Formula : $\frac{d(e^x)}{dx} = e^x$ and $\frac{d(\cos x)}{dx} = -\sin x$

According to product rule of differentiation

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [\cos 2x \ \times (3 \times e^{3x})] + [e^{3x} \times (-2\sin 2x)\ ]$$

$$= e^{3x} \times [3\cos 2x - 2\sin 2x]$$

11. Question

Differentiate each of the following w.r.t. x:

e^{-5x} cot 4x

Answer

Let y = e^{-5x} cot 4x , z = e^{-5x} and w = cot 4x

Formula : $\frac{d(e^x)}{dx} = e^x$ and $\frac{d(\cot x)}{dx} = -\mathrm{cosec}^2 x$

According to product rule of differentiation

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [\cot 4x \ \times (-5e^{-5x})] + [e^{-5x} \times (-4\,\mathrm{cosec}^2 4x)]$$

$$= -e^{-5x} \times [5\cot 4x + 4\,\mathrm{cosec}^2 4x]$$

12. Question

Differentiate each of the following w.r.t. x:

e^x log (sin 2x)

Answer

Let y = e^x log (sin 2x), z = e^x and w = log (sin 2x)

Formula : $\frac{d(e^x)}{dx} = e^x, \frac{d(\log x)}{dx} = \frac{1}{x}$ and $\frac{d(\sin x)}{dx} = \cos x$

According to product rule of differentiation

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [\ \log(\sin 2x)\ \times (e^x)] + [e^x \times \frac{1}{\sin 2x} \times 2\cos 2x\ ]$$

$= e^x \times [\log(\sin 2x) + \frac{2\cos 2x}{\sin 2x}]$

$= e^x \times [\log(\sin 2x) + 2\cot 2x]$

13. Question

Differentiate each of the following w.r.t. x:

$\log(\operatorname{cosec} x - \cot x)$

Answer

Let y = $\log(\operatorname{cosec} x - \cot x)$, z =$(\operatorname{cosec} x - \cot x)$

Formula :

$\frac{d(\operatorname{cosec} x)}{dx} = -\operatorname{cosec} x \cot x$, $\frac{d(\log x)}{dx} = \frac{1}{x}$ and $\frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$

According to chain rule of differentiation

$^{dy}/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$

$= [\frac{1}{(\operatorname{cosec} x - \cot x)}] \times [-\operatorname{cosec} x \cot x - (-\operatorname{cosec}^2 x)]$

$= [\frac{1}{(\operatorname{cosec} x - \cot x)}] \times [-\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x]$

$= [\frac{1}{(\operatorname{cosec} x - \cot x)}] \times [\operatorname{cosec} x(\operatorname{cosec} x - \cot x)]$

= cosec x

14. Question

Differentiate each of the following w.r.t. x:

$\log\left(\sec\frac{x}{2} + \tan\frac{x}{2}\right)$

Answer

Let y = $\log(\sec\frac{x}{2} + \tan\frac{x}{2})$, z =$(\sec\frac{x}{2} + \tan\frac{x}{2})$

Formula :

$\frac{d(\sec x)}{dx} = \sec x \tan x$, $\frac{d(\log x)}{dx} = \frac{1}{x}$ and $\frac{d(\tan x)}{dx} = \sec^2 x$

According to chain rule of differentiation

$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$

$= [\frac{1}{(\sec\frac{x}{2} + \tan\frac{x}{2})}] \times [(\sec\frac{x}{2}\tan\frac{x}{2} \times \frac{1}{2}) + (\sec^2\frac{x}{2} \times \frac{1}{2})]$

$= [\frac{1}{(\sec\frac{x}{2} + \tan\frac{x}{2})}] \times [\frac{1}{2}\sec\frac{x}{2}(\sec\frac{x}{2} + \tan\frac{x}{2})]$

$= \frac{1}{2}\sec\frac{x}{2}$

15. Question

Differentiate each of the following w.r.t. x:

$$\sqrt{\frac{1+e^x}{1-e^x}}$$

Answer

Let y = $\sqrt{\frac{1+e^x}{1-e^x}}$, u = $1 + e^x$, v = $1 - e^x$, z= $\frac{1+e^x}{1-e^x}$

Formula : $\frac{d(e^x)}{dx} = e^x$

According to quotient rule of differentiation

If z = $\frac{u}{v}$

$dz/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$

$= \frac{(1 - e^x) \times (e^x) - (1 + e^x) \times (-e^x)}{(1 - e^x)^2}$

$= \frac{e^x - e^{2x} + e^x + e^{2x}}{(1 - e^x)^2}$

$= \frac{2e^x}{(1 - e^x)^2}$

According to chain rule of differentiation

$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$

$= \left[\frac{1}{2} \times \left(\frac{1+e^x}{1-e^x}\right)^{\frac{1}{2}-1}\right] \times \left[\frac{2e^x}{(1-e^x)^2}\right]$

$= \left[\frac{e^x}{1} \times \left(\frac{1+e^x}{1}\right)^{-\frac{1}{2}}\right] \times \left[\frac{1}{(1-e^x)^{2-\frac{1}{2}}}\right]$

$= \left[\frac{e^x}{(1+e^x)^{\frac{1}{2}} \times (1-e^x)^{2-\frac{1}{2}}}\right]$

$= \left[\frac{e^x}{(1+e^x)^{\frac{1}{2}} \times (1-e^x)^{\frac{1}{2}} \times (1-e^x)^1}\right]$

$= \left[\frac{e^x}{((1+e^x)(1-e^x))^{\frac{1}{2}} \times (1-e^x)^1}\right]$

$= \frac{e^x}{(1-e^x)\sqrt{1-e^{2x}}}$

16. Question

Differentiate each of the following w.r.t. x:

