

ML Aggarwal Solutions Class 10 Maths

Chapter 11: Section Formula

1. Find the co-ordinates of the mid-point of the line segments joining the following pairs of points:

(i) $(2, -3), (-6, 7)$

(ii) $(5, -11), (4, 3)$

(iii) $(a + 3, 5b), (2a - 1, 3b + 4)$

Solution:

Co-ordinates of midpoint of line joining the points (x_1, y_1) and $(x_2, y_2) = \{(x_1+x_2)/2, (y_1+y_2)/2\}$

(i) \therefore Co-ordinates of midpoint of line joining the points $(2, -3)$ and $(-6, 7) = \{(2+(-6))/2, (-3+7)/2\}$

$= (-4/2, 4/2)$

$= (-2, 2)$

Hence the co-ordinates of midpoint of line joining the points $(2, -3)$ and $(-6, 7)$ is $(-2, 2)$.

(ii) Co-ordinates of midpoint of line joining the points (x_1, y_1) and $(x_2, y_2) = \{(x_1+x_2)/2, (y_1+y_2)/2\}$

\therefore Co-ordinates of midpoint of line joining the points $(5, -11)$ and $(4, 3) = \{(5+4)/2, (-11+3)/2\}$

$= (9/2, -8/2)$

$= (9/2, -4)$

Hence the co-ordinates of midpoint of line joining the points $(5, -11)$ and $(4, 3)$ is $(9/2, -4)$.

(iii) Co-ordinates of midpoint of line joining the points (x_1, y_1) and $(x_2, y_2) = \{(x_1+x_2)/2, (y_1+y_2)/2\}$

\therefore Co-ordinates of midpoint of line joining the points $(a+3, 5b)$ and $(2a-1, 3b+4) = \{(a+3+2a-1)/2, (5b+3b+4)/2\}$

$= \{(3a+2)/2, (8b+4)/2\}$

$= \{(3a+2)/2, (4b+2)\}$

Hence the co-ordinates of midpoint of line joining the points $(a+3, 5b)$ and $(2a-1, 3b+4)$ are $\{(3a+2)/2, (4b+2)\}$.

2. The co-ordinates of two points A and B are $(-3, 3)$ and $(12, -7)$ respectively. P is a point on the line segment AB such that $AP : PB = 2 : 3$. Find the co-ordinates of P.

Solution:

Let the co-ordinates of P(x, y) divides AB in the ratio m:n.

A(-3,3) and B(12,-7) are the given points.

Given m:n = 2:3

$x_1 = -3, y_1 = 3, x_2 = 12, y_2 = -7, m = 2$ and $n = 3$

\therefore By Section formula $x = (mx_2 + nx_1)/(m+n)$

$\therefore x = (2 \times 12 + 3 \times -3)/(2+3)$

$\therefore x = (24-9)/5$

$x = 15/5$

$\Rightarrow x = 3$

By Section formula $y = (my_2 + ny_1)/(m+n)$

$\therefore y = (2 \times -7 + 3 \times 3)/5$

$\therefore y = (-14+9)/5$

$\therefore y = -5/5$

$\Rightarrow y = -1$

Hence the co-ordinate of point P are $(3, -1)$.

3. P divides the distance between A $(-2, 1)$ and B $(1, 4)$ in the ratio of 2 : 1. Calculate the co-ordinates of the point P.

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Solution:

Let the co-ordinates of $P(x, y)$ divides AB in the ratio $m:n$.

$A(-2,1)$ and $B(1,4)$ are the given points.

Given $m:n = 2:1$

$x_1 = -2, y_1 = 1, x_2 = 1, y_2 = 4, m = 2$ and $n = 1$

\therefore By Section formula $x = (mx_2 + nx_1)/(m+n)$

$$\therefore x = (2 \times 1 + 1 \times -2)/(2+1)$$

$$\therefore x = (2-2)/3$$

$$x = 0/3$$

$$\Rightarrow x = 0$$

By Section formula $y = (my_2 + ny_1)/(m+n)$

$$\therefore y = (2 \times 4 + 1 \times 1)/(2+1)$$

$$\therefore y = (8+1)/3$$

$$y = 9/3$$

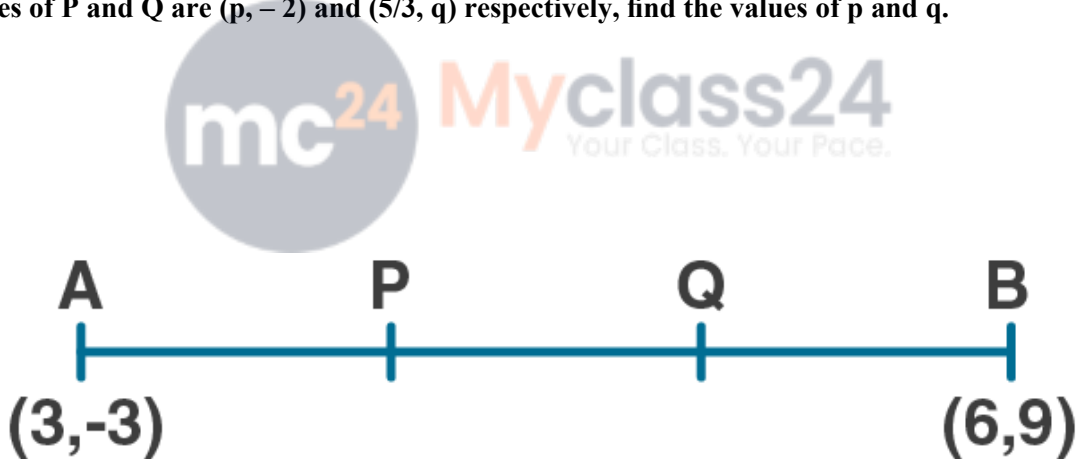
$$\Rightarrow y = 3$$

Hence the co-ordinate of point P are $(0,3)$.

4. (i) Find the co-ordinates of the points of trisection of the line segment joining the point $(3, -3)$ and $(6, 9)$.

(ii) The line segment joining the points $(3, -4)$ and $(1, 2)$ is trisected at the points P and Q . If the coordinates of P and Q are $(p, -2)$ and $(5/3, q)$ respectively, find the values of p and q .

Solution:



Let P and Q be the points of trisection of AB

i.e., $AP = PQ = QB$

Given $A(3,-3)$ and $B(6,9)$

$\therefore x_1 = 3, y_1 = -3, x_2 = 6, y_2 = 9$

$\therefore P(x, y)$ divides AB internally in the ratio $1 : 2$.

$\therefore m:n = 1:2$

\therefore By applying the section formula, the coordinates of P are as follows.

By Section formula $x = (mx_2 + nx_1)/(m+n)$

$$x = (1 \times 6 + 2 \times 3)/(1+2)$$

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$$x = (6+6)/3$$
$$x = 12/3$$
$$x = 4$$

By Section formula $y = (my_2 + ny_1)/(m+n)$

$$\therefore y = (1 \times 9 + 2 \times -3)/(2+1)$$

$$\therefore y = (9-6)/3$$

$$y = 3/3$$

$$\Rightarrow y = 1$$

Hence the co-ordinate of point P are (4,1).

Now, Q also divides AB internally in the ratio 2 : 1.

$$\therefore m:n = 2:1$$

\therefore By applying the section formula, the coordinates of P are as follows.

By Section formula $x = (mx_2 + nx_1)/(m+n)$

$$\therefore x = (2 \times 6 + 1 \times 3)/(1+2)$$

$$x = (12+3)/3$$

$$x = 15/3$$

$$x = 5$$

By Section formula $y = (my_2 + ny_1)/(m+n)$

$$\therefore y = (2 \times 9 + 1 \times -3)/(2+1)$$

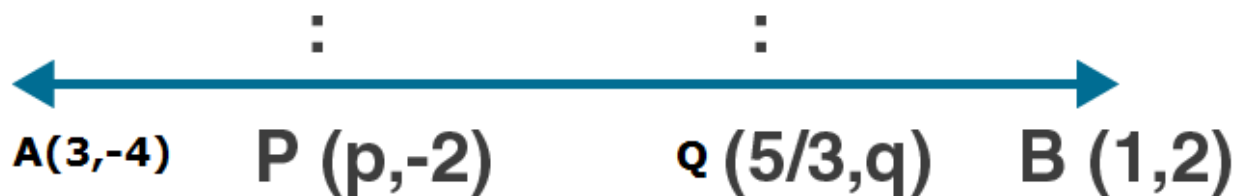
$$\therefore y = (18-3)/3$$

$$y = 15/3$$

$$\Rightarrow y = 5$$

Hence the co-ordinate of point Q are (5,5).

(ii) Let P(p,-2) and Q(5/3, q) be the points of trisection of AB



i.e., $AP = PQ = QB$

Given A(3,-4) and B(1,2)

$$\therefore x_1 = 3, y_1 = -4, x_2 = 1, y_2 = 2$$

\therefore P(p, -2) divides AB internally in the ratio 1 : 2.

By Section formula $x = (mx_2 + nx_1)/(m+n)$

$$p = (1 \times 1 + 2 \times 3)/(1+2)$$

$$p = (1+6)/3$$

$$p = 7/3$$

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Now, Q also divides AB internally in the ratio 2 : 1.

$$\therefore m:n = 2:1$$

$\therefore Q(5/3, q)$ divides AB internally in the ratio 2 : 1.

By Section formula $y = (my_2 + ny_1)/(m+n)$

$$\therefore q = (2 \times 2 + 1 \times -4)/(2+1)$$

$$\therefore q = (4-4)/3$$

$$q = 0/3$$

$$\Rightarrow q = 0$$

Hence the value of p and q are $7/3$ and 0 respectively.

5. (i) The line segment joining the points A (3, 2) and B (5, 1) is divided at the point P in the ratio 1 : 2 and it lies on the line $3x - 18y + k = 0$. Find the value of k.

(ii) A point P divides the line segment joining the points A (3, -5) and B (-4, 8) such that $AP/PB = k/1$. If P lies on the line $x + y = 0$, then find the value of k.

