$\Rightarrow \left(3x-\frac{1}{2}\right)^2 = 6+\frac{1}{4}=\frac{25}{4}=\left(\frac{5}{2}\right)^2$

$\Rightarrow 3x-\frac{1}{2}=\pm\frac{5}{2}$ (Taking square root on both sides)

$\Rightarrow 3x-\frac{1}{2}=\frac{5}{2}$ or $3x-\frac{1}{2}=-\frac{5}{2}$

$\Rightarrow 3x=\frac{5}{2}+\frac{1}{2}=\frac{6}{2}=3$ or $3x=-\frac{5}{2}+\frac{1}{2}=-\frac{4}{2}=-2$

$\Rightarrow x=1$ or $x=-\frac{2}{3}$

Hence, 1 and $-\frac{2}{3}$ are the roots of the given equation.

7. $8x^2-14x-15=0$

Sol:

$8x^2-14x-15=0$

$\Rightarrow 16x^2-28x-30=0$ (Multiplying both sides by 2)

$\Rightarrow 16x^2-28x=30$

$\Rightarrow (4x)^2-2\times 4x\times\frac{7}{2}+\left(\frac{7}{2}\right)^2=30+\left(\frac{7}{2}\right)^2$ [Adding $\left(\frac{7}{2}\right)^2$ on both sides]

$\Rightarrow \left(4x-\frac{7}{2}\right)^2=30+\frac{49}{4}=\frac{169}{4}=\left(\frac{13}{2}\right)^2$

$\Rightarrow 4x-\frac{7}{2}=\pm\frac{13}{2}$ (Taking square root on both sides)

$\Rightarrow 4x-\frac{7}{2}=\frac{13}{2}$ or $4x-\frac{7}{2}=\frac{13}{2}$

$\Rightarrow 4x=\frac{13}{2}+\frac{7}{2}=\frac{20}{2}=10$ or $4x=-\frac{13}{2}+\frac{7}{2}=-\frac{6}{2}=-3$

$\Rightarrow x=\frac{5}{2}$ or $x=-\frac{3}{4}$

Hence, $\frac{5}{2}$ and $-\frac{3}{4}$ are the roots of the given equation.

8. $7x^2+3x-4=0$

Sol:

$7x^2+3x-4=0$

$\Rightarrow 49x^2+21x-28=0$ (Multiplying both sides by 7)

$\Rightarrow 49x^2 + 21x = 28$

$\Rightarrow (7x)^2 + 2\times 7x \times \frac{3}{2} + \left(\frac{3}{2}\right)^2 = 28 + \left(\frac{3}{2}\right)^2$ [Adding $\left(\frac{3}{2}\right)^2$ on both sides]

$\Rightarrow \left(7x + \frac{3}{2}\right)^2 = 28 + \frac{9}{4} = \frac{121}{4} = \left(\frac{11}{2}\right)^2$

$\Rightarrow 7x + \frac{3}{2} = \pm\frac{11}{2}$ (Taking square root on both sides)

$\Rightarrow 7x + \frac{3}{2} = \frac{11}{2}$ *or* $7x + \frac{3}{2} = -\frac{11}{2}$

$\Rightarrow 7x = \frac{11}{2} - \frac{3}{2} = \frac{8}{2} = 4$ or $7x = -\frac{11}{2} - \frac{3}{2} = -\frac{14}{2} = -7$

$\Rightarrow x = \frac{4}{7}$ *or* $x = -1$

Hence, $\frac{4}{7}$ and -1 are the roots of the given equation.

9. $3x^2 - 2x - 1 = 0$

Sol:

$3x^2 - 2x - 1 = 0$

$\Rightarrow 9x^2 - 6x - 3 = 0$ (Multiplying both sides by 3)

$\Rightarrow 9x^2 - 6x = 3$

$\Rightarrow (3x)^2 - 2\times 3x \times 1 + 1^2 = 3 + 1^2$ [Adding 1^2 on both sides]

$\Rightarrow (3x-1)^2 = 3 + 1 = 4 = (2)^2$

$\Rightarrow 3x - 1 = \pm 2$ (Taking square root on both sides)

$\Rightarrow 3x - 1 = 2$ or $3x - 1 = -2$

$\Rightarrow 3x = 3$ or $3x = -1$

$\Rightarrow x = 1$ or $x = -\frac{1}{3}$

Hence, 1 and $-\frac{1}{3}$ are the roots of the given equation.

10. $5x^2 - 6x - 2 = 0$

Sol:

$5x^2 - 6x - 2 = 0$

$\Rightarrow 25x^2 - 30x - 10 = 0$ (Multiplying both sides by 5)

$\Rightarrow 25x^2 - 30x = 10$

$\Rightarrow (5x)^2 - 2\times 5x\times 3 + 3^2 = 10 + 3^2$ (Adding 3^2 on both sides)

$\Rightarrow (5x-3)^2 = 10 + 9 - 19$

$\Rightarrow 5x - 3 = \pm\sqrt{19}$ (Taking square root on both)

$\Rightarrow 5x - 3 = \sqrt{19}$ or $5x - 3 = -\sqrt{19}$

$\Rightarrow 5x = 3 + \sqrt{19}$ or $5x = 3 - \sqrt{19}$

$\Rightarrow x = \frac{3+\sqrt{19}}{5}$ or $x = \frac{3-\sqrt{19}}{5}$

Hence, $\frac{3+\sqrt{19}}{5}$ and $\frac{3-\sqrt{19}}{5}$ are $\frac{3-\sqrt{19}}{5}$ are the roots of the given equation.

11. $\frac{2}{x^2} - \frac{5}{x} + 2 = 0$

Sol:

$\frac{2}{x^2} - \frac{5}{x} + 2 = 0$

$\Rightarrow \frac{2 - 5x + 2x^2}{x^2} = 0$

$\Rightarrow 2x^2 - 5x + 2 = 0$

$\Rightarrow 4x^2 - 10x + 4 = 0$ (Multiplying both sides by 2)

$\Rightarrow 4x^2 - 10x = -4$

$\Rightarrow (2x)^2 - 2\times 2x\times \frac{5}{2} + \left(\frac{5}{2}\right)^2 = -4 + \left(\frac{5}{2}\right)^2$ [Adding $\left(\frac{5}{2}\right)^2$ on both sides]

$\Rightarrow \left(2x - \frac{5}{2}\right)^2 = -4 + \frac{25}{4} = \frac{9}{4} = \left(\frac{3}{2}\right)^2$

$\Rightarrow 2x - \frac{5}{2} = \pm\frac{3}{2}$ (Taking square root on both sides)

$\Rightarrow 2x - \frac{5}{2} = \frac{3}{2}$ or $2x - \frac{5}{2} = -\frac{3}{2}$

$\Rightarrow 2x = \frac{3}{2} + \frac{5}{2} = \frac{8}{2} = 4$ or $2x = -\frac{3}{2} + \frac{5}{2} = \frac{2}{2} = 1$

$\Rightarrow x = 2$ or $x = \frac{1}{2}$

Hence, 2 and $\frac{1}{2}$ are the roots of the given equation.

12. $4x^2+4bx-(a^2-b^2)=0$

Sol:

$4x^2+4bx-(a^2-b^2)=0$

$\Rightarrow 4x^2+4bx=a^2-b^2$

$\Rightarrow (2x)^2+2\times 2x\times b+b^2=a^2-b^2+b^2$ (Adding b^2 on both sides)

$\Rightarrow (2x+b)^2=a^2$

$\Rightarrow 2x+b=\pm a$ (Taking square root on both sides)

$\Rightarrow 2x+b=a \text{ or } 2x+b=-a$

$\Rightarrow 2x=a-b \text{ or } 2x=-a-b$

$\Rightarrow x=\frac{a-b}{2} \text{ or } x=-\frac{a+b}{2}$

Hence, $\frac{a-b}{2}$ and $-\frac{a+b}{2}$ are the roots of the given equation.

13. $x^2-(\sqrt{2}+1)x+\sqrt{2}=0$

Sol:

$x^2-(\sqrt{2}+1)x+\sqrt{2}=0$

$\Rightarrow x^2-(\sqrt{2}+1)x=-\sqrt{2}$

$\Rightarrow x^2-2\times x\times\left(\frac{\sqrt{2}+1}{2}\right)+\left(\frac{\sqrt{2}+1}{2}\right)^2=-\sqrt{2}+\left(\frac{\sqrt{2}+1}{2}\right)^2$

[Adding $\left(\frac{\sqrt{2}+1}{2}\right)^2$ on both sides]

$\Rightarrow \left[x-\left(\frac{\sqrt{2}+1}{2}\right)\right]^2=\frac{-4\sqrt{2}+2+1+2\sqrt{2}}{4}=\frac{2-2\sqrt{2}+1}{4}=\left(\frac{\sqrt{2}-1}{2}\right)^2$

$\Rightarrow x-\left(\frac{\sqrt{2}+1}{2}\right)=\pm\left(\frac{\sqrt{2}-1}{2}\right)$ (Taking square root on both sides)

$\Rightarrow x-\left(\frac{\sqrt{2}+1}{2}\right)=\left(\frac{\sqrt{2}-1}{2}\right) \text{ or } x-\left(\frac{\sqrt{2}+1}{2}\right)=-\left(\frac{\sqrt{2}-1}{2}\right)$

$\Rightarrow x=\frac{\sqrt{2}+1}{2}+\frac{\sqrt{2}-1}{2} \text{ or } x=\frac{\sqrt{2}+1}{2}-\frac{\sqrt{2}-1}{2}$

$\Rightarrow x = \frac{2\sqrt{2}}{2} = \sqrt{2}$ or $x = \frac{2}{2} = 1$

Hence, $\sqrt{2}$ and 1 are the roots of the given equation.

