EXERCISE 6.4

In Fig. 6.16, if ∠A = ∠C, AB = 6 cm, BP = 15 cm, AP = 12 cm and CP = 4 cm, then find the lengths of PD and CD.

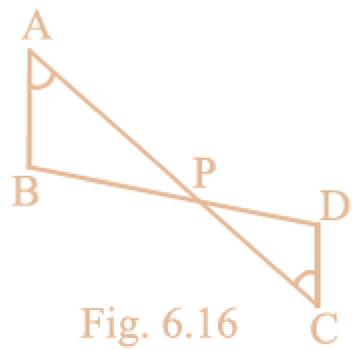


Fig. 6.16

Solution:

According to the question,
∠A = ∠C,
AB = 6 cm, BP = 15 cm,
AP = 12 cm
CP = 4 cm
From ΔAPB and ΔCPD,
∠A = ∠C
∠APB = ∠CPD [vertically opposite angles]
∴ By AAA similarity criteria,
ΔAPD ~ ΔCPD
Using basic proportionality theorem,

$$\frac{AP}{CP} = \frac{PB}{PD} = \frac{AB}{CD}$$

$$\frac{12}{4} = \frac{15}{PD} = \frac{6}{CD}$$

Considering AP/CP = PB/PD, we get,

$$\frac{12}{4} = \frac{15}{PD}$$

$$PD = \frac{15 \times 4}{12} = \frac{60}{12} = 5\text{cm}$$

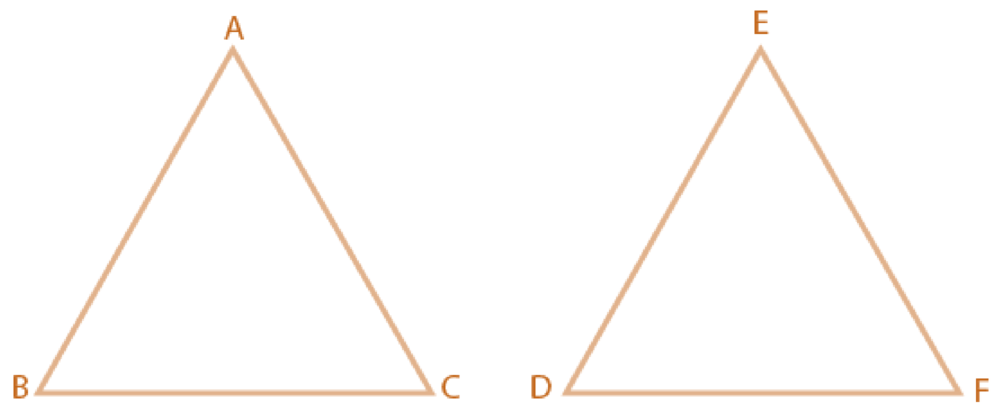
Considering, AP/CP = AB/CD

$$CD = \frac{(6 \times 4)}{12} = 2 \text{ cm}$$

Therefore,
Length of PD = 5 cm

Length of CD = 2 cm

1. It is given that Δ ABC ~ Δ EDF such that AB = 5 cm, AC = 7 cm, DF= 15 cm and DE = 12 cm. Find the lengths of the remaining sides of the triangles.
Solution:



According to the question,
ΔABC ~ ΔEDF
From property of similar triangle,
We know that, corresponding sides of ΔABC and ΔEDF are in the same ratio.
AB/ED = AC/EF = BC/DF ….(i)
According to the question,
AB = 5cm, AC = 7cm
DF = 15cm and DE = 12cm
Substituting these values in Equation (i), we get,