$\frac{e^x + e^{-x}}{e^x - e^{-x}}$

Answer

Let $y = \frac{e^x+e^{-x}}{e^x-e^{-x}}$, $u = e^x + e^{-x}$, $v = e^x - e^{-x}$

Formula : $\frac{d(e^x)}{dx} = e^x$

According to quotient rule of differentiation

If $y = \frac{u}{v}$

$$dy/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(e^x - e^{-x}) \times (e^x - e^{-x}) \; - \; (e^x + e^{-x}) \times (e^x + e^{-x})}{(e^x - e^{-x})^2}$$

$$= \frac{(e^x - e^{-x})^2 \; - \; (e^x + e^{-x})^2}{(e^x - e^{-x})^2}$$

$$= \frac{(e^x - e^{-x} + e^x + e^{-x})(e^x - e^{-x} - e^x - e^{-x})}{(e^x - e^{-x})^2}$$

$(a^2 - b^2 = (a-b)(a+b))$

$$= \frac{(2\,e^x)(-2e^{-x})}{(e^x - e^{-x})^2}$$

$$= \frac{-4}{(e^x - e^{-x})^2}$$

17. Question

Differentiate each of the following w.r.t. x:

$$xe^{\sqrt{\sin x}}$$

Answer

Let y = $xe^{\sqrt{\sin x}}$, z = x and w = $e^{\sqrt{\sin x}}$

Formula : $\frac{d(e^x)}{dx} = e^x, \frac{d(\sin x)}{dx} = \cos x$

According to product rule of differentiation

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [\,e^{\sqrt{\sin x}} \times (1)] + [x \times e^{\sqrt{\sin x}} \times \frac{1}{2} \times \frac{1}{\sqrt{\sin x}} \times \cos x\,]$$

$$= e^{\sqrt{\sin x}} \times [1 + \frac{x\cos x}{2\sqrt{\sin x}}]$$

18. Question

Differentiate each of the following w.r.t. x:

$$e^{\sin x}\sin(e^x)$$

Answer

Let y = $e^{\sin x}\sin e^x$, z = $e^{\sin x}$ and w = $\sin e^x$

Formula : $\frac{d(e^x)}{dx} = e^x, \frac{d(\sin x)}{dx} = \cos x$

According to product rule of differentiation

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$= [\sin e^x \times (e^{\sin x} \times \cos x)] + [e^{\sin x} \times \cos e^x \times e^x]$

$= e^{\sin x}[(\sin e^x \times \cos x) + (\cos e^x \times e^x)]$

$= e^{\sin x}(e^x \cos e^x + \cos x \sin e^x)$

19. Question

Differentiate each of the following w.r.t. x:

$e^{\sqrt{1-x^2}} \tan x$

Answer

Let y = $e^{\sqrt{1-x^2}} \tan x$, z = $e^{\sqrt{1-x^2}}$ and w = $\tan x$

Formula : $\frac{d(e^x)}{dx} = e^x, \frac{d(\tan x)}{dx} = \sec^2 x$

According to product rule of differentiation

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [\tan x \times \left(e^{\sqrt{1-x^2}} \times \frac{1}{2} \times \frac{1}{\sqrt{1-x^2}} \times (-2x)\right)] + [e^{\sqrt{1-x^2}} \times \sec^2 x]$$

$$= e^{\sqrt{1-x^2}} \times \left[\sec^2 x - \frac{x \tan x}{\sqrt{1-x^2}}\right]$$

20. Question

Differentiate each of the following w.r.t. x:

$$\frac{e^x}{1+\cos x}$$

Answer

Let y = $\frac{e^x}{1+\cos x}$, u = e^x, v = $1 + \cos x$

Formula: $\frac{d(e^x)}{dx} = e^x, \frac{d(\cos x)}{dx} = -\sin x$

According to quotient rule of differentiation

If y = $\frac{u}{v}$

$$dy/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1+\cos x)\times(e^x) - (e^x)\times(-\sin x)}{(1+\cos x)^2}$$

$$= \frac{e^x(1+\cos x+\sin x)}{(1+\cos x)^2}$$

21. Question

Differentiate each of the following w.r.t. x:

$$x^3 e^x \cos x$$

Answer

Let y = $x^3 e^x \cos x$, z = x^3 and w = $e^x \cos x$

Formula : $\frac{d(e^x)}{dx} = e^x$ and $\frac{d(\cos x)}{dx} = -\sin x$

$$\frac{dw}{dx} = [\cos x \times (e^x)] + [e^x \times (-\sin x)] = e^x[\cos x - \sin x]$$

According to product rule of differentiation

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [e^x \cos x \times (3x^2)] + [x^3 \times (e^x[\cos x - \sin x])]$$

$$= e^x x^2 \times [3\cos x + x\cos x - x\sin x]$$

$$= e^x x^2 (x\cos x - x\sin x + 3\cos x)$$

22. Question

Differentiate each of the following w.r.t. x:

$$e^{x\cos x}$$

Answer

Let y = $e^{x\cos x}$, z = xcos x

Formula : $\frac{d(e^x)}{dx} = e^x$ and $\frac{d(\cos x)}{dx} = -\sin x$

$\frac{dz}{dx} = [\cos x \times (1)] + [x \times (-\sin x)] = [\cos x - x\sin x]$ (Using product rule)

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$= [e^{x\cos x}] \times [\cos x - x\sin x]$

$= e^{x\cos x}(\cos x - x\sin x)$

Exercise 10C

1. Question

Differentiate each of the following w.r.t. x:

$\cos^{-1} 2x$

Answer

Formulae :

i) $\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$

ii) $\frac{d}{dx}(kx) = k$

Answer :

Let,

$y = \cos^{-1}2x$

and u = 2x

therefore, $y = \cos^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(\cos^{-1}u).\frac{d}{dx}(2x)$

$= \frac{-1}{\sqrt{1-u^2}}.2$

.... $\left(\because \frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}} \;\&\; \frac{d}{dx}(kx) = k\right)$

$= \frac{-2}{\sqrt{1-(2x)^2}}$

$= \frac{-2}{\sqrt{1-4x^2}}$

$\therefore \frac{dy}{dx} = \frac{-2}{\sqrt{1-4x^2}}$

2. Question

Differentiate each of the following w.r.t. x:

$\tan^{-1} x^2$

Answer

Formulae :

i) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

ii) $\frac{d}{dx}(x^n) = n.x^{n-1}$

Answer :

Let,

$y = \tan^{-1}x^2$

and u = x^2

therefore, $y = \tan^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(\tan^{-1}u).\frac{d}{dx}(x^2)$