14. $\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$

Sol:

$\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$

$\Rightarrow 2x^2 - 3\sqrt{2}x - 4 = 0$ (Multiplying both sides by $\sqrt{2}$)

$\Rightarrow 2x^2 - 3\sqrt{2}x = 4$

$\Rightarrow \left(\sqrt{2}x\right)^2 - 2\times\sqrt{2}x\times\frac{3}{2} + \left(\frac{3}{2}\right)^2 = 4 + \left(\frac{3}{2}\right)^2$ [Adding $\left(\frac{3}{2}\right)^2$ on both sides]

$\Rightarrow \left(\sqrt{2}x - \frac{3}{2}\right)^2 = 4 + \frac{9}{4} = \frac{25}{4} = \left(\frac{5}{2}\right)^2$

$\Rightarrow \sqrt{2}x - \frac{3}{2} = \pm\frac{5}{2}$ (Taking square root on both sides)

$\Rightarrow \sqrt{2}x - \frac{3}{2} = \frac{5}{2}$ or $\sqrt{2}x - \frac{3}{2} = -\frac{5}{2}$

$\Rightarrow \sqrt{2}x = \frac{5}{2} + \frac{3}{2} = \frac{8}{2} = 4$ or $\sqrt{2}x = -\frac{5}{2} + \frac{3}{2} = -\frac{2}{2} = -1$

$\Rightarrow x = \frac{4}{\sqrt{2}} = 2\sqrt{2}$ or $x = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

Hence, $2\sqrt{2}$ and $-\frac{\sqrt{2}}{2}$ are the roots of the given equation.

15. $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

Sol:

$\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

$\Rightarrow 3x^2 + 10\sqrt{3}x + 21 = 0$ (Multiplying both sides by $\sqrt{3}$)

$\Rightarrow 3x^2 + 10\sqrt{3}x = -21$

$\Rightarrow \left(\sqrt{3}x\right)^2 + 2\times\sqrt{3}x\times5 + 5^2 = -21 + 5^2$ (Adding 5^2 on both sides)

$\Rightarrow \left(\sqrt{3}x + 5\right)^2 = 21 + 25 = 4 = 2^2$

$\Rightarrow \sqrt{3}x + 5 = \pm2$ (Taking square root on both sides)

$\Rightarrow \sqrt{3}x + 5 = 2$ or $\sqrt{3}x + 5 = -2$

$\Rightarrow \sqrt{3}x = -3$ or $\sqrt{3}x = -7$

$\Rightarrow x = -\frac{3}{\sqrt{3}} = -\sqrt{3}$ or $x = -\frac{7}{\sqrt{3}} = -\frac{7\sqrt{3}}{3}$

Hence, $-\sqrt{3}$ and $-\frac{7\sqrt{3}}{3}$ are the roots of the given equation.

16. By using the method of completing the square, show that the equation $2x^2 + x + 4 = 0$ has no real roots.

Sol:

$2x^2 + x + 4 = 0$

$\Rightarrow 4x^2 + 2x + 8 = 0$ (Multiplying both sides by 2)

$\Rightarrow 4x^2 + 2x = -8$

$\Rightarrow (2x)^2 + 2 \times 2x \times \frac{1}{2} + \left(\frac{1}{2}\right)^2 = -8 + \left(\frac{1}{2}\right)^2$ [Adding $\left(\frac{1}{2}\right)^2$ on both sides]

$\Rightarrow \left(2x + \frac{1}{2}\right)^2 = -8 + \frac{1}{4} = -\frac{31}{4} < 0$

But, $\left(2x + \frac{1}{2}\right)^2$ cannot be negative for any real value of x.

So, there is no real value of x satisfying the given equation.

Hence, the given equation has no real roots.

Exercise -10C

1. $2x^2 - 7x + 6 = 0$

Sol:

(i) $2x^2 - 7x + 6 = 0$

Here,

$a = 2,$

$b = -7,$

$c = 6$

Discriminant D is diven by:

$D = b^2 - 4ac$

$= (-7)^2 - 4 \times 2 \times 6$

$= 49 - 48$

$= 1$

(ii) $3x^2 - 2x + 8 = 0$

Here,

$a = 3,$

$b = -2,$

$c = 8$

Discriminant D is given by:

$D = b^2 - 4ac$

$= (-2)^2 - 4 \times 3 \times 8$

$= 4 - 96$

$= -92$

(iii) $2x^2 - 5\sqrt{2x} + 4 = 0$

Here,

$a = 2,$

$b = -5\sqrt{2},$

$c = 4$

Discriminant D is given by:

$D = b^2 - 4ac$

$= \left(-5\sqrt{2}\right)^2 - 4 \times 2 \times 4$

$= (25 \times 2) - 32$

$= 50 - 32$

$= 18$

(iv) $\sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$

Here,

$a = \sqrt{3}$

$b = 2\sqrt{2},$

$c = -2\sqrt{3}$

Discriminant D is given by:

$D = b^2 - 4ac$

$= \left(2\sqrt{2}\right)^2 - 4 \times \sqrt{3} \times \left(-2\sqrt{3}\right)$

$= (4 \times 2) + (8 \times 3)$

$= 8 + 24$

$= 32$

(v) $(x-1)(2x-1) = 0$

$\Rightarrow 2x^2 - 3x + 1 = 0$

Comparing it with $ax^2 + bx + c = 0,$ we get

$a = 2, b = -3 \text{ and } c = 1$

$\therefore$ Discriminant, $D = b^2 - 4ac = (-3)^2 - 4 \times 2 \times 1 = 9 - 8 = 1$

(vi) $1 - x = 2x^2$

$\Rightarrow 2x^2 + x - 1 = 0$

Here,

$a = 2,$

$b = 1,$

$c = -1$

Discriminant D is given by:

$D = b^2 - 4ac$

$= 1^2 - 4 \times 2(-1)$

$= 1 + 8$

$= 9$

Find the roots of the each of the following equations, if they exist, by applying the quadratic formula:

2. $x^2 - 4x - 1 = 0$

Sol:

Given:

$x^2 - 4x - 1 = 0$

On comparing it with $ax^2 + bx + c = 0,$ we get:

$a = 1, b = -4 \text{ and } c = -1$

Discriminant D is given by:

$D = (b^2 - 4ac)$

$= (-4)^2 - 4 \times 1 \times (-1)$

$= 16 + 4$

$= 20$

$= 20 > 0$

Hence, the roots of the equation are real.

Roots α and β are given by:

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(-4) + \sqrt{20}}{2 \times 1} = \frac{4 + 2\sqrt{5}}{2} = \frac{2(2 + \sqrt{5})}{2} = (2 + \sqrt{5})$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(-4) - \sqrt{20}}{2} = \frac{4 - 2\sqrt{5}}{2} = \frac{2(2 - \sqrt{5})}{2} = (2 - \sqrt{5})$$

Thus, the roots of the equation are $\left(2+\sqrt{5}\right)$ and $\left(2-\sqrt{5}\right)$.

3. $x^2-6x+4=0$

Sol:

Given:

$x^2-6x+4=0$

On comparing it with $ax^2+bx+c=0,$ we get:

$a=1, b=-6 \text{ and } c=4$

Discriminant D is given by:

$D=\left(b^2-4ac\right)$

$=(-6)^2-4\times1\times4$

$=36-16$

$=20>0$

Hence, the roots of the equation are real.

Roots α and β are given by:

$$\alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-(-6)+\sqrt{20}}{2\times1}=\frac{6+2\sqrt{5}}{2}=\frac{2\left(3+\sqrt{3}\right)}{2}=\left(3+\sqrt{5}\right)$$

$$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-(-6)-\sqrt{20}}{2}=\frac{6-2\sqrt{5}}{2}=\frac{2\left(3-\sqrt{5}\right)}{2}=\left(3-\sqrt{5}\right)$$

Thus, the roots of the equation are $\left(3+2\sqrt{5}\right)$ and $\left(3-2\sqrt{5}\right)$.

4. $2x^2+x-4=0.$

Sol:

The given equation is $2x^2+x-4=0.$

Comparing it with $ax^2+bx+c=0,$ we get

$a=2, b=1$ and $c=-4$

$\therefore$ Discriminant, $D=b^2-4ac=(1)^2-4\times2\times(-4)=1+32=33>0$

So, the given equation has real roots.

Now, $\sqrt{D}=\sqrt{33}$

$$\therefore \alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-1+\sqrt{33}}{2\times2}=\frac{-1+\sqrt{33}}{4}$$

$$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-1-\sqrt{33}}{2\times2}=\frac{-1-\sqrt{33}}{4}$$

Hence, $\frac{-1+\sqrt{33}}{4}$ and $\frac{-1-\sqrt{33}}{4}$ are the roots of the given equation.

5. $25x^2+30x+7=0$

Sol:

Given:

$25x^2+30x+7=0$

On comparing it with $ax^2+bx+x=0$, we get;

$a=25, b=30 \text{ and } c=7$

Discriminant D is given by:

$D=\left(b^2-4ac\right)$

$=30^2-4\times25\times7$

$=900-700$

$=200$

$=200>0$

Hence, the roots of the equation are real.

Roots α and β are given by:

$\alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-30+\sqrt{200}}{2\times25}=\frac{-30+10\sqrt{2}}{50}=\frac{10\left(-3+\sqrt{2}\right)}{50}=\frac{\left(-3+\sqrt{2}\right)}{5}$

$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-30-\sqrt{200}}{2\times25}=\frac{-30-10\sqrt{2}}{50}=\frac{10\left(-3-\sqrt{2}\right)}{50}=\frac{\left(-3-\sqrt{2}\right)}{5}$

Thus, the roots of the equation are $\frac{\left(-3+\sqrt{2}\right)}{5}$ *and* $\frac{\left(-3-\sqrt{2}\right)}{5}$.

6. $16x^2+24x+1$

Sol:

Given:

$16x^2+24x+1$

$\Rightarrow 16x^2-24x-1=0$

On comparing it with $ax^2+bx+x=0$, we get;

$a=16, b=-24 \text{ and } c=-1$

Discriminant D is given by:

$D=\left(b^2-4ac\right)$

$=\left(-24\right)^2-4\times16\times\left(-1\right)$

$= 576 + (64)$

$= 640 > 0$

Hence, the roots of the equation are real.

