3)^{n-1}$
$2^9/36 \times 1/2\times3^2 = (-2/3)^{n-1}$
$(2/3)^8 = (-1)^{n-1}\,(2/3)^{n-1}$
8 = n – 1
n = 8 + 1
= 9
∴ 9^{th} term of the Progression is 512/729

8. Find the 4th term from the end of the G.P. ½, 1/6, 1/18, 1/54, … , 1/4374
Solution:
The nth term from the end is given by:
$a_n = l\,(1/r)^{n-1}$ where, l is the last term, r is the common ratio, n is the nth term
Given: last term, l = 1/4374
$r = t_2/t_1 = (1/6) / (1/2)$
$= 1/6 \times 2/1$
$= 1/3$
n = 4
So, $a_n = l\,(1/r)^{n-1}$
$a_4 = 1/4374\,(1/(1/3))^{4-1}$
$= 1/4374\,(3/1)^3$
$= 1/4374 \times 3^3$
$= 1/4374 \times 27$
$= 1/162$
∴ 4^{th} term from last is 1/162.