$$\frac{5}{12} = \frac{7}{EF} = \frac{BC}{15}$$

On taking 5/12 = 7/EF, we get,

$$\frac{5}{12} = \frac{7}{EF}$$

$$EF = \frac{12 \times 7}{5} = 16.8cm$$

On taking 5/12 = BC/15, we get,

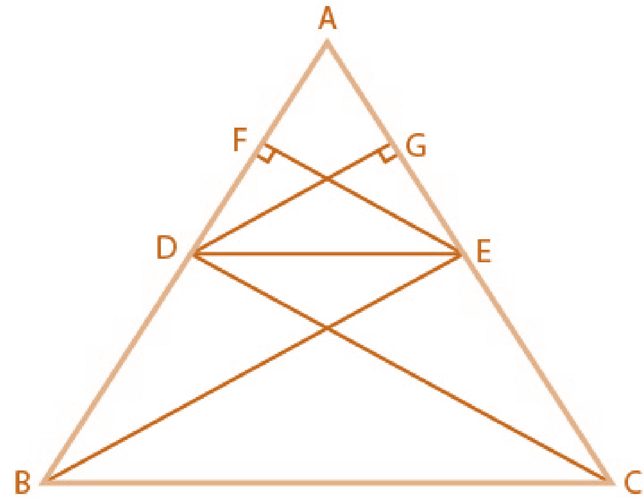
$$\frac{5}{12} = \frac{BC}{15}$$

$$BC = \frac{5 \times 15}{12} = 6.25 \text{ cm}$$

Hence, lengths of the remaining sides of the triangles are EF = 16.8 cm and BC = 6.25 cm

2. Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides, then the two sides are divided in the same ratio.
Solution:

Let a ΔABC in which a line DE parallel to BC intersects AB at D and AC at E.
To prove DE divides the two sides in the same ratio.
AD/DB = AE/EC



Construction:
Join BE, CD
Draw EF ⊥ AB and DG ⊥ AC.
We know that,
Area of triangle = ½ × base × height
Then,

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times DB \times EF}$$

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AD}{DB} \quad ...(i)$$

$$\frac{ar(\Delta ADE)}{ar(\Delta DEC)} = \frac{\frac{1}{2} \times AE \times GD}{\frac{1}{2} \times EC \times GD}$$

$$\frac{ar(\Delta ADE)}{ar(\Delta DEC)} = \frac{AE}{EC} \quad ...(ii)$$

Since,
ΔBDE and ΔDEC lie between the same parallel DE and BC and are on the same base DE.
We have,
area (ΔBDE) = area(ΔDEC) …..(iii)
From Equation (i), (ii) and (iii),
We get,
AD/DB = AE/EC
Hence proved.

3. In Fig 6.17, if PQRS is a parallelogram and AB||PS, then prove that OC||SR.

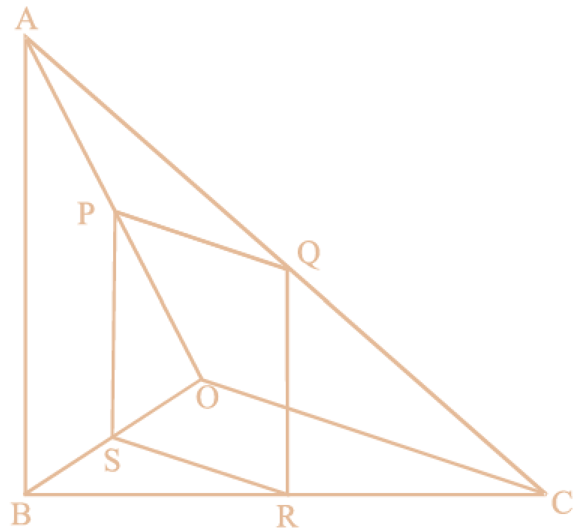
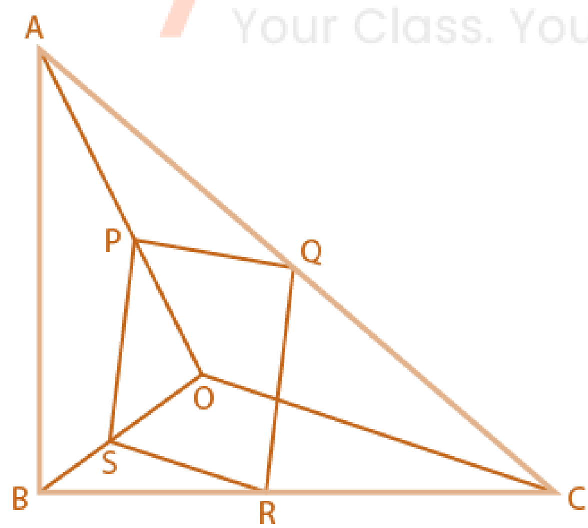


Fig. 6.17

Solution:

According to the question,
PQRS is a parallelogram,
Therefore, PQ || SR and PS || QR.
Also given, AB || PS.