Solution:

(i) Let the co-ordinates of P(x, y) divides AB in the ratio m:n.

A(3,2) and B(5,1) are the given points.

Given m:n = 1:2

$$x_1 = 3, y_1 = 2, x_2 = 5, y_2 = 1, m = 1 \text{ and } n = 2$$

\therefore By Section formula $x = (mx_2 + nx_1)/(m+n)$

$$\therefore x = (1 \times 5 + 2 \times 3)/(1+2)$$

$$\therefore x = (5+6)/3$$

$$\Rightarrow x = 11/3$$

By Section formula $y = (my_2 + ny_1)/(m+n)$

$$\therefore y = (1 \times 1 + 2 \times 2)/(1+2)$$

$$\therefore y = (1+4)/3$$

$$\Rightarrow y = 5/3$$

Given P lies on the line $3x - 18y + k = 0$

Substitute x and y in above equation

$$3 \times (11/3) - 18 \times (5/3) + k = 0$$

$$\Rightarrow 11 - 30 + k = 0$$

$$\Rightarrow -19 + k = 0$$

$$\Rightarrow k = 19$$

Hence the value of k is 19.

(ii) Let the co-ordinates of P(x, y) divides AB in the ratio m:n.

A(3,-5) and B(-4,8) are the given points.

Given $AP/PB = k/1$

$$\therefore m:n = k:1$$

$$x_1 = 3, y_1 = -5, x_2 = -4, y_2 = 8, m = k \text{ and } n = 1$$

\therefore By Section formula $x = (mx_2 + nx_1)/(m+n)$

$$\therefore x = (k \times -4 + 1 \times 3)/(k+1)$$

$$\therefore x = (-4k+3)/(k+1)$$

$$\Rightarrow x = (-4k+3)/(k+1)$$



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By Section formula $y = (my_2 + ny_1)/(m+n)$

$$\therefore y = (k \times 8 + 1 \times -5)/(k+1)$$

$$\therefore y = (-4k+3)/(k+1)$$

Co-ordinate of P is $((-4k+3)/(k+1), (8k-5)/(k+1))$

Given P lies on line $x+y = 0$

Substitute value of x and y in above equation

$$(-4k+3)/(k+1) + (8k-5)/(k+1) = 0$$

$$\Rightarrow (-4k+3) + (8k-5) = 0$$

$$\Rightarrow 4k-2 = 0$$

$$\Rightarrow 4k = 2$$

$$k = 2/4 = 1/2$$

Hence the value of k is $1/2$.

6. Find the coordinates of the point which is three-fourth of the way from A (3, 1) to B (-2, 5).

Solution:



Let P be the point which is three-fourth of the way from A(3,1) to B(-2,5).

$$\therefore AP/AB = 3/4$$

$$AB = AP + PB$$

$$\therefore AP/AB = AP/(AP + PB) = 3/4$$

$$\Rightarrow 4AP = 3AP + 3PB$$

$$\Rightarrow 4AP - 3AP = 3PB$$

$$\Rightarrow AP = 3PB$$

$$\Rightarrow AP/PB = 3/1$$

\therefore The ratio $m:n = 3:1$

$$x_1 = 3, y_1 = 1, x_2 = -2, y_2 = 5$$

\therefore By Section formula $x = (mx_2 + nx_1)/(m+n)$

$$\therefore x = (3 \times -2 + 1 \times 3)/(3+1)$$

$$\therefore x = (-6+3)/4$$

$$\Rightarrow x = -3/4$$

By Section formula $y = (my_2 + ny_1)/(m+n)$

$$\therefore y = (3 \times 5 + 1 \times 1)/(3+1)$$

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$$\therefore y = (15+1)/4$$

$$\Rightarrow y = 16/4$$

$$\Rightarrow y = 4$$

Hence the co-ordinates of P are $(-3/4, 4)$.

7. Point P (3, - 5) is reflected to P' in the x- axis. Also P on reflection in the y-axis is mapped as P''.

(i) Find the co-ordinates of P' and P''.

(ii) Compute the distance P' P''.

(iii) Find the middle point of the line segment P' P''.

(iv) On which co-ordinate axis does the middle point of the line segment P P'' lie ?

Solution:

(i) The image of P(3,-5) when reflected in X-axis will be (3,5).

When you reflect a point across the X-axis, the x-coordinate remains the same, but the y-coordinate is transformed into its opposite (its sign is changed).

\therefore Co-ordinates of P' = (3,5)

Image of P(3,-5) when reflected in Y axis will be (-3,-5).

When you reflect a point across the Y-axis, the y-coordinate remains the same, but the x-coordinate is transformed into its opposite (its sign is changed)

\therefore Co-ordinates of P'' = (-3,-5)

(ii) Let P'(x₁, y₁) and P''(x₂, y₂) be the given points

By distance formula $d(P', P'') = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Co-ordinates of P' = (3,5)

Co-ordinates of P'' = (-3,-5)

Here x₁ = 3, y₁ = 5, x₂ = -3, y₂ = -5

$\therefore d(P', P'') = \sqrt{[(-3-3)^2 + (-5-5)^2]}$

$$= \sqrt{[(-6)^2 + (-10)^2]}$$

$$= \sqrt{(36+100)}$$

$$= \sqrt{136}$$

$$= \sqrt{4 \times 34}$$

$$= 2\sqrt{34}$$

Hence the distance between P' and P'' is $2\sqrt{34}$ units.

(iii) Co-ordinates of P' = (3,5)

Co-ordinates of P'' = (-3,-5)

Here x₁ = 3, y₁ = 5, x₂ = -3, y₂ = -5

Let Q(x,y) be the midpoint of P'P''

By midpoint formula,

$$x = (x_1 + x_2)/2$$

$$y = (y_1 + y_2)/2$$

$$\therefore x = (3 + (-3))/2 = 0/2 = 0$$

$$y = (5 + (-5))/2 = 0/2 = 0$$

Hence the co-ordinate of midpoint of P'P'' is (0,0).

(iv) Co-ordinates of P = (3,-5)

Co-ordinates of P'' = (-3,-5)

Here x₁ = 3, y₁ = -5, x₂ = -3, y₂ = -5

Let R(x,y) be the midpoint of PP''

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By midpoint formula,

$$x = \frac{(x_1+x_2)}{2}$$

$$y = \frac{(y_1+y_2)}{2}$$

$$\therefore x = \frac{(3+(-3))}{2} = \frac{0}{2} = 0$$

$$y = \frac{(-5+5)}{2} = \frac{-10}{2} = -5$$

So the co-ordinate of midpoint of PP'' is (0,-5).

Here x co-ordinate is zero.

Hence the point lies on Y-axis.

8. Use graph paper for this question. Take 1 cm = 1 unit on both axes. Plot the points A(3, 0) and B(0, 4).

(i) Write down the co-ordinates of A1, the reflection of A in the y-axis.

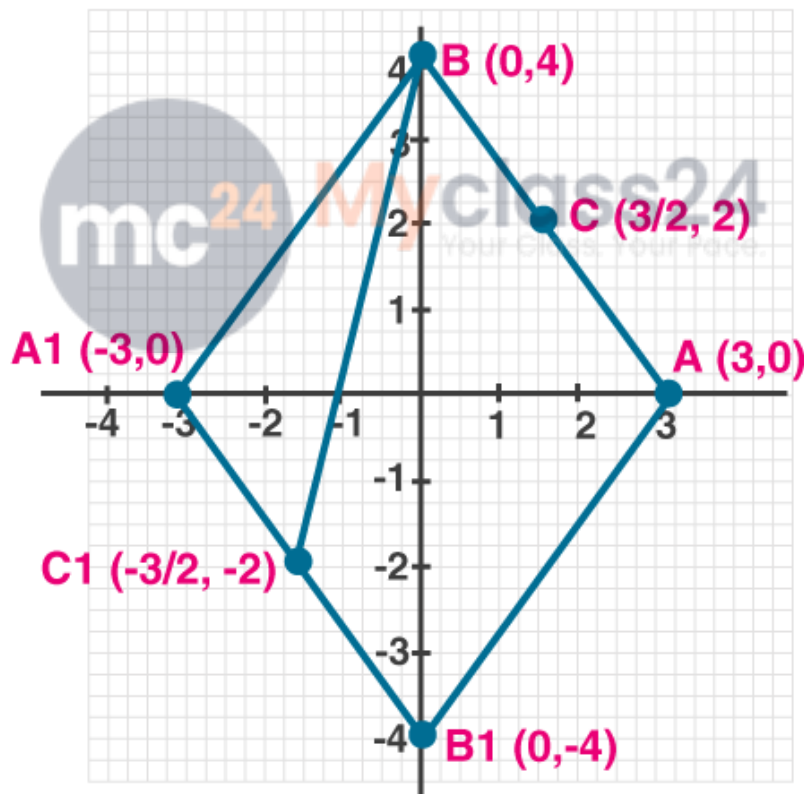
(ii) Write down the co-ordinates of B1, the reflection of B in the x-axis.

(iii) Assign the special name to the quadrilateral ABA1B1.

(iv) If C is the midpoint of AB. Write down the co-ordinates of the point C1, the reflection of C in the origin.

(v) Assign the special name to quadrilateral ABC1B1.

Solution:



(i) Co-ordinates of point A are (3,0).

When you reflect a point across the Y-axis, the y-coordinate remains the same, but the x-coordinate is transformed into its opposite (its sign is changed)

Hence the reflection of A in the Y axis is (-3,0).

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(ii) Co-ordinates of point B are (0,4).

When you reflect a point across the X-axis, the x-coordinate remains the same, but the y-coordinate is transformed into its opposite (its sign is changed).

Hence the reflection of B in the X-axis is (0,-4)

(iii) The quadrilateral ABA₁B₁ will be a rhombus.

(iv) Let C be midpoint of AB.

Co-ordinate of C = $((3+0)/2, (0+4)/2) = (3/2, 2)$ [Midpoint formula]

In a point reflection in the origin, the image of the point (x,y) is the point (-x,-y).