Roots α and β are given by:

$$\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-(-24)+\sqrt{640}}{2\times 16} = \frac{24+8\sqrt{10}}{32} = \frac{8(3+\sqrt{10})}{32} = \frac{(3+\sqrt{10})}{4}$$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-(-24)-\sqrt{640}}{2\times 16} = \frac{24-8\sqrt{10}}{32} = \frac{8(3-\sqrt{10})}{32} = \frac{(3-\sqrt{10})}{4}$$

Thus, the roots of the equation are $\frac{(3+\sqrt{10})}{4}$ *and* $\frac{(3-\sqrt{10})}{4}$.

7. $15x^2 - 28 = x$

Sol:

Given:

$15x^2 - 28 = x$

$\Rightarrow 15x^2 - x - 28 = 0$

On comparing it with $ax^2 + bx + c = 0$, we get;

$a = 25, b = -1$ *and* $c = -28$

Discriminant D is given by:

$D = (b^2 - 4ac)$

$= (-1)^2 - 4\times 15\times(-28)$

$= 1 - (-1680)$

$= 1 + 1680$

$= 1681$

$= 1681 > 0$

Hence, the roots of the equation are real.

Roots α and β are given by:

$$\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-(-1)+\sqrt{1681}}{2\times 25} = \frac{1+41}{30} = \frac{42}{30} = \frac{7}{5}$$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-(-1)-\sqrt{1681}}{2\times 25} = \frac{1-41}{30} = \frac{-40}{30} = \frac{-4}{3}$$

Thus, the roots of the equation are $\frac{7}{5}$ and $\frac{-4}{3}$.

8. $2x^2 - 2\sqrt{2}x + 1 = 0$

Sol:

The given equation is $2x^2 - 2\sqrt{2}x + 1 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 2, b = -2\sqrt{2}$ *and* $c = 1$

$\therefore$ Discriminant, $D = b^2 - 4ac = \left(-2\sqrt{2}\right)^2 - 4 \times 2 \times 1 = 8 - 8 = 0$

So, the given equation has real roots.

Now, $\sqrt{D} = 0$

$$\therefore \alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-\left(-2\sqrt{2}\right)}{2 \times 2} = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-\left(-2\sqrt{2}\right) - \sqrt{0}}{2 \times 2} = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2}$$

Hence, $\frac{\sqrt{2}}{2}$ is the repeated root of the given equation.

9. $\sqrt{2}x^2 + 7 + 5\sqrt{2} = 0.$

Sol:

The given equation is $\sqrt{2}x^2 + 7 + 5\sqrt{2} = 0.$

Comparing it with $ax^2 + bx + c = 0$, we get

$a = \sqrt{2}, b = 7$ *and* $c = 5\sqrt{2}$

$\therefore$ Discriminant, $D = b^2 - 4ac = (7)^2 - 4 \times \sqrt{2} \times 5\sqrt{2} = 49 - 40 = 9 > 0$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{9} = 3$

$$\therefore \alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-7 + 3}{2 \times \sqrt{2}} = \frac{-4}{2\sqrt{2}} = -\sqrt{2}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-7 - 3}{2 \times \sqrt{2}} = \frac{-10}{2\sqrt{2}} = -\frac{5\sqrt{2}}{2}$$

Hence, $-\sqrt{2}$ and $-\frac{5\sqrt{2}}{2}$ are the root of the given equation.

10. $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

Sol:

Given:

$\sqrt{3}x^2+10x-8\sqrt{3}=0$

On comparing it with $ax^2+bx+x=0$, we get;

$a=\sqrt{3}, b=10 \; and \; c=-8\sqrt{3}$

Discriminant D is given by:

$D=\left(b^2-4ac\right)$

$=(10)^2-4\times\sqrt{3}\times\left(-8\sqrt{3}\right)$

$=100+96$

$=196>0$

Hence, the roots of the equation are real.

Roots α and β are given by:

$$\alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-10+\sqrt{196}}{2\sqrt{3}}=\frac{-10+14}{2\sqrt{3}}=\frac{4}{2\sqrt{3}}=\frac{2}{\sqrt{3}}=\frac{2}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$$

$$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-(10)-\sqrt{196}}{2\sqrt{3}}=\frac{-10-14}{2\sqrt{3}}=\frac{-24}{2\sqrt{3}}=\frac{-12}{\sqrt{3}}=\frac{-12}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{-12\sqrt{3}}{3}=-4\sqrt{3}$$

Thus, the roots of the equation are $\frac{2\sqrt{3}}{3}$ and $-4\sqrt{3}$.

11. $\sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}=0.$

Sol:

The given equation is $\sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}=0.$

Comparing it with $ax^2+bx+c=0$, we get

$a=\sqrt{3}, b=-2\sqrt{2} \; and \; c=-2\sqrt{3}$

$\therefore$ Discriminant, $D=b^2-4ac=\left(-2\sqrt{2}\right)^2-4\times\sqrt{3}\times\left(-2\sqrt{3}\right)=8+24=32>0$

So, the given equation has real roots.

Now, $\sqrt{D}=\sqrt{32}=4\sqrt{2}$

$$\therefore \alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-\left(-2\sqrt{2}\right)+4\sqrt{2}}{2\times\sqrt{3}}=\frac{6\sqrt{2}}{2\sqrt{3}}=\sqrt{6}$$

$$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-\left(-2\sqrt{2}\right)-4\sqrt{2}}{2\times\sqrt{3}}=\frac{-2\sqrt{2}}{2\sqrt{3}}=-\frac{\sqrt{6}}{3}$$

Hence, $\sqrt{6}$ and $-\frac{\sqrt{6}}{3}$ are the root of the given equation.

12. $2x^2 + 6\sqrt{3}x - 60 = 0.$

Sol:

The given equation is $2x^2 + 6\sqrt{3}x - 60 = 0.$

Comparing it with $ax^2 + bx + c = 0,$ we get

$a = 2, b = 6\sqrt{3}$ *and* $c = -60$

$\therefore$ Discriminant, $D = b^2 - 4ac = \left(6\sqrt{3}\right)^2 - 4\times 2\times(-60) = 180 + 480 = 588 > 0$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{588} = 14\sqrt{3}$

$$\therefore \alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-6\sqrt{3}+14\sqrt{3}}{2\times 2} = \frac{8\sqrt{3}}{4} = 2\sqrt{3}$$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-6\sqrt{3}-14\sqrt{3}}{2\times 2} = \frac{-20\sqrt{3}}{4} = -5\sqrt{3}$$

Hence, $2\sqrt{3}$ and $-5\sqrt{3}$ are the root of the given equation.

13. $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

Sol:

The given equation is $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

Comparing it with $ax^2 + bx + c = 0,$ we get

$a = 4\sqrt{3}, b = 5$ *and* $c = -2\sqrt{3}$

$\therefore$ Discriminant, $D = b^2 - 4ac = 5^2 - 4\times 4\sqrt{3}\times\left(-2\sqrt{3}\right) = 25 + 96 = 121 > 0$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{121} = 11$

$$\therefore \alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-5-11}{2\times 4\sqrt{3}} = \frac{6}{8\sqrt{3}} = \frac{\sqrt{3}}{4}$$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-5-11}{2\times 4\sqrt{3}} = \frac{-16}{8\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

Hence, $\frac{\sqrt{3}}{4}$ and $-\frac{2\sqrt{3}}{3}$ are the root of the given equation.

14. $3x^2 - 2\sqrt{6}x + 2 = 0$

Sol:

The given equation is $3x^2 - 2\sqrt{6}x + 2 = 0$

Comparing it with $ax^2 + bx + c = 0,$ we get

$a = 3, b = -2\sqrt{6}$ *and* $c = 2$

$\therefore$ Discriminant, $D = b^2 - 4ac = \left(-2\sqrt{6}\right)^2 - 4 \times 3 \times 2 = 24 - 24 = 0$

So, the given equation has real roots.

Now, $\sqrt{D} = 0$

$$\therefore \alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-\left(-2\sqrt{6}\right)+0}{2\times 3} = \frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{3}$$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-\left(-2\sqrt{6}\right)}{2\times 3} = \frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{3}$$

Hence, $\frac{\sqrt{6}}{3}$ are the repeated of the given equation.

15. $2\sqrt{3}x^2 - 5x + \sqrt{3} = 0$

Sol:

The given equation is $2\sqrt{3}x^2 - 5x + \sqrt{3} = 0$

Comparing it with $ax^2 + bx + c = 0,$ we get

$a = 2\sqrt{3}, b = -5$ *and* $c = \sqrt{3}$

$\therefore$ Discriminant, $D = b^2 - 4ac = (-5)^2 - 4 \times 2\sqrt{3} \times \sqrt{3} = 25 - 25 = 1 > 0$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{1} = 1$

$$\therefore \alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-(-5+1)+1}{2\times 2\sqrt{3}} = \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-(-5)-1}{2\times 2\sqrt{3}} = \frac{4}{4\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Hence, $\frac{\sqrt{3}}{2}$ and $\frac{\sqrt{3}}{3}$ are the roots of the given equation.

16. $x^2 + x + 2 = 0.$

Sol:

The given equation is $x^2 + x + 2 = 0.$

Comparing it with $ax^2 + bx + c = 0,$ we get

$a = 1, b = 1$ and $c = 2$

$\therefore$ Discriminant $D = b^2 - 4ac = 1^2 - 4 \times 1 \times 2 = 1 - 8 = -7 < 0$

Hence, the given equation has no real roots (or real roots does not exist).

17. $2x^2+ax-a^2=0.$

Sol:

The given equation is $2x^2+ax-a^2=0.$

Comparing it with $Ax^2+Bx+C=0,$ we get

$A=2, B=a \text{ and } C=-a^2$

$\therefore$ Discriminant, $D=B^2-4AC=a^2-4\times 2\times -a^2=a^2+8a^2=9a^2\geq 0$

So, the given equation has real roots.