$= \frac{1}{1+u^2}.2x$

.... $\left(\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \; \& \; \frac{d}{dx}(x^n) = n.x^{n-1}\right)$

$= \frac{2x}{1+(x^2)^2}$

$= \frac{2x}{1+x^4}$

$\therefore \frac{dy}{dx} = \frac{2x}{1+x^4}$

3. Question

Differentiate each of the following w.r.t. x:

$\sec^{-1}\sqrt{x}$

Answer

Formulae :

i) $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$

ii) $\frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}}$

Answer :

Let,

$y = \sec^{-1}\sqrt{x}$

and $u = \sqrt{x}$

therefore, $y = \sec^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\sec^{-1}u).\frac{d}{dx}(\sqrt{x})$$

$$= \frac{1}{u\sqrt{u^2-1}}.\frac{1}{2\sqrt{x}}$$

.... $\left(\because \frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}} \;\&\; \frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}}\right)$

$$= \frac{1}{\sqrt{x}\sqrt{\left(\sqrt{x}\right)^2-1}}.\left(\frac{1}{2\sqrt{x}}\right)$$

$$= \frac{1}{2\sqrt{x}.\sqrt{x}\sqrt{x-1}}$$

$$= \frac{1}{2x\sqrt{x-1}}$$

$\therefore \frac{dy}{dx} = \frac{1}{2x\sqrt{x-1}}$

4. Question

Differentiate each of the following w.r.t. x:

$\sin^{-1}\frac{x}{a}$

Answer

Formulae :

i) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

ii) $\frac{d}{dx}(kx) = k$

Answer :

Let,

$$y = \sin^{-1}\left(\frac{x}{a}\right)$$

and $u = \frac{x}{a}$

therefore, $y = \sin^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\sin^{-1}u).\frac{d}{dx}\left(\frac{x}{a}\right)$$

$$= \frac{1}{\sqrt{1-u^2}}.\frac{1}{a}$$

.... $\left(\because \frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \ \& \ \frac{d}{dx}(kx) = k\right)$

$$= \frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}.\frac{1}{a}$$

$$= \frac{1}{\sqrt{\frac{a^2-x^2}{a^2}}}.\frac{1}{a}$$

$$= \frac{a}{\sqrt{a^2-x^2}}.\frac{1}{a}$$

$$= \frac{1}{\sqrt{a^2 - x^2}}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{a^2 - x^2}}$$

5. Question

Differentiate each of the following w.r.t. x:

$$\tan^{-1}(\log x)$$

Answer

Formulae :

i) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

ii) $\frac{d}{dx}(\log x) = \frac{1}{x}$

Answer :

Let,

$$y = \tan^{-1}(\log x)$$

and u = log x

therefore, $y = \tan^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\tan^{-1}u).\frac{d}{dx}(\log x)$$

$$= \frac{1}{1+u^2}.\frac{1}{x}$$

.... $\left(\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \;\&\; \frac{d}{dx}(\log x) = \frac{1}{x}\right)$

$$= \frac{1}{1+(\log x)^2}.\frac{1}{x}$$

$$= \frac{1}{x\{1+(\log x)^2\}}$$

$$\therefore \frac{dy}{dx} = \frac{1}{x\{1+(\log x)^2\}}$$

6. Question

Differentiate each of the following w.r.t. x:

$\cot^{-1}(e^x)$

Answer

Formulae :

i) $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$

ii) $\frac{d}{dx}(e^x) = e^x$

Answer :

Let,

$y = \cot^{-1}(e^x)$

and $u = e^x$

therefore, $y = \cot^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(\cot^{-1}u) \cdot \frac{d}{dx}(e^x)$

$= \frac{-1}{1+u^2} \cdot e^x$

.... $\left(\because \frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2} \ \& \ \frac{d}{dx}(e^x) = e^x\right)$

$= \frac{-1}{1+(e^x)^2} \cdot e^x$

$= \frac{-e^x}{1+e^{2x}}$

$\therefore \frac{dy}{dx} = \frac{-e^x}{1+e^{2x}}$

7. Question

Differentiate each of the following w.r.t. x:

$\log(\tan^{-1}x)$

Answer

Formulae :

i) $\frac{d}{dx}(\log x) = \frac{1}{x}$

ii) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

Answer :

Let,

$y = \log(\tan^{-1})$

and u = $\tan^{-1}x$

therefore, $y = \log u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\log u).\frac{d}{dx}(\tan^{-1}x)$$

$$= \frac{1}{u}.\frac{1}{1+x^2}$$

.... $\left(\because \frac{d}{dx}(\log x) = \frac{1}{x} \;\&\; \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}\right)$

$$= \frac{1}{\tan^{-1}x}.\frac{1}{1+x^2}$$

$$= \frac{1}{(1+x^2).\tan^{-1}x}$$

$$\therefore \frac{dy}{dx} = \frac{1}{(1+x^2).\tan^{-1}x}$$

8. Question

Differentiate each of the following w.r.t. x:

$\cot^{-1}x^3$

Answer

Formulae :

i) $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$

ii) $\frac{d}{dx}(x^n) = n.x^{n-1}$

Answer :

Let,

$y = \cot^{-1}(x^3)$

and u = x^3

therefore, $y = \cot^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\cot^{-1}u).\frac{d}{dx}(x^3)$$

$$= \frac{-1}{1+u^2}.3x^2$$

.... $\left(\because \frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2} \;\&\; \frac{d}{dx}(x^n) = n.x^{n-1}\right)$

$$= \frac{-1}{1+(x^3)^2}.3x^2$$

$$= \frac{-3x^2}{1+x^6}$$

$\therefore \frac{dy}{dx} = \frac{-3x^2}{1+x^6}$

9. Question

Differentiate each of the following w.r.t. x:

$\sin^{-1}(\cos x)$

Answer

Formulae :

i) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

ii) $\frac{d}{dx}(\cos x) = -\sin x$

iii) $\sin^2 x + \cos^2 x = 1$

Answer :

Let,

$y = \sin^{-1}(\cos x)$

and $u = \cos x$

therefore, $y = \sin^{-1} u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(\sin^{-1} u) . \frac{d}{dx}(\cos x)$