Hence the reflection of C in the origin is $(-3/2, -2)$

(v) In quadrilateral ABC₁B₁, AB \parallel B₁C₁

Hence the quadrilateral ABC₁B₁ is a trapezium.

9. The line segment joining A (-3, 1) and B (5, -4) is a diameter of a circle whose centre is C. find the co-ordinates of the point C. (1990)

Solution:

Given Co-ordinates of A = (-3,1)

Co-ordinates of B = (5,-4)

Here $x_1 = -3, y_1 = 1, x_2 = 5, y_2 = -4$

Let C(x,y) be the midpoint of AB

By midpoint formula,

$$x = (x_1+x_2)/2$$

$$y = (y_1+y_2)/2$$

$$\therefore x = (-3+5)/2 = 2/2 = 1$$

$$y = (1-4)/2 = -3/2$$

Hence the co-ordinate of midpoint of AB is C(1,-3/2).

10. The mid-point of the line segment joining the points (3m, 6) and (-4, 3n) is (1, 2m - 1). Find the values of m and n.

Solution:

Let the midpoint of line joining the points A(3m,6) and B(-4,3n) be C(1,2m-1).

Here $x_1 = 3m, y_1 = 6, x_2 = -4, y_2 = 3n$

$$x = 1, y = 2m-1$$

By Midpoint formula,

$$x = (x_1+x_2)/2$$

$$\therefore 1 = (3m-4)/2$$

$$\Rightarrow 3m-4 = 2$$

$$\Rightarrow 3m = 2+4$$

$$\Rightarrow 3m = 6$$

$$\Rightarrow m = 6/3 = 2$$

By Midpoint formula,

$$y = (y_1+y_2)/2$$

$$\therefore 2m-1 = (6+3n)/2$$

$$\Rightarrow 4m-2 = 6+3n$$

Put $m = 2$ in above equation

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$$4 \times 2 - 2 = 6 + 3n$$

$$8 - 2 - 6 = 3n$$

$$\Rightarrow 3n = 0$$

$$\Rightarrow n = 0$$

Hence the value of m and n are 2 and 0 respectively.

11. The co-ordinates of the mid-point of the line segment PQ are (1, -2). The co-ordinates of P are (-3, 2). Find the co-ordinates of Q.(1992)

Solution:

Let the co-ordinates of Q be (x_2, y_2) .

Given co-ordinates of P = $(-3, 2)$

Co-ordinates of midpoint = $(1, -2)$

Here $x_1 = -3, y_1 = 2, x = 1, y = -2$

By Midpoint formula,

$$x = \frac{(x_1 + x_2)}{2}$$

$$\therefore 1 = \frac{(-3 + x_2)}{2}$$

$$\Rightarrow 2 = -3 + x_2$$

$$\Rightarrow x_2 = 2 + 3 = 5$$

By Midpoint formula,

$$y = \frac{(y_1 + y_2)}{2}$$

$$\therefore -2 = \frac{(2 + y_2)}{2}$$

$$\Rightarrow -4 = 2 + y_2$$

$$\Rightarrow y_2 = -4 - 2$$

$$\Rightarrow y_2 = -6$$

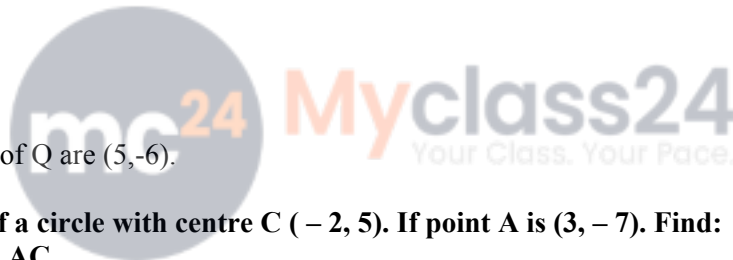
Hence the co-ordinates of Q are $(5, -6)$.

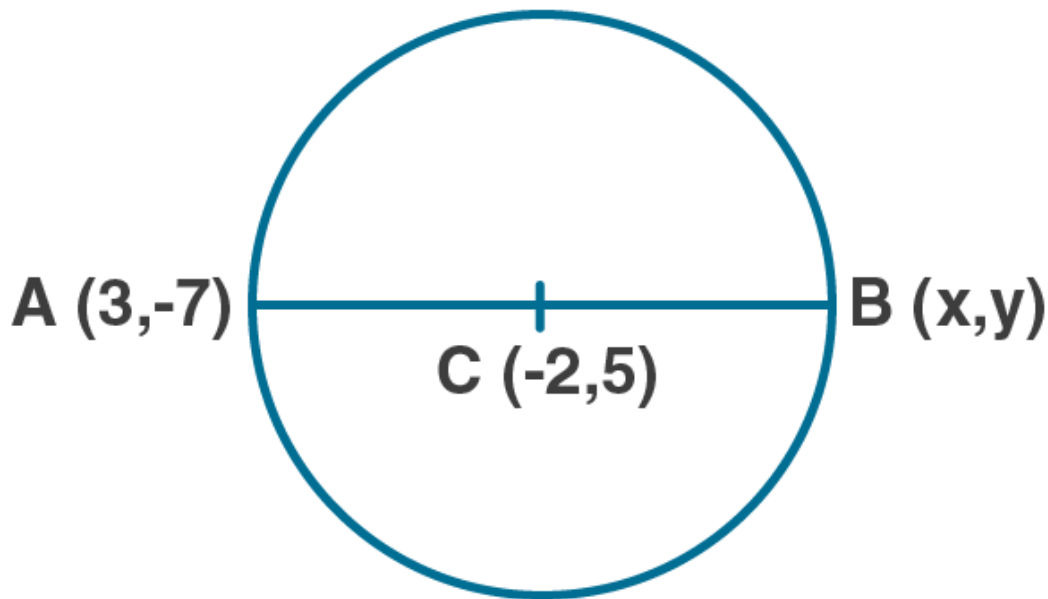
12. AB is a diameter of a circle with centre C $(-2, 5)$. If point A is $(3, -7)$. Find:

(i) the length of radius AC.

(ii) the coordinates of B.

Solution:





(i) Length of radius AC = d(A,C)

Co-ordinates of A = (3,-7)

Co-ordinates of C = (-2,5)

Here $x_1 = 3$, $y_1 = -7$, $x_2 = -2$, $y_2 = 5$

By distance formula, $d(A,C) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

$$= \sqrt{[(-2-3)^2+(5-(-7))^2]}$$

$$= \sqrt{[(-5)^2+(12)^2]}$$

$$= \sqrt{[25+144]}$$

$$= \sqrt{169}$$

$$= 13$$

Hence the radius is 13 units.

(ii) Given AB is the diameter and C is the centre of the circle.

\therefore By midpoint formula, $-2 = (x+3)/2$

$$\Rightarrow -4 = x+3$$

$$\Rightarrow x = -4-3 = -7$$

\therefore By midpoint formula, $5 = (-7+y)/2$

$$\Rightarrow 10 = -7+y$$

$$\Rightarrow y = 10+7 = 17$$

Hence the co-ordinates of B are (-7,17).

13. Find the reflection (image) of the point (5, -3) in the point (-1, 3).

Solution:

Let the co-ordinates of the image of the point P(5,-3) be

P1(x, y) in the point (-1, 3) then the point (-1, 3) will be the midpoint of PP1.

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By midpoint formula, $x = (x_1+x_2)/2$

$$\therefore -1 = (5+x_2)/2 \quad [x = -1, x_1 = 5]$$

$$-2 = 5+x_2$$

$$x_2 = -2-5 = -7$$

By midpoint formula, $y = (y_1+y_2)/2$

$$3 = (-3+y_2)/2 \quad [y = 3, y_1 = -3]$$

$$6 = -3+y_2$$

$$y_2 = 6+3 = 9$$

Hence the co-ordinates of the image of P is (-7,9).

14. The line segment joining A(-1,5/3) the points B (a, 5) is divided in the ratio 1 : 3 at P, the point where the line segment AB intersects y-axis. Calculate

(i) the value of a

(ii) the co-ordinates of P. (1994)

Solution:

(i) Let P(x,y) divides the line segment joining the points A(-1,5/3), B(a,5) in the ratio 1:3,

Here m:n = 1:3

$$x_1 = -1, y_1 = 5/3, x_2 = a, y_2 = 5$$

\therefore By Section formula $x = (mx_2+nx_1)/(m+n)$

$$\therefore x = (1 \times a + 3 \times -1)/(1+3)$$

$$\therefore x = (a-3)/4$$

$$\Rightarrow x = (a-3)/4 \quad \dots(i)$$

By Section formula $y = (my_2+ny_1)/(m+n)$

$$\therefore y = (1 \times 5 + 3 \times 5/3)/(3+1)$$

$$\therefore y = (5+5)/4$$

$$\Rightarrow y = 10/4$$

$$\Rightarrow y = 5/2 \quad \dots(ii)$$

Given P meets Y axis. So its x co-ordinate will be zero.

$$\text{i.e., } (a-3)/4 = 0$$

$$\Rightarrow a-3 = 0$$

$$\Rightarrow a = 3$$

$$(ii) x = (a-3)/4 \quad [\text{From (i)}]$$

Substitute a = 3 in above equation.

$$x = (3-3)/4 = 0$$

$$y = 5/2 \quad [\text{From (ii)}]$$

Hence the co-ordinates of P are (0,5/2).

15. The point P (-4, 1) divides the line segment joining the points A (2, -2) and B in the ratio of 3 : 5. Find the point B.

Solution:

Let the co-ordinates of B be (x_2, y_2) .