Now, $\sqrt{D}=\sqrt{9a^2}=3a$

$\therefore \alpha=\frac{-B+\sqrt{D}}{2A}=\frac{-a+3a}{2\times 2}=\frac{2a}{4}=\frac{a}{2}$

$\beta=\frac{-B-\sqrt{D}}{2A}=\frac{-a-3a}{2\times 2}=\frac{-4a}{4}=-a$

Hence, $\frac{a}{2}$ and $-a$ are the roots of the given equation.

18. $x^2-\left(\sqrt{3}+1\right)x+\sqrt{3}=0.$

Sol:

The given equation is $x^2-\left(\sqrt{3}+1\right)x+\sqrt{3}=0.$

Comparing it with $ax^2+bx+c=0,$ we get

$a=1, b=-\left(\sqrt{3}+1\right)$ and $c=\sqrt{3}$

$\therefore$ Discriminant,

$D=b^2-4ac=\left[-\left(\sqrt{3}+1\right)\right]^2-4\times 1\times\sqrt{3}=3+1+2\sqrt{3}-4\sqrt{3}=3-2\sqrt{3}+1=\left(\sqrt{3}-1\right)^2>0$

So, the given equation has real roots.

Now, $\sqrt{D}=\sqrt{\left(\sqrt{3}-1\right)^2}=\sqrt{3}-1$

$\therefore \alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-\left[-\left(\sqrt{3}+1\right)\right]+\left(\sqrt{3}-1\right)}{2\times 1}=\frac{\sqrt{3}+1+\sqrt{3}-1}{2}=\frac{2\sqrt{3}}{2}=\sqrt{3}$

$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-\left[-\left(\sqrt{3}+1\right)\right]-\left(\sqrt{3}-1\right)}{2\times 1}=\frac{\sqrt{3}+1-\sqrt{3}+1}{2}=\frac{2}{2}=1$

Hence, $\sqrt{3}$ and 1 are the roots of the given equation.

19. $2x^2+5\sqrt{3}x+6=0.$

Sol:

The given equation is $2x^2+5\sqrt{3}x+6=0.$

Comparing it with $ax^2+bx+c=0,$ we get

$a=2, b=5\sqrt{3}$ *and* $c=6$

$\therefore$ Discriminant, $D=b^2-4ac=\left(5\sqrt{3}\right)^2-4\times2\times6=75-48=27>0$

So, the given equation has real roots.

Now, $\sqrt{D}=\sqrt{27}=3\sqrt{3}$

$$\therefore \alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-5\sqrt{3}+3\sqrt{3}}{2\times2}=\frac{-2\sqrt{3}}{4}=-\frac{\sqrt{3}}{2}$$

$$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-5\sqrt{3}-3\sqrt{3}}{2\times2}=\frac{-8\sqrt{3}}{4}=-2\sqrt{3}$$

Hence, $-\frac{\sqrt{3}}{2}$ and $-2\sqrt{3}$ are the roots of the given equation.

20. $3x^2-2x+2=0.$b

Sol:

The given equation is $3x^2-2x+2=0.$

Comparing it with $ax^2+bx+c=0,$ we get

$a=3, b=-2$ and $c=2$

$\therefore$ Discriminant $D=b^2-4ac=(-2)^2-4\times3\times2=4-24=-20<0$

Hence, the given equation has no real roots (or real roots does not exist).

21. $x+\frac{1}{x}=3, x\neq0$

Sol:

The given equation is

$$x+\frac{1}{x}=3, x\neq0$$

$$\Rightarrow \frac{x^2+1}{x}=3$$

$$\Rightarrow x^2+1=3x$$

$$\Rightarrow x^2-3x+1=0$$

This equation is of the form $ax^2+bx+c=0,$ where, $a=1, b=-3$ and $c=1.$

$\therefore$ Discriminant, $D=b^2-4ac=(-3)^2-4\times1\times1=9-4=5>0$

So, the given equation has real roots.

Now, $\sqrt{D}=\sqrt{5}$

$$\therefore a=\frac{-b+\sqrt{D}}{2a}=\frac{-(-3)+\sqrt{5}}{2\times 1}=\frac{3+\sqrt{5}}{2}$$

$$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-(-3)-\sqrt{5}}{2\times 1}=\frac{3-\sqrt{5}}{2}$$

Hence, $\frac{3+\sqrt{5}}{2}$ and $\frac{3-\sqrt{5}}{2}$ are the roots of the given equation.

22. $\frac{1}{x}-\frac{1}{x-2}=3, x\neq 0,2$

Sol:

The given equation is

$$\frac{1}{x}-\frac{1}{x-2}=3, x\neq 0,2$$

$$\Rightarrow \frac{x-2-x}{x(x-2)}=3$$

$$\Rightarrow \frac{-2}{x^2-2x}=3$$

$$\Rightarrow -2=3x^2-6x$$

$$\Rightarrow 3x^2-6x+2=0$$

This equation is of the form $ax^2+bx+c=0$, where $a=3, b=-6$ and $c=2$.

$\therefore$ Discriminant, $D=b^2-4ac=(-6)^2-4\times 3\times 2=36-24=12>0$

So, the given equation has real roots.

Now, $\sqrt{D}=\sqrt{12}=2\sqrt{3}$

$$\therefore a=\frac{-b+\sqrt{D}}{2a}=\frac{-(-6)+2\sqrt{3}}{2\times 3}=\frac{6+2\sqrt{3}}{6}=\frac{3+\sqrt{3}}{3}$$

$$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-(-6)-2\sqrt{3}}{2\times 3}=\frac{6-2\sqrt{3}}{6}=\frac{3-\sqrt{3}}{3}$$

Hence, $\frac{3+\sqrt{3}}{3}$ and $\frac{3-\sqrt{3}}{3}$ are the roots of the given equation.

23. $x-\frac{1}{x}=3, x\neq 0$

Sol:

The given equation is

$$x-\frac{1}{x}=3, x\neq 0$$

$$\Rightarrow \frac{x^2-1}{x}=3$$

$$\Rightarrow x^2-1=3x$$

$$\Rightarrow x^2-3x-1=0$$

This equation is of the form $ax^2+bx+c=0,$ where $a=1, b=-3$ and $c=-1.$

$\therefore$ Discriminant, $D=b^2-4ac=(-3)^2-4\times 1\times(-1)=9+4=13>0$

So, the given equation has real roots.

Now, $\sqrt{D}=\sqrt{13}$

$$\therefore a=\frac{-b+\sqrt{D}}{2a}=\frac{-(-3)+\sqrt{13}}{2\times 1}=\frac{3+\sqrt{13}}{2}$$

$$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-(-3)-\sqrt{13}}{2\times 1}=\frac{3-\sqrt{13}}{2\times 1}=\frac{3-\sqrt{13}}{2}$$

Hence, $\frac{3+\sqrt{13}}{3}$ and $\frac{3-\sqrt{13}}{3}$ are the roots of the given equation.

24. $\frac{m}{n}x^2\frac{n}{m}=1-2x$

Sol:

The given equation is

$$\frac{m}{n}x^2\frac{n}{m}=1-2x$$

$$\Rightarrow \frac{m^2x^2+n^2}{mn}=1-2x$$

$$\Rightarrow m^2x^2+n^2=mn-2mnx$$

$$\Rightarrow m^2x^2+2mnx+n^2-mn=0$$

This equation is of the form $ax^2+bx+c=0,$ where $a=m^2, b=2mn$ and $c=n^2-mn$

$\therefore$ Discriminant,

$$D=b^2-4ac=(2mn)^2-4\times m^2\times(n^2-mn)=4m^2n^2-4m^2n^2+4m^3n^2=4m^3n>0$$

So, the given equation has real roots.

Now, $\sqrt{D}=\sqrt{4m^3n}=2m\sqrt{mn}$

$$\therefore a=\frac{-b+\sqrt{D}}{2a}=\frac{-2mn+2m\sqrt{mn}}{2\times m^2}=\frac{2mn\left(-n+\sqrt{mn}\right)}{2m^2}=\frac{-n+\sqrt{mn}}{m}$$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-2mn-2m\sqrt{mn}}{2\times m^2} = \frac{-2m\left(n+\sqrt{mn}\right)}{2m^2} = \frac{-n+\sqrt{mn}}{m}$$

Hence, $\frac{-n+\sqrt{mn}}{m}$ and $\frac{-n-\sqrt{mn}}{m}$ are the roots of the given equation.

25. $36x^2 - 12ax + \left(a^2 - b^2\right) = 0$

Sol:

The given equation is $36x^2 - 12ax + \left(a^2 - b^2\right) = 0$

Comparing it with $Ax^2 + Bx + C = 0,$ we get

$A = 36, B = -12a \text{ and } C = a^2 - b^2$

$\therefore$ Discriminant,

$D = B^2 - 4AC = (-12a)^2 - 4\times 36\times\left(a^2 - b^2\right) = 144a^2 - 144a^2 + 144b^2 = 144b^2 > 0$

So, the given equation has real roots

Now, $\sqrt{D} = \sqrt{144b^2} = 12b$

$$\therefore \alpha = \frac{-B+\sqrt{D}}{2A} = \frac{-(-12a)+12b}{2\times 36} = \frac{12(a+b)}{72} = \frac{a+b}{6}$$

$$\beta = \frac{-B-\sqrt{D}}{2A} = \frac{-(-12a)-12b}{2\times 36} = \frac{12(a-b)}{72} = \frac{a-b}{6}$$

Hence, $\frac{a+b}{6}$ and $\frac{a-b}{6}$ are the roots of the given equation.