To prove:

OC || SR

From ΔOPS and OAB,

PS||AB
∠POS = ∠AOB [common angle]
∠OSP = ∠OBA [corresponding angles]

ΔOPS ~ ΔOAB [by AAA similarity criteria]

Then,

Using basic proportionality theorem,

We get,

PS/AB = OS/OB …(i)

From ΔCQR and ΔCAB,

QR || PS || AB

∠QCR = ∠ACB [common angle]

∠CRQ = ∠CBA [corresponding angles]

ΔCQR ~ ΔCAB

Then, by basic proportionality theorem

$$\frac{QR}{AB} = \frac{CR}{CB}$$

$$\frac{PC}{AB} = \frac{CR}{CB} \quad ...(ii)$$

[PS ≅ QR Since, PQRS is a parallelogram,]

From Equation (i) and (ii),

$$\frac{OS}{OB} = \frac{CR}{CB}$$

$$\frac{OB}{OS} = \frac{CB}{CR}$$

Subtracting 1 from L.H.S and R.H.S, we get,

$$\frac{OB}{OS} - 1 = \frac{CB}{CR} - 1$$

$$\frac{OB - OS}{OS} = \frac{(CB - CR)}{CR}$$

$$\frac{BS}{OS} = \frac{BR}{CR}$$

SR || OC [By converse of basic proportionality theorem]

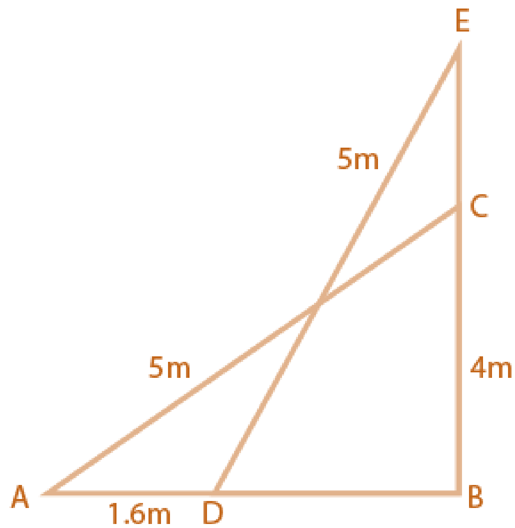
Hence proved.

4. A 5 m long ladder is placed leaning towards a vertical wall such that it reaches the wall at a point 4 m high. If the foot of the ladder is moved 1.6 m towards the wall, then find the distance by which the top of the ladder would slide upwards on the wall.

Solution:

Let the length of the ladder = AC = 5 m

Let the height of the wall on which ladder is placed = BC = 4m.

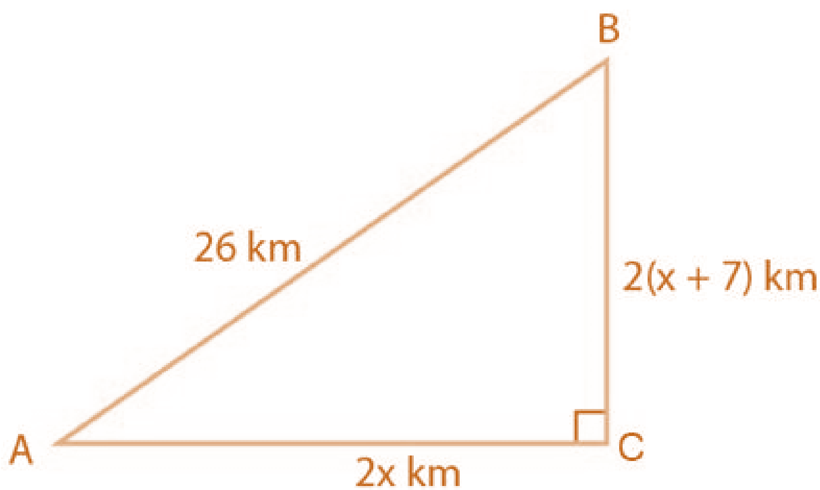


From right angled ΔEBD,
Using the Pythagoras Theorem,
$ED^2 = EB^2 + BD^2$
$(5)^2 = (EB)^2 + (14)^2$ [BD = 1.4]
$25 = (EB)^2 + 1.96$
$(EB)^2 = 25 – 1.96 = 23.04$
EB = √23.04 = 4.8
Now, we have,
EC = EB – BC = 4.8 – 4 = 0.8
Hence, the top of the ladder would slide upwards on the wall by a distance of 0.8 m.