Given co-ordinates of A = (2,-2)

Co-ordinates of P = (-4,1)

Ratio m:n = 3:5

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$$x_1 = 2, y_1 = -2, x = -4, y = 1$$

P divides AB in the ratio 3:5

$$\therefore \text{By section formula, } x = (mx_2 + nx_1)/(m+n)$$

$$\therefore -4 = (3x_2 + 5 \times 2)/(3+5)$$

$$\therefore -4 = (3x_2 + 10)/8$$

$$-32 = 3x_2 + 10$$

$$\Rightarrow 3x_2 = -32 - 10 = -42$$

$$\Rightarrow x_2 = -42/3 = -14$$

By Section formula $y = (my_2 + ny_1)/(m+n)$

$$\therefore 1 = (3y_2 + 5 \times -2)/(3+5)$$

$$\therefore 1 = (3y_2 - 10)/8$$

$$\Rightarrow 8 = 3y_2 - 10$$

$$\Rightarrow 3y_2 = 8 + 10 = 18$$

$$\Rightarrow y = 18/3 = 6$$

Hence the co-ordinates of B are (-14,6).

16. (i) In what ratio does the point (5, 4) divide the line segment joining the points (2, 1) and (7, 6) ?

(ii) In what ratio does the point (-4, b) divide the line segment joining the points P (2, -2), Q (-14, 6) ?

Hence find the value of b.

Solution:

(i) Let the ratio that the point (5,4) divide the line segment joining the points (2,1) and (7,6) be m:n,

Here $x_1 = 2, y_1 = 1, x_2 = 7, y_2 = 6, x = 5, y = 4$

$$\therefore \text{By section formula, } x = (mx_2 + nx_1)/(m+n)$$

$$\therefore 5 = (m \times 7 + n \times 2)/(m+n)$$

$$\therefore 5 = (7m + 2n)/(m+n)$$

$$\therefore 5(m+n) = 7m + 2n$$

$$\therefore 5m + 5n = 7m + 2n$$

$$\therefore 5m - 7m = 2n - 5n$$

$$\therefore -2m = -3n$$

$$\therefore m/n = -3/-2 = 3/2$$

Hence the ratio m:n is 3:2.

(ii) Let the ratio that the point (-4,b) divide the line segment joining the points (2,-2) and (-14,6) be m:n,

Here $x_1 = 2, y_1 = -2, x_2 = -14, y_2 = 6, x = -4, y = b$

$$\therefore \text{By section formula, } x = (mx_2 + nx_1)/(m+n)$$

$$\therefore -4 = (m \times -14 + n \times 2)/(m+n)$$

$$\therefore -4 = (-14m + 2n)/(m+n)$$

$$\therefore -4(m+n) = -14m + 2n$$

$$\therefore -4m - 4n = -14m + 2n$$

$$\therefore -4m + 14m = 2n + 4n$$

$$\therefore 10m = 6n$$

$$\therefore m/n = 6/10 = 3/5$$

Hence the ratio m:n is 3:5.

By Section formula $y = (my_2 + ny_1)/(m+n)$

$$\therefore b = (3 \times 6 + 5 \times -2)/(3+5)$$

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$$\therefore b = (18-10)/8$$

$$\Rightarrow b = 8/8$$

$$\Rightarrow b = 1$$

Hence the value of b is 1 and the ratio $m:n$ is 3:5.

17. The line segment joining A (2, 3) and B (6, -5) is intercepted by the x-axis at the point K. Write the ordinate of the point k. Hence, find the ratio in which K divides AB. Also, find the coordinates of the point K.

Solution:

Since the point K is on X axis, its y co-ordinate is zero.

Let the point K be $(x,0)$.

Let the point K divides the line segment joining A(2,3) and B(6,-5) in the ratio $m:n$.

Here $x_1 = 2$, $y_1 = 3$, $x_2 = 6$, $y_2 = -5$, $y = 0$

By Section formula $y = (my_2 + ny_1)/(m+n)$

$$\therefore 0 = (m \times -5 + n \times 3)/(m+n)$$

$$\therefore 0 = (-5m + 3n)/m+n$$

$$\Rightarrow -5m + 3n = 0$$

$$\Rightarrow -5m = -3n$$

$$\Rightarrow m/n = -3/-5 = 3/5$$

Hence the point K divides the line segment in the ratio 3:5.

\therefore By section formula, $x = (mx_2 + nx_1)/(m+n)$

$$\therefore x = (3 \times 6 + 5 \times 2)/(3+5)$$

$$\therefore x = (18+10)/8$$

$$\therefore x = 28/8 = 7/2$$

Hence the co-ordinates of K are $(7/2, 0)$.



18. If A (-4, 3) and B (8, -6), (i) find the length of AB. (ii) in what ratio is the line joining AB, divided by the x-axis? (2008)

Solution:

(i) Given points are A(-4,3) and B(8,-6).

Here $x_1 = -4$, $y_1 = 3$

$x_2 = 8$, $y_2 = -6$

By distance formula, $d(AB) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

$$\therefore d(AB) = \sqrt{[(8 - (-4))^2 + (-6 - 3)^2]}$$

$$\therefore d(AB) = \sqrt{[(12)^2 + (-9)^2]}$$

$$\therefore d(AB) = \sqrt{144 + 81}$$

$$\therefore d(AB) = \sqrt{225}$$

$$\therefore d(AB) = 15$$

Hence the length of AB is 15 units.

(ii) Let $m:n$ be the ratio in which the line AB is divided by the X axis.

Since the line meets X axis, its y co-ordinate is zero.

By Section formula $y = (my_2 + ny_1)/(m+n)$

$$\therefore 0 = (m \times -6 + n \times 3)/(m+n)$$

$$\therefore 0 = (-6m + 3n)/m+n$$

$$\Rightarrow -6m + 3n = 0$$

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$$\Rightarrow -6m = -3n$$

$$\Rightarrow m/n = -3/-6 = 3/6 = 1/2$$

Hence the ratio is 1:2.

19. (i) Calculate the ratio in which the line segment joining (3, 4) and (-2, 1) is divided by the y-axis.

(ii) In what ratio does the line $x - y - 2 = 0$ divide the line segment joining the points (3, -1) and (8, 9)?

Also, find the coordinates of the point of division.

Solution:

(i) Let $m:n$ be the ratio in which the line segment joining (3,4) and (-2,1) is divided by the Y axis.

Since the line meets Y axis, its x co-ordinate is zero.

Here $x_1 = 3, y_1 = 4$

$x_2 = -2, y_2 = 1$

\therefore By section formula, $x = (mx_2 + nx_1)/(m+n)$

$$\therefore 0 = (m \times -2 + n \times 3)/(m+n)$$

$$\therefore 0 = (-2m + 3n)/(m+n)$$

$$\therefore 0 = -2m + 3n$$

$$\therefore 2m = 3n$$

$$\therefore m/n = 3/2$$

Hence the ratio $m:n$ is 3:2.

(ii) Let the line $x-y-2 = 0$ divide the line segment joining the points (3,-1) and (8,9) in the ratio $m:n$ at the point $P(x,y)$

Here $x_1 = 3, y_1 = -1$

$x_2 = 8, y_2 = 9$

\therefore By section formula, $x = (mx_2 + nx_1)/(m+n)$

$$\therefore x = (m \times 8 + n \times 3)/(m+n)$$

$$\therefore x = (8m + 3n)/(m+n) \quad \dots \quad (i)$$

By Section formula $y = (my_2 + ny_1)/(m+n)$

$$\therefore y = (m \times 9 + n \times -1)/(m+n)$$

$$\therefore y = (9m - n)/(m+n) \quad \dots \quad (ii)$$

Since the point $P(x,y)$ lies on the line $x-y-2 = 0$,

eqn (i) and (ii) will satisfy the equation $x-y-2 = 0 \dots (iii)$

Substitute (i) and (ii) in (iii)

$$[(8m+3n)/(m+n)] - [(9m-n)/(m+n)] - 2 = 0$$

$$[(8m+3n)/(m+n)] - [(9m-n)/(m+n)] - [2(m+n)/(m+n)] = 0$$

$$8m+3n-(9m-n)-2(m+n) = 0$$

$$8m+3n-9m+n-2m-2n = 0$$

$$-3m+2n = 0$$

$$-3m = -2n$$

$$\therefore m/n = -2/-3 = 2/3$$

Hence the ratio $m:n$ is 2:3.

Substitute m and n in (i)

$$x = (8m+3n)/(m+n)$$

$$\therefore x = (8 \times 2 + 3 \times 3)/(2+3)$$

$$\therefore x = (16+9)/5$$

$$\therefore x = 25/5 = 5$$

Substitute m and n in (ii)

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$$y = (9m-n)/(m+n)$$
$$\therefore y = (9 \times 2 - 3)/(2+3)$$
$$\therefore y = (18-3)/5$$
$$\therefore y = 15/5 = 3$$

Hence the co-ordinates of P are (5,3).

20. Given a line segment AB joining the points A (-4, 6) and B (8, -3). Find:

- (i) the ratio in which AB is divided by the y-axis.**
- (ii) find the coordinates of the point of intersection.**
- (iii) the length of AB.**

Solution:

(i) Let m:n be the ratio in which the line segment joining A (-4,6) and B(8,-3) is divided by the Y axis. Since the line meets Y axis, its x co-ordinate is zero.

$$\text{Here } x_1 = -4, y_1 = 6$$
$$x_2 = 8, y_2 = -3$$

$$\therefore \text{By section formula, } x = (mx_2 + nx_1)/(m+n)$$
$$\therefore 0 = (m \times 8 + n \times -4)/(m+n)$$
$$\therefore 0 = (8m - 4n)/(m+n)$$
$$\therefore 0 = 8m - 4n$$
$$\therefore 8m = 4n$$
$$\therefore m/n = 4/8 = 1/2$$

Hence the ratio m:n is 1:2.

(ii) By Section formula $y = (my_2 + ny_1)/(m+n)$

Substitute m and n in above equation

$$\therefore y = (1 \times -3 + 2 \times 6)/(1+2)$$
$$\therefore y = (-3+12)/3$$
$$\therefore y = 9/3 = 3$$

So the co-ordinates of the point of intersection are (0,3).

(iii) By distance formula, $d(AB) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

$$\therefore d(AB) = \sqrt{[(8 - (-4))^2 + (-3 - 6)^2]}$$
$$\therefore d(AB) = \sqrt{[(12)^2 + (-9)^2]}$$
$$\therefore d(AB) = \sqrt{(144 + 81)}$$
$$\therefore d(AB) = \sqrt{225}$$
$$\therefore d(AB) = 15$$

Hence the length of AB is 15 units.