$= \frac{1}{\sqrt{1-u^2}} . (-\sin x)$

.... $\left(\because \frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \ \& \ \frac{d}{dx}(\cos x) = -\sin x\right)$

$= \frac{1}{\sqrt{1-(\cos x)^2}} . (-\sin x)$

$= \frac{1}{\sqrt{\sin^2 x}} . (-\sin x)$ ....$(\because \sin^2 x + \cos^2 x = 1)$

$= \frac{1}{\sin x} . (-\sin x)$

$= -1$

$\therefore \frac{dy}{dx} = -1$

10. Question

Differentiate each of the following w.r.t. x:

$(1 + x^2) \tan^{-1} x$

Answer

Formulae :

i) $\frac{d}{dx}(u.v) = u \frac{dv}{dx} + v \frac{du}{dx}$

ii) $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$

iii) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

iv) $\frac{d}{dx}(k) = 0$

v) $\frac{d}{dx}(x^n) = n.x^{n-1}$

Answer :

Let,

$y = (1+x^2)\tan^{-1}x$

Let, $u = (1+x^2)$ and $v = \tan^{-1}x$

therefore, $y = u.v$

$\therefore \frac{dy}{dx} = (1+x^2).\frac{d}{dx}(\tan^{-1}x) + (\tan^{-1}u).\frac{d}{dx}(1+x^2)$

.........$\left(\because \frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx}\right)$

$= (1+x^2).\frac{1}{1+x^2} + (\tan^{-1}x)\left\{\frac{d}{dx}(1) + \frac{d}{dx}(x^2)\right\}$

.... $\left(\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}\ \&\ \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

$= 1 + (\tan^{-1}x)(0+2x)$

.... $\left(\because \frac{d}{dx}(k) = 0\ \&\ \frac{d}{dx}(x^n) = n.x^{n-1}\right)$

$= 1 + 2x\tan^{-1}x$

$\therefore \frac{dy}{dx} = 1 + 2x\,tan^{-1}x$

11. Question

Differentiate each of the following w.r.t. x:

$\tan^{-1}(\cot x)$

Answer

Formulae :

i) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

ii) $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

iii) $1+\cot^2 x=\operatorname{cosec}^2 x$

Answer :

Let,

$y=\tan^{-1}(\cot x)$

and u = cot x

therefore, $y=\tan^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx}=\frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx}=\frac{d}{du}(\tan^{-1}u).\frac{d}{dx}(\cot x)$$

$$=\frac{1}{1+u^2}.(-\operatorname{cosec}^2 x)$$

.... $\left(\because \frac{d}{dx}(\tan^{-1}x)=\frac{1}{1+x^2}\ \&\ \frac{d}{dx}(\cot x)=-\operatorname{cosec}^2 x\right)$

$$=\frac{-\operatorname{cosec}^2 x}{1+(\cot x)^2}$$

$=\frac{-\operatorname{cosec}^2 x}{\operatorname{cosec}^2 x}$ ....$(\because 1+\cot^2 x=\operatorname{cosec}^2 x)$

= -1

$\therefore \frac{dy}{dx}=-1$

12. Question

Differentiate each of the following w.r.t. x:

$\log(\sin^{-1}x^4)$

Answer

Formulae :

i) $\frac{d}{dx}(\log x)=\frac{1}{x}$

ii) $\frac{d}{dx}(\sin^{-1}x)=\frac{1}{\sqrt{1-x^2}}$

iii) $\frac{d}{dx}(x^n)=n.x^{n-1}$

Answer :

Let,

$y = \log(\sin^{-1}x^4)$

and u = x^4

therefore, $y = \log(\sin^{-1}u)$

let, $v = \sin^{-1}u$

therefore, y= log v

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ ………… By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{dv}(\log v).\frac{d}{du}(\sin^{-1}u).\frac{d}{dx}(x^4)$

$= \frac{1}{v}.\left(\frac{1}{\sqrt{1-u^2}}\right).4x^3$

………… $\left(\because \frac{d}{dx}(\log x) = \frac{1}{x}, \frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \& \frac{d}{dx}(x^n) = n.x^{n-1}\right)$

$= \frac{1}{\sin^{-1}u}.\left(\frac{1}{\sqrt{1-(x^4)^2}}\right).4x^3$

$= \frac{1}{\sin^{-1}x^4}.\left(\frac{1}{\sqrt{1-x^8}}\right).4x^3$

$= \frac{4x^3}{\sin^{-1}x^4.\sqrt{1-x^8}}$

$\therefore \frac{dy}{dx} = \frac{4x^3}{\sin^{-1}x^4.\sqrt{1-x^8}}$

13. Question

Differentiate each of the following w.r.t. x:

$(\cot^{-1}x^2)^3$

Answer

Formulae :

i) $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$

ii) $\frac{d}{dx}(x^n) = n.x^{n-1}$

Answer :

Let,

$y = (\cot^{-1}x^2)^3$

and u = x^2

therefore, $y = (\cot^{-1}u)^3$

let, $v = \cot^{-1}u$

therefore, $y = v^3$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ ………… By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(v^3).\frac{d}{du}(\cot^{-1}u).\frac{d}{dx}(x^2)$$

$$= 3v^2.\left(\frac{-1}{1+u^2}\right).2x$$

………… $\left(\because \frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2} \ \& \ \frac{d}{dx}(x^n) = n.x^{n-1}\right)$

$$= 3(\cot^{-1}u)^2.\left(\frac{-1}{1+(x^2)^2}\right).2x$$

$$= \left(\cot^{-1}(x^2)\right)^2.\frac{-6x}{1+(x^2)^2}$$

$$= \frac{-6x\left(\cot^{-1}(x^2)\right)^2}{1+x^4}$$

$$\therefore \frac{dy}{dx} = \frac{-6x\left(\cot^{-1}(x^2)\right)^2}{1+x^4}$$

14. Question

Differentiate each of the following w.r.t. x:

$\tan^{-1}(\cos\sqrt{x})$

Answer

Formulae :

i) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

ii) $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

iii) $\frac{d}{dx}(\cos x) = -\sin x$

Answer :