26. $x^2 - 2ax + \left(a^2 - b^2\right) = 0$

Sol:

Given:

$x^2 - 2ax + \left(a^2 - b^2\right) = 0$

On comparing it with $Ax^2 + Bx + C = 0,$ we get:

$A = 1, B = -2a$ and $C = \left(a^2 - b^2\right)$

Discriminant D is given by:

$D = B^2 - 4AC$

$= (-2a)^2 - 4\times 1\times\left(a^2 - b^2\right)$

$= 4a^2 - 4a^2 + 4b^2$

$= 4b^2 > 0$

Hence, the roots of the equation are real.

Roots α and β are given by:

$$\alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-(-2a)+\sqrt{4b^2}}{2\times1}=\frac{2a+2b}{2}=\frac{2(a+b}{2}=(a+b)$$

$$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-(-2a)-\sqrt{4b^2}}{2\times1}=\frac{2a-2b}{2}=\frac{2(a-b)}{2}=(a-b)$$

Hence, the roots of the equation are $(a+b)$ and $(a-b)$.

27. $x^2-2ax-(4b^2-a^2)=0$

Sol:

The given equation is $x^2-2ax-(4b^2-a^2)=0$

Comparing it with $Ax^2+Bx+C=0,$ we get

$A=1, B=-2a \text{ and } C=-(4b^2-a^2)$

$\therefore$ Discriminant,

$B^2-4AC=(-2a)^2-4\times1\times\left[-(4b^2-a^2)\right]=4a^2+16b^2-4a^2=16b^2>0$

So, the given equation has real roots

Now, $\sqrt{D}=\sqrt{16b^2}=4b$

$$\therefore\alpha=\frac{-B+\sqrt{D}}{2A}=\frac{-(-2a)+4b}{2\times1}=\frac{2(a+2b)}{2}=a+2b$$

$$\beta=\frac{-B-\sqrt{D}}{2A}=\frac{-(-2a)-4b}{2\times1}=\frac{2(a-2b)}{2}=a-2b$$

Hence, $a+2b$ and $a-2b$ are the roots of the given equation.

28. $x^2+6x-(a^2+2a-8)=0.$

Sol:

The given equation is $x^2+6x-(a^2+2a-8)=0.$

Comparing it with $Ax^2+Bx+C=0,$ we get

$A=1, B=6 \text{ and } C=-(a^2+2a-8)$

$\therefore$ Discriminant, $D=$

$B^2-4AC=6^2-4\times1\times\left[-(a^2+2a-8)\right]=36+4a^2+8a-32=4a^2+8a-32=4a^2+8a+4$

$=4(a^2+2a+1)=4(a+1)^2>0$

So, the given equation has real roots

Now, $\sqrt{D}=\sqrt{4(a+1)^2}=2(a+1)$

$\therefore \alpha = \dfrac{-B+\sqrt{D}}{2A} = \dfrac{-6+2(a+1)}{2\times 1} = \dfrac{2a-4}{2} = a-2$

$\beta = \dfrac{-B-\sqrt{D}}{2A} = \dfrac{-6-2(a+1)}{2\times 1} = \dfrac{-2a-8}{2} = -a-4 = -(a+4)$

Hence, $(a-2)$ and $-(a+4)$ are the roots of the given equation.

29. $x^2+5x-(a^2+a-6)=0.$

Sol:

The given equation is $x^2+5x-(a^2+a-6)=0.$

Comparing it with $Ax^2+Bx+C=0,$ we get

$A=1, B=5 \ and \ C=-(a^2+a-8)$

$\therefore$ Discriminant, $D=$

$B^2-4AC=5^2-4\times 1\times\left[-(a^2+a-6)\right]=25+4a^2+4a-24=4a^2+4a^2+4a+1$

$=(2a+1)^2>0$

So, the given equation has real roots

Now, $\sqrt{D}=\sqrt{(2a+1)^2}=2a+1$

$\therefore \alpha = \dfrac{-B+\sqrt{D}}{2A} = \dfrac{-5+2a+1}{2\times 1} = \dfrac{2a-4}{2} = a-2$

$\beta = \dfrac{-B-\sqrt{D}}{2A} = \dfrac{-5-(2a+1)}{2\times 1} = \dfrac{-2a-6}{2} = -a-3 = -(a+3)$

Hence, $(a-2)$ and $-(a+3)$ are the roots of the given equation.

30. $x^2-4ax-b^2+4a^2=0.$

Sol:

The given equation is $x^2-4ax-b^2+4a^2=0.$

Comparing it with $Ax^2+Bx+C=0,$ we get

$A=1, B=-4a \ and \ C=-b^2+4a^2$

$\therefore$ Discriminant, $D=B^2-4AC=(-4a)^2-4\times 1\times(-b^2+4a^2)=16a^2+4b^2-16a^2=4b^2>0$

So, the given equation has real roots

Now, $\sqrt{D}=\sqrt{4b^2}=2b$

$\therefore \alpha = \dfrac{-B+\sqrt{D}}{2A} = \dfrac{-(-4a)+2b}{2\times 1} = \dfrac{4a+2b}{2} = 2a+b$

$$\beta = \frac{-B-\sqrt{D}}{2A} = \frac{-(-4a)-2b}{2\times 1} = \frac{4a-2b}{2} = 2a-b$$

Hence, $(2a+b)$ and $(2a-b)$ are the roots of the given equation.

31. $4x^2 - 4a^2x + (a^4 - b^4) = 0.$

Sol:

The given equation is $4x^2 - 4a^2x + (a^4 - b^4) = 0.$

Comparing it with $Ax^2 + Bx + C = 0,$ we get

$A = 4, B = -4a^2 \text{ and } C = a^4 - b^4$

$\therefore$ Discriminant, $B^2 - 4AC = (-4a^2)^2 - 4\times 4\times(a^2 - b^4) = 16a^4 - 16a^4 + 16b^4 = 16b^4 > 0$

So, the given equation has real roots

Now, $\sqrt{D} = \sqrt{16b^4} = 4b^2$

$$\therefore \alpha = \frac{-B+\sqrt{D}}{2A} = \frac{-(-4a^2)+4b^2}{2\times 4} = \frac{4(a^2+b^2)}{8} = \frac{a^2+b^2}{2}$$

$$\beta = \frac{-B-\sqrt{D}}{2A} = \frac{-(-4a^2)-4b^2}{2\times 4} = \frac{4(a^2-b^2)}{8} = \frac{a^2-b^2}{2}$$

Hence, $\frac{1}{2}(a^2+b^2)$ and $\frac{1}{2}(a^2-b^2)$ are the roots of the given equation.

32. $4x^2 - 4bx - (a^2 - b^2) = 0.$

Sol:

The given equation is $4x^2 - 4bx - (a^2 - b^2) = 0.$

Comparing it with $Ax^2 + Bx + C = 0,$ we get

$A = 4, B = 4b \text{ and } C = -(a^2 - b^2)$

$\therefore$ Discriminant,

$D = B^2 - 4AC = (4b)^2 - 4\times 4\times\left[-(a^2 - b^2)\right] = 16b^2 + 16a^2 - 16b^2 = 16a^2 > 0$

So, the given equation has real roots

Now, $\sqrt{D} = \sqrt{16a^2} = 4a$

$$\therefore \alpha = \frac{-B+\sqrt{D}}{2A} = \frac{-4b+4a}{2\times 4} = \frac{4(a-b)}{8} = \frac{a-b}{2}$$

$$\beta = \frac{-B-\sqrt{D}}{2A} = \frac{-4b-4a}{2\times 4} = \frac{-4(a+b)}{8} = -\frac{a+b}{2}$$

Hence, $\frac{1}{2}(a-b)$ and $-\frac{1}{2}(a+b)$ are the roots of the given equation.

33. $x^2-(2b-1)x+(b^2-b-20)=0.$

Sol:

The given equation is $x^2-(2b-1)x+(b^2-b-20)=0.$

Comparing it with $Ax^2+Bx+C=0,$ we get

$A=1, B=-(2b-1)$ *and* $C=b^2-b-20$

$\therefore$ Discriminant,

$D=B^2-4AC=[-(2b-1)]^2-4\times1\times(b^2-b-20)=4b^2-4b+1-4b^2+4b+80=81>0$

So, the given equation has real roots

Now, $\sqrt{D}=\sqrt{18}=9$

$\therefore \alpha=\frac{-B+\sqrt{D}}{2A}=\frac{-[-(2b-1)]+9}{2\times1}=\frac{2b+8}{2}=b+4$

$\beta=\frac{-B-\sqrt{D}}{2A}=\frac{-[-(2b-1)]-9}{2\times1}=\frac{2b-10}{2}=b-5$

Hence, $(b+4)$ and $(b-5)$ are the roots of the given equation.

34. $3a^2x^2+8abx+4b^2=0$

Sol:

Given:

$3a^2x^2+8abx+4b^2=0$

On comparing it with $Ax^2+Bx+C=0,$ we get:

$A=3a^2, B=8ab$ *and* $C=4b^2$

Discriminant D is given by:

$D=(B^2-4AC)$

$=(8ab)^2-4\times3a^2\times4b^2$

$=16a^2b^2>0$

Hence, the roots of the equation are real.

Roots α and β are given by:

$\alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-8ab+\sqrt{16a^2b^2}}{2\times3a^2}=\frac{-8ab+4ab}{6a^2}=\frac{-4ab}{6a^2}=\frac{-2b}{3a}$

$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-8ab-\sqrt{16a^2b^2}}{2\times3a^2}=\frac{-8ab-4ab}{6a^2}=\frac{-12ab}{6a^2}=\frac{-2b}{a}$

Thus, the roots of the equation are $\frac{-2b}{3a}$ *and* $\frac{-2b}{a}$.