21. (i) Write down the co-ordinates of the point P that divides the line joining A (-4, 1) and B (17,10) in the ratio 1 : 2.

(ii) Calculate the distance OP where O is the origin.

(iii) In what ratio does the y-axis divide the line AB ?

Solution:

(i) Let P(x,y) divides the line segment joining the points A(-4,1), B(17,10) in the ratio 1:2,

$$\text{Here } x_1 = -4, y_1 = 1$$
$$x_2 = 17, y_2 = 10$$

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$$m:n = 1:2$$

$$\therefore \text{By section formula, } x = \frac{mx_2 + nx_1}{m+n}$$

$$\therefore x = \frac{1 \times 17 + 2 \times -4}{1+2}$$

$$\therefore x = \frac{17 + -8}{3}$$

$$\therefore x = \frac{9}{3}$$

$$\therefore x = 3$$

$$\text{By Section formula } y = \frac{my_2 + ny_1}{m+n}$$

$$\therefore y = \frac{1 \times 10 + 2 \times 1}{1+2}$$

$$\therefore y = \frac{10+2}{3}$$

$$\therefore y = \frac{12}{3} = 4$$

Hence the co-ordinates of the point P are (3,4).

(ii) Since O is the origin, the co-ordinates of O are (0,0).

$$\text{By distance formula, } d(OP) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$$

$$\therefore d(OP) = \sqrt{[(0-3)^2 + (0-4)^2]}$$

$$\therefore d(OP) = \sqrt{[(3)^2 + (4)^2]}$$

$$\therefore d(OP) = \sqrt{9+16}$$

$$\therefore d(OP) = \sqrt{25} = 5$$

Hence the distance OP is 5 units.

(iii) Let $m:n$ be the ratio in which Y axis divide line AB.

Since AB touches Y axis, its x co-ordinate will be zero.

$$\text{Here } x_1 = -4, y_1 = 1$$

$$x_2 = 17, y_2 = 10$$

$$\therefore \text{By section formula, } x = \frac{mx_2 + nx_1}{m+n}$$

$$\therefore 0 = \frac{m \times 17 + n \times -4}{m+n}$$

$$\therefore 0 = \frac{17m - 4n}{m+n}$$

$$\Rightarrow 17m - 4n = 0$$

$$\Rightarrow 17m = 4n$$

$$\Rightarrow m/n = 4/17$$

$$\therefore m:n = 4:17$$

Hence the ratio in which Y axis divide line AB is 4:17.

22. Calculate the length of the median through the vertex A of the triangle ABC with vertices A (7, -3), B (5, 3) and C (3, -1).

Solution:

Let M(x,y) be the median of $\triangle ABC$ through A to BC.

M will be the midpoint of BC.

$$x_1 = 5, y_1 = 3$$

$$x_2 = 3, y_2 = -1$$

By midpoint formula, $x = \frac{x_1 + x_2}{2}$

$$\therefore x = \frac{5+3}{2} = \frac{8}{2} = 4$$

By midpoint formula, $y = \frac{y_1 + y_2}{2}$

$$\therefore y = \frac{3+(-1)}{2} = \frac{2}{2} = 1$$

Hence the co-ordinates of M are (4,1).

By distance formula, $d(AM) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

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$$x_1 = 7, y_1 = -3$$

$$x_2 = 4, y_2 = 1$$

$$\therefore d(AM) = \sqrt{[(4-7)^2 + (1-(-3))^2]}$$

$$\therefore d(AM) = \sqrt{[(-3)^2 + (4)^2]}$$

$$\therefore d(AM) = \sqrt{9+16}$$

$$\therefore d(AM) = \sqrt{25} = 5$$

Hence the length of the median AM is 5 units.

23. Three consecutive vertices of a parallelogram ABCD are A (1, 2), B (1, 0) and C (4, 0). Find the fourth vertex D.

Solution:

Let M be the midpoint of the diagonals of the parallelogram ABCD.

Co-ordinate of M will be the midpoint of diagonal AC.

Given points are A(1,2), B(1,0) and C(4,0).

Consider line AC.

$$x_1 = 1, y_1 = 2$$

$$x_2 = 4, y_2 = 0$$

By midpoint formula, $x = (x_1+x_2)/2$

$$\therefore x = (1+4)/2 = 5/2$$

By midpoint formula, $y = (y_1+y_2)/2$

$$\therefore y = (2+0)/2 = 2/2 = 1$$

Hence the co-ordinates of M are (5/2, 1).

M is also the midpoint of diagonal BD.

Consider line BD and M as midpoint.

$$x_1 = 1, y_1 = 0$$

$$x = 5/2, y = 1$$

By midpoint formula, $x = (x_1+x_2)/2$

$$\therefore 5/2 = (1+x_2)/2$$

$$\therefore 5 = 1+x_2$$

$$\therefore x_2 = 5-1 = 4$$

By midpoint formula, $y = (y_1+y_2)/2$

$$\therefore 1 = (0+y_2)/2$$

$$\therefore 1 = y_2/2$$

$$\Rightarrow y_2 = 2$$

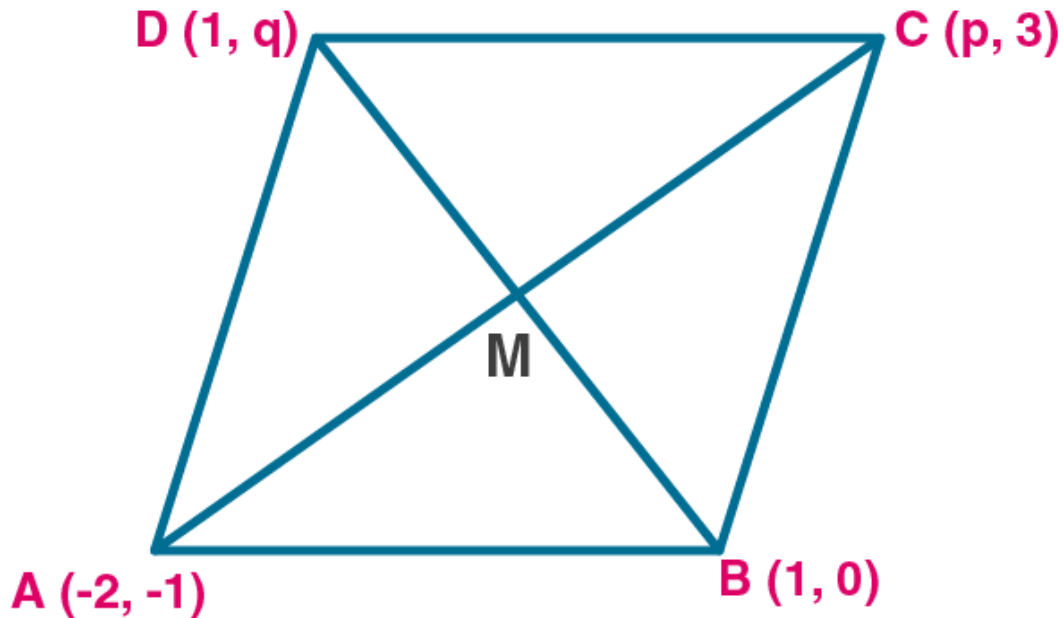
Hence the co-ordinates of D are (4,2).

24. If the points A (- 2, - 1), B (1, 0), C (p, 3) and D (1, q) form a parallelogram ABCD, find the values of p and q.

Solution:



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Given vertices of the parallelogram are A(-2,-1), B(1, 0), C(p,3) and D(1,q).

Let M(x,y) be the midpoint of the diagonals of the parallelogram ABCD.

Diagonals AC and BD bisect each other at M.

When M is the midpoint of AC

By midpoint formula,

$$x = \frac{-2+p}{2} = \frac{p-2}{2} \quad \dots(i)$$

$$y = \frac{-1+3}{2} = \frac{2}{2} = 1 \quad \dots(ii)$$

When M is the midpoint of BD

By midpoint formula,

$$x = \frac{1+1}{2} = \frac{2}{2} = 1 \quad \dots(iii)$$

$$y = \frac{q+0}{2} = \frac{q}{2} \quad \dots(iv)$$

Equating (i) and (iii), we get

$$\frac{p-2}{2} = 1$$

$$\Rightarrow p-2 = 2$$

$$\Rightarrow p = 2+2 = 4$$

Equating (ii) and (iv), we get

$$\frac{q}{2} = 1$$

$$\Rightarrow q = 2$$

Hence the value of p and q are 4 and 2 respectively.

25. If two vertices of a parallelogram are (3, 2) (-1, 0) and its diagonals meet at (2, -5), find the other two vertices of the parallelogram.

Solution:

Let A(3,2) and B(-1,0) be the two vertices of the parallelogram ABCD.

Let M(2,-5) be the point where diagonals meet.

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Since the diagonals of the parallelogram bisect each other, M is the midpoint of AC and BD.

Consider A-M-C

Let co-ordinate of C be (x_2, y_2)

$$x_1 = 3, y_1 = 2$$

$$x = 2, y = -5$$

By midpoint formula, $x = (x_1 + x_2)/2$

$$2 = (3 + x_2)/2$$

$$\Rightarrow 3 + x_2 = 4$$

$$\Rightarrow x_2 = 4 - 3 = 1$$

By midpoint formula, $y = (y_1 + y_2)/2$

$$-5 = (2 + y_2)/2$$

$$-10 = 2 + y_2$$

$$\Rightarrow y_2 = -10 - 2 = -12$$

Hence the co-ordinates of the point C are $(1, -12)$.

