NCERT Solutions for Class-XII Maths

Chapter-10.2

NCERT Chemistry Class 12

1. Compute the magnitude of the following vectors:

$\vec{a} = \hat{i} + \hat{j} + k; \vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}; \vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$

1. The given vectors are:

$\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k};\ \vec{b} = 2\hat{\imath} - 7\hat{\jmath} - 3\hat{k};\ \vec{c} = \frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} - \frac{1}{\sqrt{3}}\hat{k}$

$|\vec{a}| = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3}$

$|\vec{b}| = \sqrt{(2)^2 + (-7)^2 + (-3)^2}$

$= \sqrt{4 + 49 + 9}$

$= \sqrt{62}$

$|\vec{c}\,| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(-\frac{1}{\sqrt{3}}\right)^2}$

$= \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = 1$

2. Write two different vectors having same magnitude.

2. Let

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$ And $\vec{b} = \hat{i} - \hat{j} - \hat{k}$

We can see clearly $\vec{a} \neq \vec{b}$ because the all the coefficients of $\vec{a}$ and $\vec{b}$ are not same. In $\hat{j}$ and $\hat{k}$, the coefficients are different. Now, we check the magnitude of both,

$|\vec{a}| = \sqrt{x^2 + y^2 + z^2} = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$ And

$|\vec{b}| = \sqrt{x^2 + y^2 + z^2} = \sqrt{1^2 + (-1)^2 + (-1)^2} = \sqrt{3}$

So, magnitude of both vectors is same but they are different.

3. Write two different vectors having same direction.

3. Consider $\vec{p} = \left(\hat{i} + \hat{j} + \hat{k}\right)$ and $\vec{q} = \left(2\hat{i} + 2\hat{j} + 2\hat{k}\right)$.

The direction cosines of $\vec{p}$ are given by,

$I=\frac{1}{\sqrt{1^2+1^2+1^2}}=\frac{1}{\sqrt{3}}, m=\frac{1}{\sqrt{1^2+1^2+1^2}}=\frac{1}{\sqrt{3}}$, and $n=\frac{1}{\sqrt{1^2+1^2+1^2}}=\frac{1}{\sqrt{3}}$.

The direction cosines of $\vec{q}$ are given by

$$I=\frac{2}{\sqrt{2^2+2^2+2^2}}=\frac{2}{2\sqrt{3}}=\frac{1}{\sqrt{3}}, m=\frac{2}{\sqrt{2^2+2^2+2^2}}=\frac{2}{2\sqrt{3}}=\frac{1}{\sqrt{3}},$$

and $n=\frac{2}{\sqrt{2^2+2^2+2^2}}=\frac{2}{2\sqrt{3}}=\frac{1}{\sqrt{3}}$

The direction cosines of $\vec{p}$ and $\vec{q}$ are the same. Hence, the two vectors have the same direction.

4. Find the values of x and y so that the vectors $2\hat{i}+3\hat{j}$ and $x\hat{i}+y\hat{j}$ are equal.

4. For two vectors to be equal, the coefficients of both vectors should be equal.

Comparing the $\hat{i}$-coefficient, we get x=2

5. Find the scalar and vector components of the vector with initial point (2, 1) and terminal point (– 5, 7).

5. The vector with the initial point $P(2,1)$ and terminal point $Q(-5,7)$ can be given by,

$\overrightarrow{PQ}=(-5-2)\hat{i}+(7-1)\hat{j}$

$\Rightarrow \overrightarrow{PQ}=-7\hat{i}+6\hat{j}$

Hence, the required scalar components are -7 and 6 while the vector components are $-7\hat{i}$ and $6\hat{j}$

6. Find the sum of the vectors $\vec{a}=\hat{i}-2\hat{j}+\hat{k}, \vec{b}=-2\hat{i}+4\hat{j}+5\hat{k}$, and $\vec{c}=\hat{i}-6\hat{j}-7\hat{k}$.

6. $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$

$\vec{b}=-2\hat{i}+4\hat{j}+5\hat{k}$ And $\vec{c}=\hat{i}-6\hat{j}-7\hat{k}$

We want to find $\vec{a}+\vec{b}+\vec{c}$

To find the sum, we add coefficients of $\hat{i},\hat{j},\hat{k}$ to each other.