35. $a^2b^2x^2 - \left(4b^4 - 3a^4\right)x - 12a^2b^2 = 0, a \neq 0 \, and \, b \neq 0$

Sol:

The given equation is $a^2b^2x^2 - \left(4b^4 - 3a^4\right)x - 12a^2b^2 = 0.$

Comparing it with $Ax^2 + Bx + C = 0,$ we get

$A = a^2b^2, B = -\left(4b^4 - 3a^4\right) \text{ and } c = -12a^2b^2$

$\therefore$ Discriminant,

$$B^2 - 4AC = \left[-\left(4b^4 - 3a^4\right)\right]^2 - 4 \times a^2b^2 \times \left(-12a^2b^2\right) = 16b^8 - 24a^4b^4 + 9a^8 + 48a^4b^4$$

$$= 16b^8 + 24a^4b^4 + 9a^8 = \left(4b^4 + 3a^4\right)^2 > 0$$

So, the given equation has real roots

Now, $\sqrt{D} = \sqrt{\left(4b^4 + 3a^4\right)^2} = 4b^4 + 3a^4$

$$\therefore \alpha = \frac{-B + \sqrt{D}}{2A} = \frac{-\left[-\left(4b^4 - 3a^4\right)\right] + \left(4b^4 + 3a^4\right)}{2 \times a^2b^2} = \frac{8b^4}{2a^2b^2} = \frac{4b^2}{a^2}$$

$$\beta = \frac{-B - \sqrt{D}}{2A} = \frac{-\left[-\left(4b^4 - 3a^4\right)\right] - \left(4b^4 + 3a^4\right)}{2 \times a^2b^2} = \frac{-6a^4}{2a^2b^2} = -\frac{3a^2}{b^2}$$

Hence, $\frac{4b^2}{a^2}$ and $-\frac{3a^2}{b^2}$ are the roots of the given equation.

36. $12abx^2 - \left(9a^2 - 8b^2\right)x - 6ab = 0, where \, a \neq 0 \, and \, b \neq 0$

Sol:

Given:

$12abx^2 - \left(9a^2 - 8b^2\right)x - 6ab = 0$

On comparing it with $Ax^2 + Bx + C = 0,$ we get:

$A = 12ab, B = -\left(9a^2 - 8b^2\right)$ and $C = -6ab$

Discriminant D is given by:

$$D = B^2 - 4AC$$

$$= \left[-\left(9a^2 - 8b^2\right)\right]^2 - 4 \times 12ab \times \left(-6ab\right)$$

$$= 81a^4 - 144a^2b^2 + 64b^4 + 288a^2b^2$$

$$= 81a^4 + 144a^2b^2 + 64b^4$$

$$= \left(9a^2 + 8b^2\right)^2 > 0$$

Hence, the roots of the equation are equal.
Roots α and β are given by:

$$\alpha=\frac{-B+\sqrt{D}}{2A}=\frac{-\left[-\left(9a^2-8b^2\right)\right]+\sqrt{\left(9a^2+8b^2\right)^2}}{2\times 12ab}=\frac{9a^2-8b^2+9a^2+8b^2}{24ab}=\frac{18a^2}{24ab}=\frac{3a}{4b}$$

$$\beta=\frac{-B-\sqrt{D}}{2A}=\frac{-\left[-\left(9a^2-8b^2\right)\right]-\sqrt{\left(9a^2+8b^2\right)^2}}{2\times 12ab}=\frac{9a^2-8b^2-9a^2-8b^2}{24ab}=\frac{-16a^2}{24ab}=\frac{-2b}{3a}$$

Thus, the roots of the equation are $\frac{3a}{4b}$ and $\frac{-2b}{3a}$.

Exercise - 10D

1. Find the nature of roots of the following quadratic equations:

(i) $2x^2-8x+5=0.$

(ii) $3x^2-2\sqrt{6}x+2=0.$

(iii) $5x^2-4x+1=0.$

(iv) $5x(x-2)+6=0$

(v) $12x^2-4\sqrt{15}x+5=0$

(vi) $x^2-x+2=0.$

Sol:

(i) The given equation is $2x^2-8x+5=0.$

This is of the form $ax^2+bx+c=0,$ where $a=2, b=-8$ *and* $c=5.$

$\therefore$ Discriminant, $D=b^2-4ac=(-8)^2-4\times 2\times 5=64-40=24>0$

Hence, the given equation has real and unequal roots.

(ii) The given equation is $3x^2-2\sqrt{6}x+2=0.$

This is of the form $ax^2+bx+c=0,$ where $a=3, b=-2\sqrt{6}$ *and* $c=2.$

$\therefore$ Discriminant, $D=b^2-4ac=\left(-2\sqrt{6}\right)^2-4\times 3\times 2=24-24=0$

Hence, the given equation has real and equal roots.

(iii) The given equation is $5x^2-4x+1=0.$

This is of the form $ax^2+bx+c=0,$ where $a=5, b=-4$ *and* $c=1.$

$\therefore$ Discriminant, $D=b^2-4ac=(-4)^2-4\times 5\times 1=16-20=-4<0$

Hence, the given equation has no real roots.

(iv) The given equation is

$5x(x-2)+6=0$

$\Rightarrow 5x^2-10x+6=0$

This is of the form $ax^2+bx+c=0,$ where $a=5, b=-10$ *and* $c=6.$

$\therefore$ Discriminant, $D=b^2-4ac=(-10)^2-4\times5\times6=100-120=-20<0$

Hence, the given equation has no real roots.

(v) The given equation is $12x^2-4\sqrt{15}x+5=0$

This is of the form $ax^2+bx+c=0,$ where $a=12, b=-4\sqrt{15}$ *and* $c=5.$

$\therefore$ Discriminant, $D=b^2-4ac=\left(-4\sqrt{15}\right)^2-4\times12\times5=240-240=0$

Hence, the given equation has real and equal roots.

(vi) The given equation is $x^2-x+2=0.$

This is of the form $ax^2+bx+c=0,$ where $a=1, b=-1$ *and* $c=2.$

$\therefore$ Discriminant, $D=b^2-4ac=(-1)^2-4\times1\times2=1-8=-7<0$

Hence, the given equation has no real roots.

2. If a and b are distinct real numbers, show that the quadratic equations $2(a^2+b^2)x^2+2(a+b)x+1=0.$ has no real roots.

Sol:

The given equation is $2(a^2+b^2)x^2+2(a+b)x+1=0.$

$\therefore D=\left[2(a+b)\right]^2-4\times2(a^2+b^2)\times1$

$=4(a^2+2ab+b^2)-8(a^2+b^2)$

$=4a^2+8ab+4b^2-8a^2-8b^2$

$=-4a^2+8ab-4b^2$

$=-4(a^2-2ab+b^2)$

$=-4(a-b)^2<0$

Hence, the given equation has no real roots.

3. Show that the roots of the equation $x^2+px-q^2=0$ are real for all real values of p and q.

Sol:

Given:

$x^2+px-q^2=0$

Here,

$a=1, b=p$ *and* $c=-q^2$

Discriminant D is given by:

$D=\left(b^2-4ac\right)$

$=p^2-4\times1\times\left(-q^2\right)$

$=\left(p^2+4q^2\right)>0$

$D>0$ for all real values of p and q.

Thus, the roots of the equation are real.

4. For what values of k are the roots of the quadratic equation $3x^2+2kx+27=0$ real and equal?

Sol:

Given:

$3x^2+2kx+27=0$

Here,

$a=3, b=2k \text{ and } c=27$

It is given that the roots of the equation are real and equal; therefore, we have:

$D=0$

$\Rightarrow(2k)^2-4\times3\times27=0$

$\Rightarrow4k^2-324=0$

$\Rightarrow4k^2=324$

$\Rightarrow k^2=81$

$\Rightarrow k=\pm9$

$\therefore k=9 \text{ or } k=-9$

5. For what value of k are the roots of the quadratic equation $kx\left(x-2\sqrt{5}\right)+10=0$ real and equal.

Sol:

The given equation is

$kx\left(x-2\sqrt{5}\right)+10=0$

$\Rightarrow kx^2-2\sqrt{5}kx+10=0$

This is of the form $ax^2+bx+c=0,$ where $a=k, b=-2\sqrt{5}k \text{ and } c=10$.

$\therefore D=b^2-4ac=\left(-2\sqrt{5}k\right)^2-4\times k\times10=20k^2-40k$

The given equation will have real and equal roots if $D=0$.

$\therefore 20k^2 - 40k = 0$

$\Rightarrow 20k(k-2) = 0$

$\Rightarrow k = 0 \text{ or } k - 2 = 0$

$\Rightarrow k = 0 \text{ or } k = 2$

But, for $k = 0$, we get $10 = 0$, which is not true

Hence, 2 is the required value of k.

6. For what values of p are the roots of the equation $4x^2 + px + 3 = 0$. real and equal?

Sol:

The given equation is $4x^2 + px + 3 = 0$.

This is of the form $ax^2 + bx + c = 0$, where $a = 4, b = p \text{ and } c = 3$.

$\therefore D = b^2 - 4ac = p^2 - 4 \times 4 \times 3 = p^2 - 48$

The given equation will have real and equal roots if $D = 0$.

$\therefore p^2 - 48 = 0$

$\Rightarrow p^2 = 48$

$\Rightarrow p = \pm\sqrt{48} = \pm 4\sqrt{3}$

Hence, $4\sqrt{3}$ and $-4\sqrt{3}$ are the required values of p.

7. Find the nonzero value of k for which the roots of the quadratic equation $9x^2 - 3kx + k = 0$. are real and equal.

Sol:

The given equation is $9x^2 - 3kx + k = 0$.

This is of the form $ax^2 + bx + c = 0$, where $a = 9, b = -3k \text{ and } c = k$.

$\therefore D = b^2 - 4ac = (-3k)^2 - 4 \times 9 \times k = 9k^2 - 36k$

The given equation will have real and equal roots if $D = 0$.

$\therefore 9k^2 - 36k = 0$

$\Rightarrow 9k(k-4) = 0$

$\Rightarrow k = 0 \text{ or } k - 4 = 0$

$\Rightarrow k = 0 \text{ or } k = 4$

But, $k \neq 0$ (Given)

Hence, the required values of k is 4.