Consider B-M-D

Let co-ordinate of D be (x_2, y_2)

$$x_1 = -1, y_1 = 0$$

$$x = 2, y = -5$$

By midpoint formula, $x = (x_1 + x_2)/2$

$$2 = (-1 + x_2)/2$$

$$\Rightarrow -1 + x_2 = 4$$

$$\Rightarrow x_2 = 4 + 1 = 5$$

By midpoint formula, $y = (y_1 + y_2)/2$

$$-5 = (0 + y_2)/2$$

$$-10 = 0 + y_2$$

$$\Rightarrow y_2 = -10$$

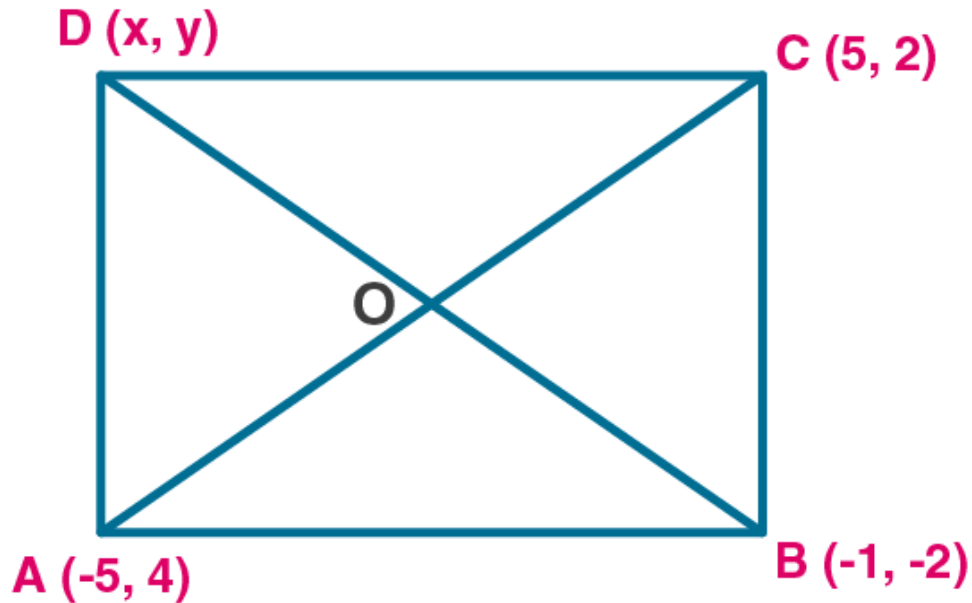
Hence the co-ordinates of the point D are $(5, -10)$.



26. Prove that the points A $(-5, 4)$, B $(-1, -2)$ and C $(5, 2)$ are the vertices of an isosceles right angled triangle. Find the co-ordinates of D so that ABCD is a square.

Solution:

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Given points are A(-5,4), B(-1,-2) and C(5,2) are given.

Since these are vertices of an isosceles triangle ABC then $AB = BC$.

By distance formula, $d(AB) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

Here $x_1 = -5$, $y_1 = 4$

$x_2 = -1$, $y_2 = -2$

$$\therefore d(AB) = \sqrt{[(-1-(-5))^2+(-2-4)^2]}$$

$$\therefore d(AB) = \sqrt{[(4)^2+(6)^2]}$$

$$\therefore d(AB) = \sqrt{(16+36)}$$

$$\therefore d(AB) = \sqrt{52} \quad \dots(i)$$

By distance formula, $d(BC) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

Here $x_1 = -1$, $y_1 = -2$

$x_2 = 5$, $y_2 = 2$

$$\therefore d(BC) = \sqrt{[(5-(-1))^2+(2-(-2))^2]}$$

$$\therefore d(BC) = \sqrt{[(6)^2+(4)^2]}$$

$$\therefore d(BC) = \sqrt{(36+16)}$$

$$\therefore d(BC) = \sqrt{52} \quad \dots(ii)$$

From (i) and (ii) $AB = BC$

So given points are the vertices of isosceles triangle.

By distance formula, $d(AC) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

Here $x_1 = -5$, $y_1 = 4$

$x_2 = 5$, $y_2 = 2$

$$\therefore d(AC) = \sqrt{[(5-(-5))^2+(2-4)^2]}$$

$$\therefore d(AC) = \sqrt{[(10)^2+(-2)^2]}$$

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$$\therefore d(AC) = \sqrt{(100+4)}$$

$$\therefore d(AC) = \sqrt{104} \quad \dots(\text{iii})$$

Apply Pythagoras theorem to triangle ABC

$$AB^2 + BC^2 = (\sqrt{52})^2 + (\sqrt{52})^2$$

$$= 52 + 52$$

$$= 104 \quad \dots(\text{iv})$$

$$AC^2 = (\sqrt{104})^2 = 104 \dots(\text{v})$$

From (iv) and (v) we got

$$AB^2 + BC^2 = AC^2$$

So Pythagoras theorem is satisfied.

So the triangle is an isosceles right angled triangle.

Hence proved.

If ABCD is a square, let the diagonals meet at O.

Diagonals of a square bisect each other. So, O is the midpoint of AC and BD.

Consider A-O-C

$$x_1 = -5, y_1 = 4$$

$$x_2 = 5, y_2 = 2$$

By midpoint formula, $x = (x_1 + x_2)/2$

$$x = (-5 + 5)/2 = 0/2 = 0$$

By midpoint formula, $y = (y_1 + y_2)/2$

$$y = (4 + 2)/2 = 6/2 = 3$$

So co-ordinate of O is (0,3).

Consider B-O-D

Let co-ordinate of D be (x_2, y_2)

$$x_1 = -1, y_1 = -2$$

$$x = 0, y = 3$$

By midpoint formula, $x = (x_1 + x_2)/2$

$$0 = (-1 + x_2)/2$$

$$\Rightarrow -1 + x_2 = 0$$

$$\Rightarrow x_2 = 1$$

By midpoint formula, $y = (y_1 + y_2)/2$

$$3 = (-2 + y_2)/2$$

$$6 = -2 + y_2$$

$$\Rightarrow y_2 = 6 + 2 = 8$$

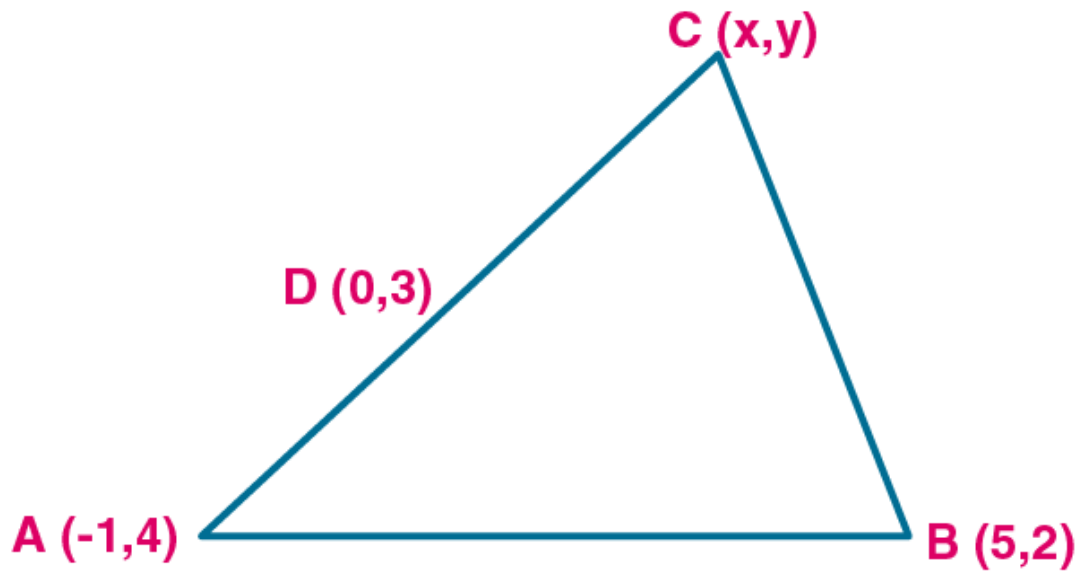
Hence the co-ordinates of the point D are (1,8).

27. Find the third vertex of a triangle if its two vertices are $(-1, 4)$ and $(5, 2)$ and midpoint of one sides is $(0, 3)$.

Solution:



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Let A (-1,4) and B(5,2) are the vertices of the triangle and let D(0,3) is the midpoint of side AC.

Let co-ordinate of C be (x,y).

Consider D(0,3) as midpoint of AC

By midpoint formula,

$$\frac{-1+x}{2} = 0$$

$$\therefore -1 + x = 0$$

$$\therefore x = 1$$

By midpoint formula,

$$\frac{4+y}{2} = 3$$

$$\therefore 4+y = 6$$

$$\therefore y = 6-4 = 2$$

So the co-ordinates of C are (1,2).