So,

$\vec{a}+\vec{b}+\vec{c}=(\hat{i}-2\hat{j}+\hat{k})+(-2\hat{i}+4\hat{j}+5\hat{k})+(\hat{i}-6\hat{j}-7\hat{k})$

$\Rightarrow \vec{a}+\vec{b}+\vec{c}=0\hat{i}-4\hat{j}-\hat{k}$

7. Find the unit vector in the direction of the vector $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$.

7. The unit vector $\hat{a}$ in the direction of vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$

$|\vec{a}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{1+1+4} = \sqrt{6}$

$$\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}} = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$$

8. Find the unit vector in the direction of vector $\overrightarrow{PQ}$, where P and Q are the points (1, 2, 3) and (4, 5, 6), respectively.

8. So, $\vec{P} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{Q} = 4\hat{i} + 5\hat{j} + 6\hat{k}$

Now we want to find the $\overrightarrow{PQ}$

$\overrightarrow{PQ} = \vec{Q} - \vec{P}$

$\Rightarrow \overrightarrow{PQ} = \left(4\hat{i} + 5\hat{j} + 6\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

$\Rightarrow \overrightarrow{PQ} = \left(3\hat{i} + 3\hat{j} + 3\hat{k}\right)$

Now, we have to find the unit direction in the direction of $\overrightarrow{PQ}$.We know that unit vector means that the magnitude of the vector is 1(unit).

It is defined as $\widehat{PQ} = \frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|}$

So, we find the magnitude of $\vec{a}$ first.

$\Rightarrow |\overrightarrow{PQ}| = \sqrt{x^2 + y^2 + z^2} = \sqrt{3^2 + 3^2 + 3^2} = \sqrt{27} = 3\sqrt{3}$

$$\widehat{PQ} = \frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} = \frac{\left(3\hat{i} + 3\hat{j} + 3\hat{k}\right)}{3\sqrt{3}} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

9. For given vectors, $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$, and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$, find the unit vector in the direction of the vector $\vec{a} + \vec{b}$.

9. The given vectors are $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$.

$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$

$\vec{b} = -\hat{i} + \hat{j} - \hat{k}$

$\therefore \vec{a} + \vec{b} = (2-1)\hat{i} + (-1+1)\hat{j} + (2-1)\hat{k} = 1\hat{i} + 0\hat{j} + 1\hat{k} = \hat{i} + \hat{k}$

$|\vec{a} + \vec{b}| = \sqrt{1^2 + 1^2} = \sqrt{2}$

Hence, the unit vector in the direction of $\left(\vec{a}+\vec{b}\right)$ is

$$\frac{\left(\vec{a}+\vec{b}\right)}{\left|\vec{a}+\vec{b}\right|}=\frac{\hat{i}+\hat{k}}{\sqrt{2}}=\frac{1}{2}\hat{i}+\frac{1}{\sqrt{2}}\hat{k}$$

10. Find a vector in the direction of vector $5\hat{i}-\hat{j}+2\hat{k}$. which has magnitude 8 units.

10. Let $\vec{a}=5\hat{i}-\hat{j}+2\hat{k}$

The vector in the direction of $\vec{a}$ having unit magnitude is $\hat{a}$.

So, The vector in the direction of $\vec{a}$ having magnitude 8 units $=8\hat{a}$.

$$\Rightarrow\left|\vec{a}\right|=\sqrt{x^2+y^2+z^2}=\sqrt{5^2+(-1)^2+2^2}=\sqrt{30}$$

$$8\hat{a}=8\frac{\vec{a}}{\left|\vec{a}\right|}=8\frac{\left(5\hat{i}-\hat{j}+2\hat{k}\right)}{\sqrt{30}}=\frac{40}{\sqrt{30}}\hat{i}-\frac{8}{\sqrt{30}}\hat{j}+\frac{16}{\sqrt{30}}\hat{k}$$

11. Show that the vectors $2\hat{i}-3\hat{j}+4\hat{k}$, and $-4\hat{i}+6\hat{j}-8\hat{k}$ are collinear.

11. Let $\vec{a}=2\hat{i}-3\hat{j}+4\hat{k}$ and $\vec{b}=-4\hat{i}+6\hat{j}-8\hat{k}$.

It is observed that $\vec{b}=-4\hat{i}+6\hat{j}-8\hat{k}=-2\left(2\hat{i}-3\hat{j}+4\hat{k}\right)=-2\vec{a}\ \therefore\vec{b}=\lambda\vec{a}$

where,

$\lambda=-2$

Hence, the given vectors are collinear.