Let,

$y = \tan^{-1}(\cos\sqrt{x})$

and $u = \sqrt{x}$

therefore, $y = \tan^{-1}(\cos u)$

let, $v = \cos u$

therefore, $y = \tan^{-1}v$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(\tan^{-1}v).\frac{d}{du}(\cos u).\frac{d}{dx}(\sqrt{x})$$

$$= \frac{1}{1+v^2}.(-\sin u).\frac{1}{2\sqrt{x}}$$

.... $\left(\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}, \frac{d}{dx}(\cos x) = -\sin x \ \& \ \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}\right)$

$$= \frac{1}{1+(\cos u)^2}.(-\sin\sqrt{x}).\frac{1}{2\sqrt{x}}$$

$$= \frac{1}{1+(\cos\sqrt{x})^2}.(-\sin\sqrt{x}).\frac{1}{2\sqrt{x}}$$

$$= \frac{-\sin\sqrt{x}}{2\sqrt{x}\left(1+(\cos\sqrt{x})^2\right)}$$

$$\therefore \frac{dy}{dx} = \frac{-\sin\sqrt{x}}{2\sqrt{x}\left(1+(\cos\sqrt{x})^2\right)}$$

15. Question

Differentiate each of the following w.r.t. x:

$$\tan(\sin^{-1} x)$$

Answer

Formulae :

i) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

ii) $\frac{d}{dx}(\tan x) = \sec^2 x$

Answer :

Let,

$y = \tan(\sin^{-1}x)$

and u = $\sin^{-1}x$

therefore, $y = \tan u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(\tan u).\frac{d}{dx}(\sin^{-1}x)$

$= \sec^2 u.\frac{1}{\sqrt{1-x^2}}$

.... $\left(\because \frac{d}{dx}(\tan x) = \sec^2 x \,\&\, \frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}\right)$

$= \sec^2(\sin^{-1}x).\frac{1}{\sqrt{1-x^2}}$

$= \frac{\sec^2(\sin^{-1}x)}{\sqrt{1-x^2}}$

$\therefore \frac{dy}{dx} = \frac{\sec^2(\sin^{-1}x)}{\sqrt{1-x^2}}$

16. Question

Differentiate each of the following w.r.t. x:

$$e^{\tan^{-1}\sqrt{x}}$$

Answer

Formulae :

i) $\frac{d}{dx}\left(\tan^{-1}x\right) = \frac{1}{1+x^2}$

ii) $\frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}}$

iii) $\frac{d}{dx}(e^x) = e^x$

Answer :

Let,

$y = e^{\tan^{-1}\sqrt{x}}$

and $u = \sqrt{x}$

therefore, $y = e^{\tan^{-1}u}$

let, $v = \tan^{-1}u$

therefore, $y = e^v$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(e^v).\frac{d}{du}(\tan^{-1}u).\frac{d}{dx}\left(\sqrt{x}\right)$$

$$= e^v.\left(\frac{1}{1+u^2}\right).\frac{1}{2\sqrt{x}}$$

.... $\left(\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}, \frac{d}{dx}(e^x) = e^x \ \& \ \frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}}\right)$

$$= e^{\tan^{-1}u}.\left(\frac{1}{1+\left(\sqrt{x}\right)^2}\right).\frac{1}{2\sqrt{x}}$$

$$= e^{\tan^{-1}\sqrt{x}}.\left(\frac{1}{1+x}\right).\frac{1}{2\sqrt{x}}$$

$$= \frac{e^{\tan^{-1}\sqrt{x}}}{2\sqrt{x}\,(1+x)}$$

$\therefore \frac{dy}{dx} = \frac{e^{tan^{-1}\sqrt{x}}}{2\sqrt{x}\,(1+x)}$

17. Question

Differentiate each of the following w.r.t. x:

$\sqrt{\sin^{-1} x^2}$

Answer

Formulae :

i) $\frac{d}{dx}\,(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

ii) $\frac{d}{dx}\,(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

iii) $\frac{d}{dx}(x^n) = n.\,x^{n-1}$

Answer :

Let,

$y = \sqrt{\sin^{-1}x^2}$

and $u = x^2$

therefore, $y = \sqrt{\sin^{-1}u}$

let, $v = \sin^{-1}u$

therefore, $y = \sqrt{v}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{dv}\,(\sqrt{v}).\frac{d}{du}\,(\sin^{-1}u).\frac{d}{dx}\,(x^2)$

$= \frac{1}{2\sqrt{v}}.\left(\frac{1}{\sqrt{1-u^2}}\right).\,2x$

.... $\left(\because \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}, \frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}\ \&\ \frac{d}{dx}\,(x^n) = n.x^{n-1}\right)$

$= \frac{1}{2\sqrt{\sin^{-1}u}}.\left(\frac{1}{\sqrt{1-(x^2)^2}}\right).\,2x$

$$= \frac{1}{\sqrt{\sin^{-1}(x^2)}} \cdot \left(\frac{1}{\sqrt{1-x^4}}\right) . x$$

$$= \frac{x}{\sqrt{\sin^{-1}(x^2)}\,\left(\sqrt{1-x^4}\right)}$$

$$\therefore \frac{dy}{dx} = \frac{x}{\sqrt{\sin^{-1}(x^2)}\,\left(\sqrt{1-x^4}\right)}$$

18. Question

If $y = \sin^{-1}(\cos x) + \cos^{-1}(\sin x)$, prove that $\frac{dy}{dx} = -2$.