Consider D(0,3) as midpoint of BC

By midpoint formula,

$$\frac{5+x}{2} = 0$$

$$\therefore 5 + x = 0$$

$$\therefore x = -5$$

By midpoint formula,

$$\frac{2+y}{2} = 3$$

$$\therefore 2+y = 6$$

$$\therefore y = 6-2 = 4$$

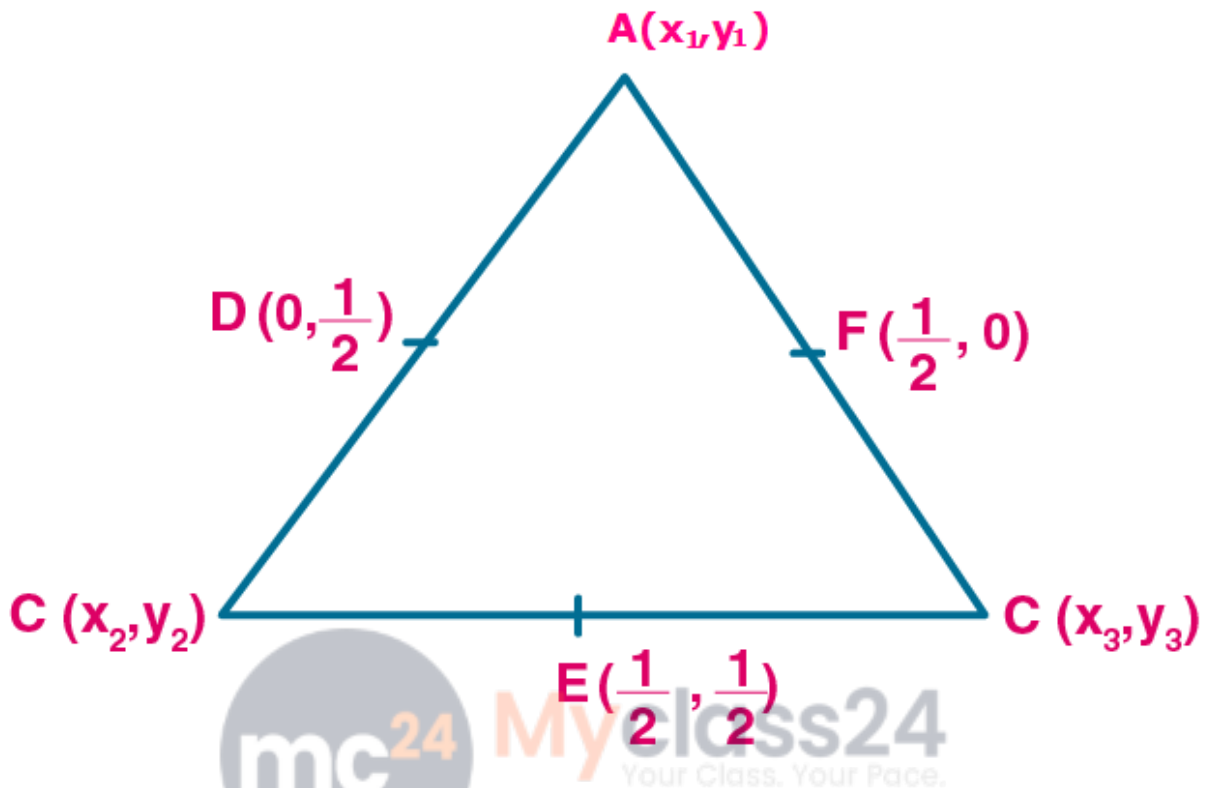
So the co-ordinates of C are (-5,4).

Hence the co-ordinates of the point C will be (1,2) or (-5,4).

28. Find the coordinates of the vertices of the triangle the middle points of whose sides are $(0, \frac{1}{2})$, $(\frac{1}{2}, \frac{1}{2})$ and $(\frac{1}{2}, 0)$.

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Solution:



Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of the triangle ABC .

Consider AB

By midpoint formula, $(x_1+x_2)/2 = 0$

$$\therefore x_1+x_2 = 0$$

$$\therefore x_1 = -x_2 \quad \dots(i)$$

By midpoint formula, $(y_1+y_2)/2 = \frac{1}{2}$

$$\therefore y_1+y_2 = 1 \quad \dots(ii)$$

Consider AC

By midpoint formula, $(x_1+x_3)/2 = \frac{1}{2}$

$$\therefore x_1+x_3 = 1 \quad \dots(iii)$$

By midpoint formula, $(y_1+y_3)/2 = 0$

$$\therefore y_1+y_3 = 0$$

$$\therefore y_1 = -y_3 \quad \dots(iv)$$

Consider BC

By midpoint formula, $(x_2+x_3)/2 = \frac{1}{2}$

$$\therefore x_2+x_3 = 1 \quad \dots(v)$$

By midpoint formula, $(y_2+y_3)/2 = \frac{1}{2}$

$$\therefore y_2+y_3 = 1 \quad \dots(vi)$$

Substitute (i) in (iii)

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Then (iii) becomes $-x_2+x_3 = 1$

Equation (v) $\Rightarrow x_2+x_3 = 1$

Adding above two equations, we get

$$2x_3 = 2$$

$$\therefore x_3 = 2/2 = 1$$

Substitute $x_3 = 1$ in (iii), we get $x_1 = 0$

$$\therefore x_2 = 0 \quad [\text{From (i)}]$$

So $x_1 = 0, x_2 = 0, x_3 = 1$

Substitute (iv) in (ii)

Then (ii) becomes $-y_3+y_2 = 1$

Equation (vi) $\Rightarrow y_2+y_3 = 1$

Adding above two equations, we get

$$2y_2 = 2$$

$$\therefore y_2 = 2/2 = 1$$

Substitute $y_2 = 1$ in (i), we get $y_1 = 0$

$$\therefore y_3 = 0$$

So $y_1 = 0, y_2 = 1, y_3 = 0$

Hence the Co-ordinates of vertices are A(0,0), B(0,1) and C(1,0).

29. Show by section formula that the points (3, - 2), (5, 2) and (8, 8) are collinear.

Solution:

Let the point B(5,2) divides the line joining A(3,-2) and C(8,8) in the ratio m:n.

Then by section formula, $x = (mx_2+nx_1)/(m+n)$

$$\therefore 5 = (m \times 8 + n \times 3)/(m+n)$$

$$\therefore 5 = (8m+3n)/(m+n)$$

$$5m+5n = 8m+3n$$

$$2n = 3m$$

$$\therefore m/n = 2/3 \quad \dots(i)$$

By section formula, $y = (my_2+ny_1)/(m+n)$

$$\therefore 2 = (m \times 8 + n \times -2)/(m+n)$$

$$\therefore 2 = (8m-2n)/(m+n)$$

$$2m+2n = 8m-2n$$

$$6m = 4n$$

$$m/n = 4/6 = 2/3 \quad \dots(ii)$$

Here ratios are same.

So the points are collinear.

30. Find the value of p for which the points (- 5, 1), (1, p) and (4, - 2) are collinear.

Solution:

Let A(-5,1) divides the line joining (1,p) and (4,-2) in the ratio m:n

Then by section formula, $x = (mx_2+nx_1)/(m+n)$

$$\therefore -5 = (m \times 4 + n \times 1)/(m+n)$$

$$\therefore -5 = (4m+n)/(m+n)$$

$$-5m-5n = 4m+n$$

$$-9m = 6n$$

$$\therefore m/n = -9/6 = -2/3 \quad \dots(i)$$

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By section formula, $y = (my_2 + ny_1)/(m+n)$

$$\therefore 1 = (m \times -2 + n \times p)/(m+n)$$

$$\therefore 1 = (-2m + pn)/(m+n)$$

$$m+n = -2m+pn$$

$$3m = (p-1)n$$

$$m/n = (p-1)/3 \quad \dots(ii)$$

Equating (i) and (ii)

$$(p-1)/3 = -2/3$$

$$p-1 = -2$$

$$p = -2+1 = -1$$

Hence the value of p is -1.

31. A (10, 5), B (6, -3) and C (2, 1) are the vertices of triangle ABC. L is the mid point of AB, M is the mid-point of AC. Write down the co-ordinates of L and M. Show that $LM = \frac{1}{2} BC$.

Solution:

Given points are A(10,5), B(6,-3) and C(2,1).

Let L(x,y) be the midpoint of AB.

$$\text{Here } x_1 = 10, y_1 = 5$$

$$x_2 = 6, y_2 = -3$$

By midpoint formula, $x = (x_1 + x_2)/2$

$$\therefore x = (10+6)/2 = 16/2 = 8$$

By midpoint formula, $y = (y_1 + y_2)/2$

$$\therefore y = (5-3)/2 = 2/2 = 1$$

So co-ordinates of L are (8,1).

Let M(x,y) be the midpoint of AC.

$$\text{Here } x_1 = 10, y_1 = 5$$

$$x_2 = 2, y_2 = 1$$

By midpoint formula, $x = (x_1 + x_2)/2$

$$\therefore x = (10+2)/2 = 12/2 = 6$$

By midpoint formula, $y = (y_1 + y_2)/2$

$$\therefore y = (5+1)/2 = 6/2 = 3$$

So co-ordinates of M are (6,3).

By distance formula, $d(LM) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

The points are L(8,1) and M(6,3)

$$\text{So } x_1 = 8, y_1 = 1$$

$$x_2 = 6, y_2 = 3$$

$$\therefore d(LM) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$$

$$\therefore d(LM) = \sqrt{[(6-8)^2 + (3-1)^2]}$$

$$\therefore d(LM) = \sqrt{[(-2)^2 + (2)^2]}$$

$$\therefore d(LM) = \sqrt{4+4}$$

$$\therefore d(LM) = \sqrt{8} = 2\sqrt{2} \quad \dots(i)$$

By distance formula, $d(BC) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

The points are B(6,-3) and C(2,1).

$$\text{So } x_1 = 6, y_1 = -3$$

$$x_2 = 2, y_2 = 1$$

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$$\begin{aligned}\therefore d(BC) &= \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]} \\ \therefore d(BC) &= \sqrt{[(2-6)^2+(1-(-3))^2]} \\ \therefore d(BC) &= \sqrt{[(-4)^2+(4)^2]} \\ \therefore d(BC) &= \sqrt{(16+16)} \\ \therefore d(BC) &= \sqrt{32} = 4\sqrt{2} \quad \dots(ii)\end{aligned}$$

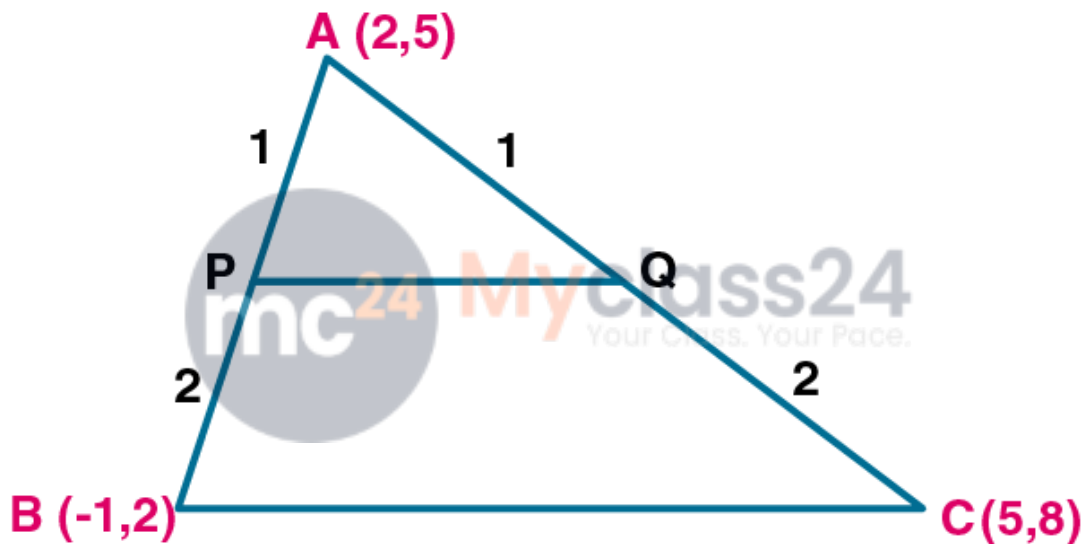
From (i) and (ii), $LM = \frac{1}{2} BC$

32. A (2, 5), B (-1, 2) and C (5, 8) are the vertices of a triangle ABC. P and Q are points on AB and AC respectively such that $AP : PB = AQ : QC = 1 : 2$.