12. Find the direction cosines of the vector $\hat{i}+2\hat{j}+3\hat{k}$.

12. The direction cosines of a vector are defined as the coefficients of $\hat{i},\hat{j},\hat{k}$ in the unit vector in the direction of the vector.

So, first we find the unit vector in the direction of the vector.

Let $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$

$$\Rightarrow\left|\vec{a}\right|=\sqrt{x^2+y^2+z^2}=\sqrt{1^2+2^2+3^2}=\sqrt{14}$$

$$\hat{a}=\frac{\vec{a}}{\left|\vec{a}\right|}=\frac{\left(\hat{i}+2\hat{j}+3\hat{k}\right)}{\sqrt{14}}=\frac{1}{\sqrt{14}}\hat{i}+\frac{2}{\sqrt{14}}\hat{j}+\frac{3}{\sqrt{14}}\hat{k}$$

Therefore, The direction cosines of the given vector are $\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}$.

13. Find the direction cosines of the vector joining the points A(1, 2, –3) and B(–1, –2, 1), directed from A to B.

13. The given points are $A(1,2,-3)$ and $B(-1,-2,1)$.

$\therefore \overrightarrow{AB} = (-1-1)\hat{i} + (-2-2)\hat{j} + \{1-(-3)\}\hat{k}$

$$\Rightarrow \overrightarrow{AB} = -2\hat{i} - 4\hat{j} + 4\hat{k}$$

$\therefore \left|\overrightarrow{AB}\right| = \sqrt{(-2)^2 + (-4)^2 + 4^2} = \sqrt{4+16+16} = \sqrt{36} = 6$

Hence, the direction cosines of $\overrightarrow{AB}$ are $\left(-\frac{2}{6}, -\frac{4}{6}, \frac{4}{6}\right) = \left(-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right)$.

14. Show that the vector $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the axes OX, OY and OZ.

14. To find the inclination of the vector with OX,OY,OZ. We find the direction cosines of the vector.

We know that the direction cosines of a vector are defined as the coefficients of $\hat{i}, \hat{j}, \hat{k}$ in the unit vector in the direction of the vector.

So, first we find the unit vector in the direction of the vector.

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$\Rightarrow |\vec{a}| = \sqrt{x^2 + y^2 + z^2} = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{(\hat{i} + \hat{j} + \hat{k})}{\sqrt{3}} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$

Therefore, The direction cosines of the given vector are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$.

Let θ_1 be the angle between $\vec{a}$ and OX.

Therefore, $\cos\theta_1 = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta_1 = \cos^{-1}\frac{1}{\sqrt{3}}$

Similarly, Let θ_2 be the angle between $\vec{a}$ and OY.

Therefore, $\cos\theta_2 = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta_2 = \cos^{-1}\frac{1}{\sqrt{3}}$

And, θ_3 be the angle between $\vec{a}$ and OZ.

Therefore, $\cos\theta_3 = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta_3 = \cos^{-1}\frac{1}{\sqrt{3}}$

Therefore, $\theta_1 = \theta_2 = \theta_3$

Hence proved that the vector is equally inclined with the axes OX, OY, and OZ.

15. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i}+2\hat{j}-\hat{k}$ and $-\hat{i}+\hat{j}+\hat{k}$ respectively, in the ratio 2 : 1

(i) internally (ii) externally

15. The position vector of point R dividing the line segment joining two points P and Q in the ratio $m : n$ is given by:

i. Internally: $\dfrac{m\vec{b}+n\vec{a}}{m+n}$

ii. Externally: $\dfrac{m\vec{b}-n\vec{a}}{m-n}$

Position vectors of P and Q are given as:

$\overrightarrow{OP}=\hat{i}+2\hat{j}-\hat{k}$ and $\overrightarrow{OQ}=-\hat{i}+\hat{j}+\hat{k}$

(i) The position vector of point R which divides the line joining two points P and Q internally in the ratio $2:1$ is given by,

$$\overline{OR}=\frac{2\left(-\hat{i}+\hat{j}+\hat{k}\right)+1\left(\hat{i}+2\hat{j}-\hat{k}\right)}{2+1}=\frac{\left(-2\hat{i}+2\hat{j}+2\hat{k}\right)+\left(\hat{i}+2\hat{j}-\hat{k}\right)}{3}$$

$$=\frac{-\hat{i}+4\hat{j}+\hat{k}}{3}=-\frac{1}{3}\hat{i}+\frac{4}{3}\hat{j}+\frac{1}{3}\hat{k}$$

(ii) The position vector of point R which divides the line joining two points P and Q externally in the ratio $2:1$ is given by,

$$\overrightarrow{OR}=\frac{2\left(-\hat{i}+\hat{j}+\hat{k}\right)-1\left(\hat{i}+2\hat{j}-\hat{k}\right)}{2-1}=\left(-2\hat{i}+2\hat{j}+2\hat{k}\right)-\left(\hat{i}+2\hat{j}-\hat{k}\right)$$

$$=-3\hat{i}+3\hat{k}$$

16. Find the position vector of the mid point of the vector joining the points P(2, 3, 4) and Q(4, 1, –2).

16. So, $\vec{P}=2\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{Q}=4\hat{i}+1\hat{j}-2\hat{k}$

Let $\vec{R}$ be the mid point of $\overrightarrow{PQ}$.

$$\vec{R}=\frac{\vec{P}+\vec{Q}}{2}$$

$$\Rightarrow\vec{R}=\frac{\left(2\hat{i}+3\hat{j}+4\hat{k}\right)+\left(4\hat{i}+1\hat{j}-2\hat{k}\right)}{2}$$

$$\Rightarrow \vec{R} = \frac{\left(6\hat{i}+4\hat{j}+2\hat{k}\right)}{2}$$

$$\Rightarrow \vec{R} = 3\hat{i}+2\hat{j}+\hat{k}$$

17. Show that the points A, B and C with position vectors, $\vec{a} = 3\hat{i}-4\hat{j}-4\hat{k}$, $\vec{b} = 2\hat{i}-\hat{j}+\hat{k}$, and $\vec{c} = \hat{i}-3\hat{j}-5\hat{k}$, respectively form the vertices of a right angled triangle.

17. O be the origin,

Let $\vec{a} = \overrightarrow{OA} = 3\hat{i}-4\hat{j}-4\hat{k}$

$\vec{b} = \overrightarrow{OB} = 2\hat{i}-\hat{j}+\hat{k}$

And

$\vec{c} = \overrightarrow{OC} = \hat{i}-3\hat{j}-5\hat{k}$

Now we find the vectors $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CA}$

$\overrightarrow{AB} = \vec{B}-\vec{A}$

$\Rightarrow \overrightarrow{AB} = \left(2\hat{i}-\hat{j}+\hat{k}\right)-\left(3\hat{i}-4\hat{j}-4\hat{k}\right)$

$\Rightarrow \overrightarrow{AB} = \left(-\hat{i}+3\hat{j}+5\hat{k}\right)$ ……….(1)

$\overrightarrow{BC} = \vec{C}-\vec{B}$

$\Rightarrow \overrightarrow{BC} = \left(\hat{i}-3\hat{j}-5\hat{k}\right)-\left(2\hat{i}-\hat{j}+\hat{k}\right)$

$\Rightarrow \overrightarrow{BC} = \left(-\hat{i}-2\hat{j}-6\hat{k}\right)$ ………….(2)

$\overrightarrow{CA} = \vec{A}-\vec{C}$

$\Rightarrow \overrightarrow{CA} = \left(3\hat{i}-4\hat{j}-4\hat{k}\right)-\left(\hat{i}-3\hat{j}-5\hat{k}\right)$

$\Rightarrow \overrightarrow{CA} = \left(2\hat{i}-\hat{j}+\hat{k}\right)$ ……….(3)

Adding equations (1), (2) and (3)

$\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA} = \left(-\hat{i}+3\hat{j}+5\hat{k}\right)+\left(-\hat{i}-2\hat{j}-6\hat{k}\right)+\left(2\hat{i}-\hat{j}+\hat{k}\right)$

$\Rightarrow \overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA} = 0\hat{i}+0\hat{j}+0\hat{k}$

Therefore, ABC form a triangle.