5. For going to a city B from city A, there is a route via city C such that AC ⊥ CB, AC = 2 x km and CB = 2 (x + 7) km. It is proposed to construct a 26 km highway which directly connects the two cities A and B. Find how much distance will be saved in reaching city B from city A after the construction of the highway.

Solution:

According to the question,
AC⊥CB,
AC = 2x km,
CB = 2 (x + 7) km and AB = 26 km
Thus, we get Δ ACB right angled at C.
Now, from ΔACB,
Using Pythagoras Theorem,
$AB^2 = AC^2 + BC^2$
$\Rightarrow (26)^2 = (2x)^2 + \{2(x + 7)\}^2$
$\Rightarrow 676 = 4x^2 + 4(x^2 + 196 + 14x)$
$\Rightarrow 676 = 4x^2 + 4x^2 + 196 + 56x$
$\Rightarrow 676 = 8^2 + 56x + 196$
$\Rightarrow 8x^2 + 56x – 480 = 0$



Dividing the equation by 8, we get,

$x^2 + 7x – 60 = 0$

$x^2 + 12x - 5x - 60 = 0$

$x(x + 12)-5(x + 12) = 0$

$(x + 12)(x - 5) = 0$

$\therefore x = -12$ or $x = 5$

Since the distance can't be negative, we neglect x = -12

$\therefore x = 5$

Now,

AC = 2x = 10km

BC = 2(x + 7) = 2(5 + 7) = 24 km

Thus, the distance covered to city B from city A via city C = AC + BC

AC + BC = 10 + 24

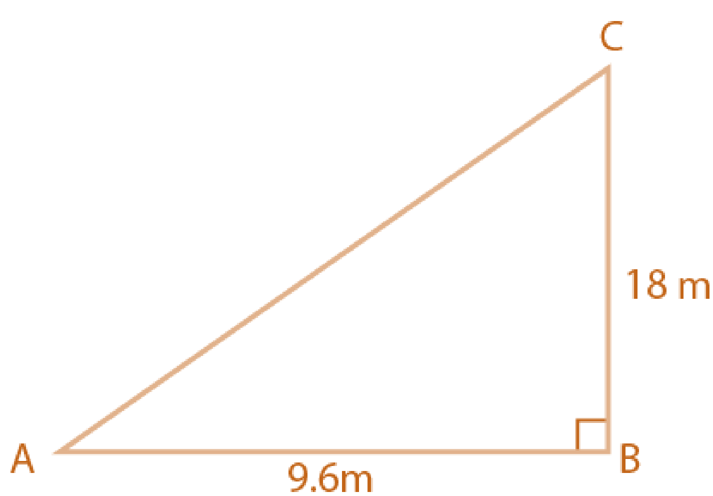
= 34 km

Distance covered to city B from city A after the highway was constructed = BA = 26 km

Therefore, the distance saved = 34 – 26 = 8 km.

6. A flag pole 18 m high casts a shadow 9.6 m long. Find the distance of the top of the pole from the far end of the shadow.

Solution:



Let MN = 18 m be the flag pole and its shadow be LM = 9.6 m.
The distance of the top of the pole, N from the far end, L of the shadow is LN.
In right angled ΔLMN,
$LN^2 = LM^2 + MN^2$ [by Pythagoras theorem]
$\Rightarrow LN^2 = (9.6)^2 + (18)^2$
$\Rightarrow LN^2 = 9.216 + 324$
$\Rightarrow LN^2 = 416.16$
$\therefore LN = \sqrt{416.16} = 20.4$ m
Hence, the required distance is 20.4 m