8. Find the values of k for which the quadratic equation $(3k+1)x^2 + 2(k+1)x + 1 = 0$. has real and equal roots.

Sol:

The given equation is $(3k+1)x^2+2(k+1)x+1=0.$

This is of the form $ax^2+bx+c=0,$ where $a=3k+1, b=2(k+1)$ *and* $c=1.$

$\therefore D=b^2-4ac$

$=[2(k+1)]^2-4\times(3k+1)\times 1$

$=4(k^2+2k+1)-4(3k+1)$

$=4k^2+8k+4-12k-4$

$=4k^2-4k$

The given equation will have real and equal roots if $D=0.$

$\therefore 4k^2-4k=0$

$\Rightarrow 4k(k-1)=0$

$\Rightarrow k=0 \text{ or } k-1=0$

$\Rightarrow k=0 \text{ or } k=1$

Hence, 0 and 1are the required values of k.

9. Find the value of p for which the quadratic equation $(2p+1)x^2-(7p+2)x+(7p-3)=0.$ has real and equal roots.

Sol:

The given equation is $(2p+1)x^2-(7p+2)x+(7p-3)=0.$

This is of the form $ax^2+bx+c=0,$ where $a=2p+1, b=-(7p+2)$ *and* $c=7p-3.$

$\therefore D=b^2-4ac$

$=-[-(7p+2)]^2-4\times(2p+1)\times(7p-3)$

$=(49p^2+28p+4)-4(14p^2+p-3)$

$=49p^2+28p+4-56p^2-4p+12$

$=-7p^2+24p+16$

The given equation will have real and equal roots if $D=0.$

$\therefore -7p^2+24p+16=0$

$\Rightarrow 7p^2-24p-16=0$

$\Rightarrow 7p^2-28p+4p-16=0$

$\Rightarrow 7p(p-4)+4(p-4)=0$

$\Rightarrow (p-4)(7p+4)=0$

$\Rightarrow p-4=0 \text{ or } 7p+4=0$

$\Rightarrow p = 4 \ or \ p = -\frac{4}{7}$

Hence, 4 and $-\frac{4}{7}$ are the required values of p.

10. Find the values of p for which the quadratic equation $(p+1)x^2 - 6(p+1)x + 3(p+9) = 0.$, $p \neq -1$ has equal roots. Hence find the roots of the equation.

Sol:

The given equation is $(p+1)x^2 - 6(p+1)x + 3(p+9) = 0.$

This is of the form $ax^2 + bx + c = 0,$ where $a = p+1, b = -6(p+1) \ and \ c = 3(p+9).$

$\therefore D = b^2 - 4ac$

$= [-6(p+1)]^2 - 4 \times (p+1) \times 3(p+9)$

$= 12(p+1)[3(p+1) - (p+9)]$

$= 12(p+1)(2p-6)$

The given equation will have real and equal roots if $D = 0.$

$\therefore 12(p+1)(2p-6) = 0$

$\Rightarrow p+1 = 0 \ or \ 2p - 6 = 0$

$\Rightarrow p = -1 \ or \ p = 3$

But, $p \neq -1$ (Given)

Thus, the value of p is 3

Putting $p = 3,$ the given equation becomes $4x^2 - 24x + 36 = 0$

$4x^2 - 24x + 36 = 0$

$\Rightarrow 4(x^2 - 6x + 9) = 0$

$\Rightarrow (x-3)^2 = 0$

$\Rightarrow x - 3 = 0$

$\Rightarrow x = 3$

Hence, 3 is the repeated root of this equation.

11. If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0.$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k.

Sol:

It is given that -5 is a root of the quadratic equation $2x^2 + px - 15 = 0.$

$\therefore 2(-5)^2 + p \times (-5) - 15 = 0$

$\Rightarrow -5p+35=0$

$\Rightarrow p=7$

The roots of the equation $px^2+px+k=0=0$ are equal.

$\therefore D=0$

$\Rightarrow p^2-4pk=0$

$\Rightarrow (7)^2-4\times 7\times k=0$

$\Rightarrow 49-28k=0$

$\Rightarrow k=\frac{49}{28}=\frac{7}{4}$

Thus, the value of k is $\frac{7}{4}$.

12. If 3 is a root of the quadratic equation $x^2-x+k=0.$, find the value of p so that the roots of the equation $x^2+2kx+\left(k^2+2k+p\right)=0$ are equal.

Sol:

It is given that 3 is a root of the quadratic equation $x^2-x+k=0.$

$\therefore (3)^2-3+k=0$

$\Rightarrow k+6=0$

$\Rightarrow k=-6$

The roots of the equation $x^2+2kx+\left(k^2+2k+p\right)=0$ are equal.

$\therefore D=0$

$\Rightarrow (2k)^2-4\times 1\times\left(k^2+2k+p\right)=0$

$\Rightarrow 4k^2-4k^2-8k-4p=0$

$\Rightarrow -8k-4p=0$

$\Rightarrow p=\frac{8k}{-4}=-2k$

$\Rightarrow p=-2\times(-6)=12$

Hence, the value of p is 12.

13. If -4 is a root of the equation $x^2+2x+4p=0.$ find the value of k for the which the quadratic equation $x^2+px(1+3k)+7(3+2k)=0$ has equal roots.

Sol:

It is given that -4 is a root of the quadratic equation $x^2+2x+4p=0.$

$\therefore (-4)^2 + 2\times(-4) + 4p = 0$
$\Rightarrow 16 - 8 + 4p = 0$
$\Rightarrow 4p + 8 = 0$
$\Rightarrow p = -2$
The equation $x^2 + px(1+3k) + 7(3+2k) = 0$ has real roots.
$\therefore D = 0$
$\Rightarrow \left[p(1+3k)\right]^2 - 4\times1\times7(3+2k) = 0$
$\Rightarrow \left[-2(1+3k)\right]^2 - 28(3+2k) = 0$
$\Rightarrow 4(1+6k+9k^2) - 28(3+2k) = 0$
$\Rightarrow 4(1+6k+9k^2 - 21 - 14k) = 0$
$\Rightarrow 9k^2 - 8k - 20 = 0$
$\Rightarrow 9k^2 - 18k + 10k - 20 = 0$
$\Rightarrow 9k(k-2) + 10(k-2) = 0$
$\Rightarrow (k-2)(9k+10) = 0$
$\Rightarrow k - 2 = 0 \ or \ 9k + 10 = 0$
$\Rightarrow k = 2 \ or \ k = -\frac{10}{9}$
Hence, the required value of k is 2 or $-\frac{10}{9}$.

14. If the quadratic equation $(1+m^2)x^2 + 2mcx + (c^2 - a^2) = 0$ has equal roots, prove that $c^2 = a^2(1+m^2)$.

Sol:
Given:
$(1+m^2)x^2 + 2mcx + (c^2 - a^2) = 0$
Here,
$a = (1+m^2), b = 2mc \ and \ c = (c^2 - a^2)$
It is given that the roots of the equation are equal; therefore, we have:
$D = 0$
$\Rightarrow (b^2 - 4ac) = 0$
$\Rightarrow (2mc)^2 - 4\times(1+m^2)\times(c^2 - a^2) = 0$

$\Rightarrow 4m^2c^2 - 4\left(c^2 - a^2 + m^2c^2 - m^2a^2\right) = 0$

$\Rightarrow 4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4m^2a^2 = 0$

$\Rightarrow -4c^2 + 4a^2 + 4m^2a^2 = 0$

$\Rightarrow a^2 + m^2a^2 = c^2$

$\Rightarrow a^2\left(1 + m^2\right) = c^2$

$\Rightarrow c^2 = a^2\left(1 + m^2\right)$

Hence proved.

15. If the roots of the quadratic equation $\left(c^2 - ab\right)x^2 - 2\left(a^2 - bc\right)x + \left(b^2 - ac\right) = 0$ are real and equal, show that either a = 0 or $\left(a^3 + b^3 + c^3 = 3abc\right)$

Sol:

Given:

$\left(c^2 - ab\right)x^2 - 2\left(a^2 - bc\right)x + \left(b^2 - ac\right) = 0$

Here,

$a = \left(c^2 - ab\right), b = -2\left(a^2 - bc\right), c = \left(b^2 - ac\right)$

It is given that the roots of the equation are real and equal; therefore, we have:

$D = 0$

$\Rightarrow \left(b^2 - 4ac\right) = 0$

$\Rightarrow \left\{-2\left(a^2 - bc\right)\right\}^2 - 4 \times \left(c^2 - ab\right) \times \left(b^2 - ac\right) = 0$

$\Rightarrow 4\left(a^4 - 2a^2bc + b^2c^2\right) - 4\left(b^2c^2 - ac^3 - ab^3 + a^2bc\right) = 0$

$\Rightarrow a^4 - 2a^2bc + b^2c^2 - b^2c^2 + ac^3 + ab^3 - a^2bc = 0$

$\Rightarrow a^4 - 3a^2bc + ac^3 + ab^3 = 0$

$\Rightarrow a\left(a^3 - 3abc + c^3 + b^3\right) = 0$

Now,

$a = 0 \text{ or } a^3 - 3abc + c^3 + b^3 = 0$

$a = 0 \text{ or } a^3 + b^3 + c^3 = 3abc$

16. Find the value of p for which the quadratic equation $2x^2 + px + 8 = 0$ has real roots.

Sol:

Given:

$2x^2 + px + 8 = 0$

Here,

$a = 2, b = p \ and \ c = 8$

Discriminant D is given by:

$D = (b^2 - 4ac)$

$= p^2 - 4 \times 2 \times 8$

$= (p^2 - 64)$

If $D \geq 0$, the roots of the equation will be real

$\Rightarrow (p^2 - 64) \geq 0$

$\Rightarrow (p+8)(p-8) \geq 0$

$\Rightarrow p \geq 8 \ and \ p \leq -8$

Thus, the roots of the equation are real for $p \geq 8$ and $p \leq -8$.