Answer

Given : $y = \sin^{-1}(\cos x) + \cos^{-1}(\sin x)$

To Prove : $\frac{dy}{dx} = -2$

Formulae :

i) $\frac{d}{dx}\ (\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

ii) $\frac{d}{dx}\ (\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$

iii) $\frac{d}{dx}\ (\cos x) = -\sin x$

iv) $\frac{d}{dx}\ (\sin x) = \cos x$

v) $\sin^2 x + \cos^2 x = 1$

vi) $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$

Answer :

Given equation,

$y = \sin^{-1}(\cos x) + \cos^{-1}(\sin x)$

Let $s = \sin^{-1}(\cos x)$ & $t = \cos^{-1}(\sin x)$

Therefore, y = s + t ....eq(1)

I. For $\sin^{-1}(\cos x)$

let $u = \cos x$

therefore, $s = \sin^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{ds}{dx} = \frac{ds}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{ds}{dx} = \frac{d}{du}(\sin^{-1}u).\frac{d}{dx}(\cos x)$

$= \frac{1}{\sqrt{1-u^2}}.(-\sin x)$

.... $\left(\because \frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \;\&\; \frac{d}{dx}(\cos x) = -\sin x\right)$

$= \frac{1}{\sqrt{1-(\cos x)^2}}.(-\sin x)$

$= \frac{1}{\sqrt{\sin^2 x}}.(-\sin x)$ ....$(\because \sin^2 x + \cos^2 x = 1)$

$= \frac{1}{\sin x}.(-\sin x)$

$= -1$

$\therefore \frac{ds}{dx} = -1$ ....eq(2)

II. For $\cos^{-1}(\sin x)$

let $u = \sin x$

therefore, $t = \cos^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dt}{dx} = \frac{dt}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dt}{dx} = \frac{d}{du}(\cos^{-1}u).\frac{d}{dx}(\sin x)$

$= \frac{-1}{\sqrt{1-u^2}}.(\cos x)$

.... $\left(\because \frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}} \;\&\; \frac{d}{dx}(\sin x) = \cos x\right)$

$= \frac{-1}{\sqrt{1-(\sin x)^2}}.(\cos x)$

$= \frac{-1}{\sqrt{\cos^2 x}}.(\cos x)$ ....($\because \sin^2 x + \cos^2 x = 1$)

$= \frac{-1}{\cos x}.(\cos x)$

$= -1$

$\therefore \frac{dt}{dx} = -1$ ....eq(2)

Differentiating eq(1) w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{d}{dx}(s+t)$

$= \frac{ds}{dx} + \frac{dt}{dx}$ ....$\left(\because \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

= -1 -1 ....from eq(2) and eq(3)

= -2

$\therefore \frac{dy}{dx} = -2$

Hence proved !!!

19. Question

Prove that $\frac{d}{dx}\left\{2x \tan^{-1}x - \log\left(1+x^2\right)\right\} = 2 \tan^{-1}x$.

Answer

To Prove : $\frac{d}{dx}\{2x\tan^{-1}x - \log(1+x^2)\} = 2\tan^{-1}x$

Formulae :

i) $\frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx}$

ii) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

iii) $\frac{d}{dx}(kx) = k$

iv) $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

v) $\frac{d}{dx}(kx) = 0$

vi) $\frac{d}{dx}(x^n) = n.x^{n-1}$

vii) $\frac{d}{dx}(u - v) = \frac{du}{dx} - \frac{dv}{dx}$

Answer :

Let,

$y = 2x\tan^{-1}x - \log(1 + x^2)$

Let $s = 2x\tan^{-1}x$ & $t = \log(1 + x^2)$

Therefore, y = s - t ....eq(1)

I. For $2x\tan^{-1}x$

let $u = 2x$ & $v = \tan^{-1}x$

therefore, $s = u.v$

Differentiating above equation w.r.t. x,

$\therefore \frac{ds}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ....$\left(\because \frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx}\right)$

$\therefore \frac{ds}{dx} = 2x\frac{d}{dx}(\tan^{-1}x) + \tan^{-1}x\frac{d}{dx}(2x)$

$= 2x.\frac{1}{1 + x^2} + \tan^{-1}x.2$

.... $\left(\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \ \& \ \frac{d}{dx}(kx) = k\right)$

$= \frac{2x}{1 + x^2} + 2\tan^{-1}x$

$\therefore \frac{ds}{dx} = \frac{2x}{1+x^2} + 2\tan^{-1}x$ ....eq(2)

II. For $\log(1 + x^2)$

let $u = (1 + x^2)$

therefore, $t = \log u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dt}{dx} = \frac{dt}{du} . \frac{du}{dx}$ .... By chain rule

$\therefore \frac{dt}{dx} = \frac{d}{du}(\log u) . \frac{d}{dx}(1 + x^2)$

$= \frac{1}{u} . \left(\frac{d}{dx}(1) + \frac{d}{dx}(x^2)\right)$ .... $\left(\because \frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

$= \frac{1}{(1 + x^2)} . (0 + 2x)$

.... $\left(\because \frac{d}{dx}(k) = 0 \ \& \ \frac{d}{dx}(x^n) = n . x^{n-1}\right)$

$= \frac{2x}{1 + x^2}$

$\therefore \frac{dt}{dx} = \frac{2x}{1+x^2}$ ....eq(3)

Differentiating eq(1) w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{d}{dx}(s - t)$

$= \frac{ds}{dx} - \frac{dt}{dx}$ ....$\left(\because \frac{d}{dx}(u - v) = \frac{du}{dx} - \frac{dv}{dx}\right)$

$= \frac{2x}{1+x^2} + 2\tan^{-1}x - \frac{2x}{1+x^2}$ ....from eq(2) and eq(3)

= 2 $\tan^{-1}x$

$\therefore \frac{dy}{dx} = 2\tan^{-1}x$

Hence proved !!!

Exercise 10D

1. Question

Differentiate each of the following w.r.t x:

$\sin^{-1}\left\{\sqrt{\frac{1 - \cos x}{2}}\right\}$

Answer

To find: Value of $\sin^{-1}\left\{\sqrt{\frac{1\text{-}\cos x}{2}}\right\}$

Formula used: (i) $\cos\theta = 2\sin^2\frac{\theta}{2}$

We have, $\sin^{-1}\left\{\sqrt{\frac{1-\cos x}{2}}\right\}$

$\Rightarrow \sin^{-1}\left\{\sqrt{\frac{2\sin^2\frac{x}{2}}{2}}\right\}$

$\Rightarrow \sin^{-1}\left\{\sqrt{\sin^2\frac{x}{2}}\right\}$

$\Rightarrow \sin^{-1}\left\{\sin\frac{x}{2}\right\}$

$\Rightarrow \frac{x}{2}$

Now, we can see that $\sin^{-1}\left\{\sqrt{\frac{1-\cos x}{2}}\right\} = \frac{x}{2}$