(i) Find the co-ordinates of P and Q.

(ii) Show that $PQ = \frac{1}{3} BC$

Solution:



(i) Given vertices of the $\triangle ABC$ are A(2,5), B(-1,2) and C(5,8).

P and Q are points on AB and AC respectively such that $AP:PB = AQ :QC = 1:2$.

P(x,y) divides AB in the ratio 1:2.

$$x_1 = 2, y_1 = 5$$

$$x_2 = -1, y_2 = 2$$

$$m:n = 1:2$$

By section formula, $x = \frac{mx_2+nx_1}{(m+n)}$

$$\therefore x = \frac{(1 \times -1 + 2 \times 2)}{(1+2)}$$

$$\therefore x = \frac{(-1+4)}{(3)}$$

$$\therefore x = \frac{3}{3} = 1$$

By section formula, $y = \frac{my_2+ny_1}{(m+n)}$

$$\therefore y = \frac{(1 \times 2 + 2 \times 5)}{(1+2)}$$

$$\therefore y = \frac{(2+10)}{(3)}$$

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$$\therefore y = 12/3 = 4$$

\therefore Co-ordinates of P are (1,4).

Q(x,y) divides AC in the ratio 1:2.

$$x_1 = 2, y_1 = 5$$

$$x_2 = 5, y_2 = 8$$

$$m:n = 1:2$$

By section formula, $x = (mx_2 + nx_1)/(m+n)$

$$\therefore x = (1 \times 5 + 2 \times 2)/(1+2)$$

$$\therefore x = (5+4)/(3)$$

$$\therefore x = 9/3 = 3$$

By section formula, $y = (my_2 + ny_1)/(m+n)$

$$\therefore y = (1 \times 8 + 2 \times 5)/(1+2)$$

$$\therefore y = (8+10)/(3)$$

$$\therefore y = 18/3 = 6$$

\therefore Co-ordinates of Q are (3,6).

(ii) By distance formula, $d(PQ) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

Points are P(1,4) and Q(3,6).

$$\text{So } x_1 = 1, y_1 = 4$$

$$x_2 = 3, y_2 = 6$$

$$\therefore d(PQ) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$$

$$\therefore d(PQ) = \sqrt{[(3-1)^2 + (6-4)^2]}$$

$$\therefore d(PQ) = \sqrt{[(2)^2 + (2)^2]}$$

$$\therefore d(PQ) = \sqrt{4+4}$$

$$\therefore d(PQ) = \sqrt{8} = 2\sqrt{2} \quad \dots(i)$$

By distance formula, $d(BC) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

Points are B(-1,2) and C(5,8).

$$\text{So } x_1 = -1, y_1 = 2$$

$$x_2 = 5, y_2 = 8$$

$$\therefore d(BC) = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$$

$$\therefore d(BC) = \sqrt{[(5 - (-1))^2 + (8 - 2)^2]}$$

$$\therefore d(BC) = \sqrt{[(6)^2 + (6)^2]}$$

$$\therefore d(BC) = \sqrt{36+36}$$

$$\therefore d(BC) = \sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2} \quad \dots(ii)$$

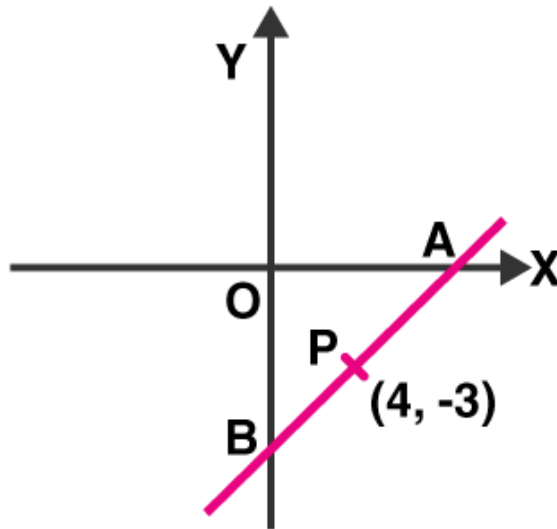
$$BC/3 = 6\sqrt{2}/3 = 2\sqrt{2} = PQ$$

$$\therefore PQ = 1/3 BC.$$

Hence proved.

33. The mid-point of the line segment AB shown in the adjoining diagram is (4, -3). Write down the co-ordinates of A and B.

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Solution:

Let $P(4, -3)$ be the midpoint of line joining the points A and B.

Since A lies on X axis, its co-ordinates are $(x_2, 0)$

Since B lies on Y axis, its co-ordinates are $(0, y_1)$

By midpoint formula, $x = (x_1 + x_2)/2$

$$\therefore 4 = (0 + x_2)/2$$

$$\therefore x_2 = 4 \times 2 = 8$$

By midpoint formula, $y = (y_1 + y_2)/2$

$$\therefore -3 = (y_1 + 0)/2$$

$$\therefore y_1 = -3 \times 2 = -6$$

Hence the co-ordinates of A and B are $(8, 0)$ and $(0, -6)$ respectively.

34. Find the co-ordinates of the centroid of a triangle whose vertices are A $(-1, 3)$, B $(1, -1)$ and C $(5, 1)$ (2006)

Solution:

Given vertices of the triangle are A $(-1, 3)$, B $(1, -1)$ and C $(5, 1)$

Co-ordinates of the centroid of a triangle, whose vertices are (x_1, y_1) , (x_2, y_2) and (x_3, y_3) are

$$[(x_1 + x_2 + x_3)/3, (y_1 + y_2 + y_3)/3]$$

$$(x_1, y_1) = (-1, 3)$$

$$(x_2, y_2) = (1, -1)$$

$$(x_3, y_3) = (5, 1)$$

$$\therefore (x_1 + x_2 + x_3)/3 = (-1 + 1 + 5)/3 = 5/3$$

$$\therefore (y_1 + y_2 + y_3)/3 = (3 - 1 + 1)/3 = 3/3 = 1$$

Hence the co-ordinates of centroid are $(5/3, 1)$.

35. Two vertices of a triangle are $(3, -5)$ and $(-7, 4)$. Find the third vertex given that the centroid is $(2, -1)$.

Solution:

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Let third vertex be $C(x_3, y_3)$.

$$\text{Given } (x_1, y_1) = (3, -5)$$

$$(x_2, y_2) = (-7, 4)$$

Co-ordinates of centroid are $(2, -1)$

Co-ordinates of the centroid of a triangle, whose vertices are (x_1, y_1) , (x_2, y_2) and (x_3, y_3) are

$$\left[\frac{(x_1 + x_2 + x_3)}{3}, \frac{(y_1 + y_2 + y_3)}{3} \right]$$

$$\therefore \frac{(x_1 + x_2 + x_3)}{3} = \frac{(3 + -7 + x_3)}{3} = 2 \quad [\text{x co-ordinate of centroid}]$$

$$\therefore -4 + x_3 = 2 \times 3$$

$$\therefore -4 + x_3 = 6$$

$$\therefore x_3 = 6 + 4$$

$$\therefore x_3 = 10$$

$$\frac{(y_1 + y_2 + y_3)}{3} = -1 \quad [\text{y co-ordinate of centroid}]$$

$$-5 + 4 + y_3 = -1 \times 3$$

$$-1 + y_3 = -3$$

$$\therefore y_3 = -3 + 1$$

$$\therefore y_3 = -2$$

Hence the third vertex is $(10, -2)$.

36. The vertices of a triangle are A $(-5, 3)$, B $(p, -1)$ and C $(6, q)$. Find the values of p and q if the centroid of the triangle ABC is the point $(1, -1)$.

Solution:

Given vertices of the triangle are A $(-5, 3)$, B $(p, -1)$ and C $(6, q)$.

Co-ordinates of centroid are $(1, -1)$.

Co-ordinates of the centroid of a triangle, whose vertices are (x_1, y_1) , (x_2, y_2) and (x_3, y_3) are

$$\left[\frac{(x_1 + x_2 + x_3)}{3}, \frac{(y_1 + y_2 + y_3)}{3} \right]$$

$$(x_1, y_1) = (-5, 3)$$

$$(x_2, y_2) = (p, -1)$$

$$(x_3, y_3) = (6, q)$$

$$\therefore \text{x co-ordinate of centroid, } \frac{(x_1 + x_2 + x_3)}{3} = \frac{(-5 + p + 6)}{3} = 1$$

$$\therefore p + 1 = 3$$

$$\therefore p = 3 - 1$$

$$\therefore p = 2$$

$$\therefore \text{y co-ordinate of centroid, } \frac{(y_1 + y_2 + y_3)}{3} = \frac{(3 - 1 + q)}{3} = -1$$

$$\therefore 2 + q = 3 \times -1$$

$$\therefore 2 + q = -3$$

$$\therefore q = -3 - 2$$

$$\therefore q = -5$$

Hence the value of p and q are 2 and -5 respectively.

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