17. Find the value of a for which the equation $(\alpha - 12)x^2 + 2(\alpha - 12)x + 2 = 0$ has equal roots.

Sol:

Given:

$(\alpha - 12)x^2 + 2(\alpha - 12)x + 2 = 0$

Here,

$a = (\alpha = 12), b = 2(\alpha - 12) \ and \ c = 2$

It is given that the roots of the equation are equal; therefore, we have

$D = 0$

$\Rightarrow (b^2 - 4ac) = 0$

$\Rightarrow \{2(\alpha - 12)\}^2 - 4 \times (\alpha - 12) \times 2 = 0$

$\Rightarrow 4(\alpha^2 - 24\alpha + 144) - 8\alpha + 96 = 0$

$\Rightarrow 4\alpha^2 - 96\alpha + 576 - 8\alpha + 96 = 0$

$\Rightarrow 4\alpha^2 - 104\alpha + 672 = 0$

$\Rightarrow \alpha^2 - 26\alpha + 168 = 0$

$\Rightarrow \alpha^2 - 14\alpha - 12\alpha + 168 = 0$

$\Rightarrow \alpha(\alpha - 14) - 12(\alpha - 14) = 0$

$\Rightarrow (\alpha - 14)(\alpha - 12) = 0$

$\therefore \alpha = 14 \ or \ \alpha = 12$

If the value of α is 12, the given equation becomes non-quadratic.

Therefore, the value of α will be 14 for the equation to have equal roots.

18. Find the value of k for which the roots of $9x^2+8kx+16=0$ are real and equal

Sol:

Given:

$9x^2+8kx+16=0$

Here,

$a=9, b=8k \text{ and } c=16$

It is given that the roots of the equation are real and equal; therefore, we have:

$D=0$

$\Rightarrow (b^2-4ac)=0$

$\Rightarrow (8k)^2-4\times 9\times 16=0$

$\Rightarrow 64k^2-576=0$

$\Rightarrow 64k^2=576$

$\Rightarrow k^2=9$

$\Rightarrow k=\pm 3$

$\therefore k=3 \text{ or } k=-3$

19. Find the values of k for which the given quadratic equation has real and distinct roots:

(i) $kx^2+6x+1=0.$ (ii) $x^2-kx+9=0.$

(iii) $9x^2+3kx+4=0.$ (iv) $5x^2-kx+1=0.$

Sol:

(i) The given equation is $kx^2+6x+1=0.$

$\therefore D=6^2-4\times k\times 1=36-4k$

The given equation has real and distinct roots if $D>0.$

$\therefore 36-4k>0$

$\Rightarrow 4k<36$

$\Rightarrow k<9$

(ii) The given equation is $x^2-kx+9=0.$

$\therefore D=(-k)^2-4\times 1\times 9=k^2-36$

The given equation has real and distinct roots if $D>0.$

$\therefore k^2-36>0$

$\Rightarrow (k-6)(k+6)>0$

$\Rightarrow k<-6 \text{ or } k>6$

(iii) The given equation is $9x^2+3kx+4=0.$

$\therefore D=(3k)^2-4\times 9\times 4=9k^2-144$

The given equation has real and distinct roots if $D>0.$

$\therefore 9k^2 - 144 > 0$

$\Rightarrow 9\left(k^2 - 16\right) > 0$

$\Rightarrow (k-4)(k+4) > 0$

$\Rightarrow k < -4 \ or \ k > 4$

(iv) The given equation is $5x^2 - kx + 1 = 0$.

$\therefore D = (-k)^2 - 4\times5\times1 = k^2 - 20$

The given equation has real and distinct roots if $D > 0$.

$\therefore k^2 - 20 > 0$

$\Rightarrow k^2 - \left(2\sqrt{5}\right)^2 > 0$

$\Rightarrow \left(k - 2\sqrt{5}\right)\left(k + 2\sqrt{5}\right) > 0$

$\Rightarrow k < -2\sqrt{5} \ or \ k > 2\sqrt{5}$

20. If a and b are real and $a \neq b$ then show that the roots of the equation $(a-b)x^2 + 5(a+b)x - 2(a-b) = 0$. are equal and unequal.

Sol:

The given equation is $(a-b)x^2 + 5(a+b)x - 2(a-b) = 0$.

$\therefore D = \left[5(a+b)\right]^2 - 4\times(a-b)\times\left[-2(a-b)\right]$

$= 25(a+b)^2 + 8(a-b)^2$

Since a and b are real and $a \neq b$, so $(a-b)^2 > 0 \ and \ (a+b)^2 > 0$.

$\therefore 8(a-b)^2 > 0$ ....(1) (Product of two positive numbers is always positive)

Also, $25(a+b)^2 > 0$ ....(2) (Product of two positive numbers is always positive)

Adding (1) and (2), we get

$25(a+b)^2 + 8(a-b)^2 > 0$ (Sum of two positive numbers is always positive)

$\Rightarrow D > 0$

Hence, the roots of the given equation are real and unequal.

21. If the roots of the equation $\left(a^2+b^2\right)x^2 - 2(ac+bd)x + \left(c^2+d^2\right) = 0$ are equal, prove that $\frac{a}{b} = \frac{c}{d}$

Sol:

It is given that the roots of the equation $\left(a^2+b^2\right)x^2 - 2(ac+bd)x + \left(c^2+d^2\right) = 0$ are equal.

$\therefore D = 0$

$\Rightarrow \left[-2(ac+bd)\right]^2 - 4(a^2+b^2)(c^2+d^2) = 0$

$\Rightarrow 4(a^2c^2+b^2d^2+2abcd) - 4(a^2c^2+a^2d^2+b^2c^2+b^2d^2) = 0$

$\Rightarrow 4(a^2c^2+b^2d^2+2abcd-a^2c^2-a^2d^2-b^2c^2-b^2d^2) = 0$

$\Rightarrow (-a^2d^2+2abcd-b^2c^2) = 0$

$\Rightarrow -(a^2d^2-2abcd+b^2c^2) = 0$

$\Rightarrow (ad-bc)^2 = 0$

$\Rightarrow ad-bc = 0$

$\Rightarrow ad = bc$

$\Rightarrow \frac{a}{b} = \frac{c}{d}$

Hence proved.

22. If the roots of the equations $ax^2+2bx+c=0$ and $bx^2-2\sqrt{acx}+b=0$ are simultaneously real then prove that $b^2=ac$

Sol:

It is given that the roots of the equation $ax^2+2bx+c=0$ are real.

$\therefore D_1 = (2b)^2 - 4\times a\times c \geq 0$

$\Rightarrow 4(b^2-ac) \geq 0$

$\Rightarrow b^2-ac \geq 0$ ....(1)

Also, the roots of the equation $bx^2-2\sqrt{ac}x+b=0$ are real.

$\therefore D_2 = \left(-2\sqrt{ac}\right)^2 - 4\times b\times b \geq 0$

$\Rightarrow 4(ac-b^2) \geq 0$

$\Rightarrow -4(b^2-ac) \geq 0$

$\Rightarrow b^2-ac \geq 0$ ....(2)

The roots of the given equations are simultaneously real if (1) and (2) holds true together. This is possible if

$b^2-ac=0$

$\Rightarrow b^2 = ac$

Exercise 10E

1. The sum of a natural number and its square is 156. Find the number.

Sol:

Let the required natural number be x.

According to the given condition,

$x+x^2=156$

$\Rightarrow x^2+x-156=0$

$\Rightarrow x^2+13x-12x-156=0$

$\Rightarrow x(x+13)-12(x+13)=0$

$\Rightarrow (x+13)(x-12)=0$

$\Rightarrow x+13=0 \text{ or } x-12=0$

$\Rightarrow x=-13 \text{ or } x=12$

$\therefore x=12$ (x cannot be negative)

Hence, the required natural number is 12.

2. The sum of natural number and its positive square root is 132. Find the number.

Sol:

Let the required natural number be x.

According to the given condition,

$x+\sqrt{x}=132$

Putting $\sqrt{x}=y \text{ or } x=y^2$, we get

$y^2+y=132$

$\Rightarrow y^2+y-132=0$

$\Rightarrow y^2+12y-11y-132=0$

$\Rightarrow y(y+12)-11(y+12)=0$

$\Rightarrow (y+12)(y-11)=0$

$\Rightarrow y+12=0 \text{ or } y-11=0$

$\Rightarrow y=-12 \text{ or } y=11$

$\therefore y=11$ (y cannot be negative)

Now,

$\sqrt{x}=11$

$\Rightarrow x=(11)^2=121$

Hence, the required natural number is 121.

3. The sum of two natural number is 28 and their product is 192. Find the numbers.

Sol:

Let the required number be x and $(28-x)$.

According to the given condition,

$x(28-x)=192$

$\Rightarrow 28x-x^2=192$

$\Rightarrow x^2-28x+192=0$

$\Rightarrow x^2-16x-12x+192=0$

$\Rightarrow x(x-16)-12(x-16)=0$

$\Rightarrow (x-12)(x-16)=0$

$\Rightarrow x-12=0 \text{ or } x-16=0$

$\Rightarrow x=12 \text{ or } x=16$

When $x=12$,

$28-x=28-12=16$

When $x=16$,

$28-x=28-16=12$

Hence, the required numbers are 12 and 16.

4. The sum of the squares of two consecutive positive integers is 365. Find the integers.

Sol:

Let the required two consecutive positive integers be x and $(x+1)$.

According to the given condition,

$x^2+(x+1)^2=365$

$\Rightarrow x^2+x^2+2x+1=365$

$\Rightarrow 2x^2+2x-364=0$

$\Rightarrow x^2+x-182=0$

$\Rightarrow x^2+14x-13x-182=0$

$\Rightarrow x(x+14)-13(x+14)=0$

$\Rightarrow (x+14)(x-13)=0$

$\Rightarrow x+14=0 \text{ or } x-13=0$

$\Rightarrow x=-14 \text{ or } x=13$

$\therefore x=13$ (x is a positive integers)

When $x=13$,

$x+1=13+1=14$