Now, we want to prove that it is a right angled triangle.

$\left|\overrightarrow{AB}\right| = \sqrt{x^2+y^2+z^2} = \sqrt{(-1)^2+(3)^2+5^2} = \sqrt{35}$

$\left|\overrightarrow{BC}\right| = \sqrt{x^2+y^2+z^2} = \sqrt{(-1)^2+(2)^2+(-6)^2} = \sqrt{41}$

$\left|\overrightarrow{CA}\right| = \sqrt{x^2+y^2+z^2} = \sqrt{(2)^2+(-1)^2+1^2} = \sqrt{6}$

$$\Rightarrow \left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{CA}\right|^2 = \left(\sqrt{35}\right)^2 + \left(\sqrt{6}\right)^2 = 35 + 6 = 41$$

$$\Rightarrow \left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{CA}\right|^2 = 41 = \left(\sqrt{41}\right)^2 = \left|\overrightarrow{BC}\right|^2$$

Therefore, the triangle satisfies the Pythagoras theorem.Hence proved the given vectors form a right angled triangle.

18. In triangle ABC (Fig 10.18), which of the following is not true:

A. $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

B. $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$

C. $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \vec{0}$

D. $\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$

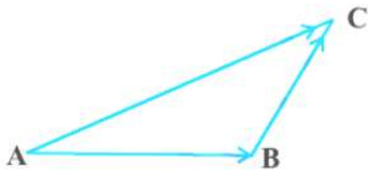
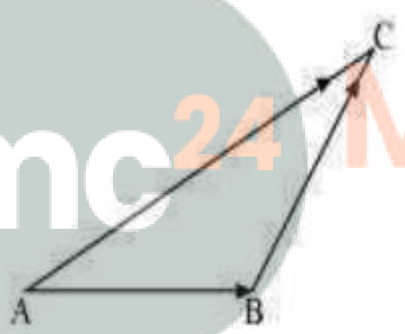


Fig 10.18

18.



On applying the triangle law of addition in the given triangle, we have:

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} = -\overrightarrow{CA}$

$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

$\therefore$ The equation given in alternative A is true.

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$

$\therefore$ The equation given in alternative B is true.

From equation (2), we have:

$\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$

$\therefore$ The equation given in alternative D is true.

Now, consider the equation given in alternative C :

From equations (1) and (3), we have:

$\overrightarrow{AC} = \overrightarrow{CA}$

$\Rightarrow \overrightarrow{AC} = -\overrightarrow{AC}$

$\Rightarrow \overrightarrow{AC} + \overrightarrow{AC} = \vec{0}$

$\Rightarrow 2\overrightarrow{AC} = \vec{0}$

$\Rightarrow \overrightarrow{AC} = \vec{0}$, which is not true.

Hence, the equation given in alternative C is incorrect.

The correct answer is C.

19. $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following are incorrect:

A. $\vec{b} = \lambda\vec{a}$, for some scalar λ

B. $\vec{a} = \pm\vec{b}$

C. the respective components of $\vec{a}$ and $\vec{b}$ are not proportional

D. both the vectors $\vec{a}$ and $\vec{b}$ have same direction, but different magnitudes.

19. We know that two vectors are collinear if they have the same direction or are parallel or anti-parallel. They can be expressed in the form $\vec{b} = m\vec{a}$ where a and b are vectors and ' m ' is a scalar quantity.

Therefore,(A) is true.

In (B), $m = \pm 1$

So, (B) is also true.

The vectors $\vec{a}$ and $\vec{b}$ are proportional,

Therefore,(C) is not true.

The vectors $\vec{a}$ and $\vec{b}$ can have different magnitude as well as different direction.

Therefore, (D) is not true.

We are asked the options which are not true.

$\therefore$ The correct answer is (C) and (D).