Now differentiating ,

$\Rightarrow \frac{d\left(\frac{x}{2}\right)}{dx}$

$\Rightarrow \frac{1}{2}\frac{dx}{dx}$

$\Rightarrow \frac{1}{2}$

Ans) $\frac{1}{2}$

2. Question

Differentiate each of the following w.r.t x:

$\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$

Answer

To find: Value of $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$

Formula used: (i) $\sin 2\theta = 2\sin\theta\cos\theta$

(ii) $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$

We have, $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\sin x}{2\cos^2\frac{x}{2}}\right)$

$\Rightarrow \tan^{-1}\left(\frac{2\sin\frac{x}{2}\ \cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right)$

$\Rightarrow \tan^{-1}\left(\tan\frac{x}{2}\right)$

$\Rightarrow \frac{x}{2}$

Now, we can see that $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right) = \frac{x}{2}$

Now differentiating ,

$\Rightarrow \frac{d\left(\frac{x}{2}\right)}{dx}$

$\Rightarrow \frac{1}{2}\frac{dx}{dx}$

$\Rightarrow \frac{1}{2}$

Ans) $\frac{1}{2}$

3. Question

Differentiate each of the following w.r.t x:

$$\cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$$

Answer

To find: Value of $\cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$

Formula used: (i) $\sin 2\theta = 2\sin\theta\cos\theta$

(ii) $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$

We have, $\cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$

$\Rightarrow \cot^{-1}\left(\frac{2\cos^2\frac{x}{2}}{\sin x}\right)$

$\Rightarrow \cot^{-1}\left(\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$

$\Rightarrow \cot^{-1}\left(\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\right)$

$\Rightarrow \cot^{-1}\left(\cot\frac{x}{2}\right)$

$\Rightarrow \frac{x}{2}$

Now, we can see that $\cot^{-1}\left(\frac{1+\cos x}{\sin x}\right) = \frac{x}{2}$

Now differentiating ,

$\Rightarrow \frac{d\left(\frac{x}{2}\right)}{dx}$

$\Rightarrow \frac{1}{2}\frac{dx}{dx}$

$\Rightarrow \frac{1}{2}$

Ans) $\frac{1}{2}$

4. Question

Differentiate each of the following w.r.t x:

$$\cot^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right)$$

Answer

To find: Value of $\cot^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right)$

Formula used: (i) $\sin 2\theta = 2\sin\theta\cos\theta$

(ii) $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$

We have, $\cot^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right)$

$$\Rightarrow \cot^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\sqrt{\frac{1+\cos x}{1+\cos x}}\right)$$

$$\Rightarrow \cot^{-1}\left(\sqrt{\frac{(1+\cos x)^2}{1-\cos^2 x}}\right)$$

$$\Rightarrow \cot^{-1}\left(\sqrt{\frac{(1+\cos x)^2}{\sin^2 x}}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{2\cos^2 \frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\right)$$

$$\Rightarrow \cot^{-1}\left(\cot\frac{x}{2}\right)$$

$$\Rightarrow \frac{x}{2}$$

Now, we can see that $\cot^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right) = \frac{x}{2}$

Now differentiating ,

$$\Rightarrow \frac{d\left(\frac{x}{2}\right)}{dx}$$

$$\Rightarrow \frac{1}{2}\frac{dx}{dx}$$

$$\Rightarrow \frac{1}{2}$$

Ans) $\frac{1}{2}$

5. Question

Differentiate each of the following w.r.t x:

$$\tan^{-1}\left(\frac{\cos x+\sin x}{\cos x-\sin x}\right)$$

Answer

To find: Value of $\tan^{-1}\left(\frac{\cos x+\sin x}{\cos x-\sin x}\right)$

Formula used: (i) $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$

We have, $\tan^{-1}\left(\frac{\cos x+\sin x}{\cos x-\sin x}\right)$

Dividing numerator and denominator by cosx

$$\Rightarrow \tan^{-1}\left(\frac{\frac{\cos x+\sin x}{\cos x}}{\frac{\cos x-\sin x}{\cos x}}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{1+\tan x}{1-\tan x}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{\tan\frac{\pi}{4}+\tan x}{1-\tan x\tan\frac{\pi}{4}}\right)$$

$$\Rightarrow \tan^{-1}\left(\tan\left(\frac{\pi}{4}+x\right)\right)$$

$$\Rightarrow \frac{\pi}{4}+x$$

Now, we can see that $\tan^{-1}\left(\frac{\cos x+\sin x}{\cos x-\sin x}\right)=\frac{\pi}{4}+x$

Now differentiating ,

$$\Rightarrow \frac{d\left(\frac{\pi}{4}+x\right)}{dx}$$

$$\Rightarrow \frac{d\left(\frac{\pi}{4}\right)}{dx}+\frac{dx}{dx}$$

$\Rightarrow 0 + 1$

$\Rightarrow 1$

Ans) 1

6. Question

Differentiate each of the following w.r.t x:

$$\cot^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$$

Answer

To find: Value of $\cot^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$

Formula used: (i) $\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$

We have, $\cot^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$

Dividing numerator and denominator by cosx

$$\Rightarrow \cot^{-1}\left(\frac{\frac{\cos x-\sin x}{\cos x}}{\frac{\cos x+\sin x}{\cos x}}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{1-\tan x}{1+\tan x}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan x\tan\frac{\pi}{4}}\right)$$

$$\Rightarrow \cot^{-1}\left(\tan\left(\frac{\pi}{4}-x\right)\right)$$

$$\Rightarrow \cot^{-1}\left(\cot\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-x\right)\right)\right)$$

$$\Rightarrow \cot^{-1}\left(\cot\left(\frac{\pi}{4}+x\right)\right)$$

$$\Rightarrow \frac{\pi}{4}+x$$

Now, we can see that $\cot^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)=\frac{\pi}{4}+x$

Now differentiating ,

$$\Rightarrow \frac{d\left(\frac{\pi}{4}+x\right)}{dx}$$

$$\Rightarrow \frac{d\left(\frac{\pi}{4}\right)}{dx} + \frac{dx}{dx}$$

$\Rightarrow 0 + 1$

$\Rightarrow 1$

Ans) 1

7. Question

Differentiate each of the following w.r.t x:

$$\cot^{-1}\left(\sqrt{\frac{1+\cos 3x}{1-\cos 3x}}\right)$$

Answer

To find: Value of $\cot^{-1}\left(\sqrt{\frac{1+\cos 3x}{1-\cos 3x}}\right)$

Formula used: (i) $1 - \cos\theta = 2\sin^2\frac{\theta}{2}$

(ii) $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$

We have, $\cot^{-1}\left(\sqrt{\frac{1+\cos 3x}{1-\cos 3x}}\right)$

$$\Rightarrow \cot^{-1}\left(\sqrt{\frac{1+\cos 3x}{2\sin^2\frac{3x}{2}}}\right)$$

$$\Rightarrow \cot^{-1}\left(\sqrt{\frac{2\cos^2\frac{3x}{2}}{2\sin^2\frac{3x}{2}}}\right)$$

$$\Rightarrow \cot^{-1}\left(\sqrt{\cot^2\left(\frac{3x}{2}\right)}\right)$$

$$\Rightarrow \cot^{-1}\left(\cot\left(\frac{3x}{2}\right)\right)$$

$\Rightarrow \frac{3x}{2}$

Now, we can see that $\cot^{-1}\left(\sqrt{\frac{1+\cos 3x}{1-\cos 3x}}\right) = \frac{3x}{2}$

Now differentiating ,

$\Rightarrow \frac{d\left(\frac{3x}{2}\right)}{dx}$

$\Rightarrow \frac{3}{2}\frac{dx}{dx}$

$\Rightarrow \frac{3}{2}$

Ans) $\frac{3}{2}$

8. Question

Differentiate each of the following w.r.t x:

$\sec^{-1}\left(\frac{1+\tan^2 x}{1-\tan^2 x}\right)$

Answer

To find: Value of $\sec^{-1}\left(\frac{1+\tan^2 x}{1-\tan^2 x}\right)$

Formula used: (i) $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

We have, $\sec^{-1}\left(\frac{1+\tan^2 x}{1-\tan^2 x}\right)$

Dividing numerator and denominator by $1+\tan^2 x$

$\Rightarrow \sec^{-1}\left(\frac{\left(\frac{1+\tan^2 x}{1+\tan^2 x}\right)}{\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)}\right)$

$\Rightarrow \sec^{-1}\left(\frac{1}{\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)}\right)$

$\Rightarrow \sec^{-1}\left(\frac{1}{\cos 2x}\right)$

$\Rightarrow \sec^{-1}(\sec 2x)$

$\Rightarrow 2x$

Now, we can see that $\sec^{-1}\left(\frac{1+\tan^2 x}{1-\tan^2 x}\right) = 2x$

Now differentiating ,

$\Rightarrow \frac{d(2x)}{dx}$

$\Rightarrow 2\ \frac{dx}{dx}$

$\Rightarrow 2$

Ans) 2

9. Question

Differentiate each of the following w.r.t x:

$\sin^{-1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)$

Answer

To find: Value of $\sin^{-1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)$

Formula used: (i) $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

We have, $\sin^{-1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)$

$\Rightarrow \sin^{-1}(\cos 2x)$

$\Rightarrow \sin^{-1}\left(\sin\ \left(\frac{\pi}{2}-2x\right)\right)$

$\Rightarrow \frac{\pi}{2}-2x$

Now, we can see that $\sin^{-1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right) = \frac{\pi}{2}-2x$

Now differentiating ,

$$\Rightarrow \frac{d\left(\frac{\pi}{2}-2x\right)}{dx}$$

$$\Rightarrow \frac{d\left(\frac{\pi}{2}\right)}{dx} - 2\frac{dx}{dx}$$

⇒ 0 - 2

⇒ -2

Ans) -2

10. Question

Differentiate each of the following w.r.t x:

$$\operatorname{cosec}^{-1}\left(\frac{1+\tan^2 x}{2\tan x}\right)$$

Answer

To find: Value of $\operatorname{cosec}^{-1}\left(\frac{1+\tan^2 x}{2\tan x}\right)$

Formula used: (i) $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

We have, $\operatorname{cosec}^{-1}\left(\frac{1+\tan^2 x}{2\tan x}\right)$

Dividing Numerator and Denominator with $1+\tan^2 x$

$$\Rightarrow \operatorname{cosec}^{-1}\left(\frac{\left(\frac{1+\tan^2 x}{1+\tan^2 x}\right)}{\left(\frac{2\tan x}{1+\tan^2 x}\right)}\right)$$

$$\Rightarrow \operatorname{cosec}^{-1}\left(\frac{(1)}{\left(\frac{2\tan x}{1+\tan^2 x}\right)}\right)$$

$$\Rightarrow \operatorname{cosec}^{-1}\left(\frac{1}{\sin 2x}\right)$$

$$\Rightarrow \operatorname{cosec}^{-1}(\operatorname{cosec} 2x)$$

⇒ 2x

Now, we can see that $\operatorname{cosec}^{-1}\left(\frac{1+\tan^2 x}{2\tan x}\right) = 2x$

Now differentiating ,

$\Rightarrow \frac{d(2x)}{dx}$

$\Rightarrow 2\frac{dx}{dx}$

$\Rightarrow 2$

Ans) 2

11. Question

Differentiate each of the following w.r.t x:

$\cot^{-1}(\cos ecx + \cot x)$

Answer

To find: Value of $\cot^{-1}(\operatorname{cosec} x + \cot x)$

Formula used: (i) $\sin 2\theta = 2\sin\theta\cos\theta$

(ii) $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$

We have, $\cot^{-1}(\operatorname{cosec} x + \cot x)$

$\Rightarrow \cot^{-1}\left(\frac{1}{\sin x} + \frac{\cos x}{\sin x}\right)$

$\Rightarrow \cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$

$\Rightarrow \cot^{-1}\left(\frac{2\cos^2\frac{x}{2}}{\sin x}\right)$

$\Rightarrow \cot^{-1}\left(\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$

$\Rightarrow \cot^{-1}\left(\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\right)$

$\Rightarrow \cot^{-1}\left(\cot\frac{x}{2}\right)$

$\Rightarrow \frac{x}{2}$