NCERT Solutions for Class-XII Maths

Chapter-13.4

NCERT Chemistry Class 12

1. State which of the following are not the probability distributions of a random variable. Give reasons for your answer.

(i)

X	0	1	2
P(X)	0.4	0.4	0.2

1. A given table with values for X and P(X).

As we know the sum of all the probabilities in a probability distribution of a random variable must be one.

i.e. $\sum_{i=1}^{n} p_i = 1, \text{ where } p_i > 1 \text{ and } i = 0,1,2,\ldots,n$

Hence the sum of probabilities of given table = 0.4 + 0.4 + 0.2

= 1

Hence, the given table is the probability distributions of a random variable.

(ii)

X	0	1	2	3	4
P(X)	0.4	0.5	0.2	-0.1	0.3

Solution: Given: A given table with values for X and P(X).

As we see from the table that P(X) = -0.1 for X = 3.

It is known that probability of any observation must always be positive that it can't be negative.

Hence, the given table is not the probability distributions of a random variable.

(iii)

Y	-1	0	1
P(Y)	0.6	0.1	0.2

Solution: Given: A given table with values for X and P(X).

As we know the sum of all the probabilities in a probability distribution of a random variable must be one.

i.e. $\sum_{i=1}^{n} p_i = 1$, where $p_i > 1$ and $i = 0, 1, 2, \ldots, n$

Hence the sum of probabilities of given table = 0.6 + 0.1 + 0.2

$= 0.9 \neq 1$

Hence, the given table is not the probability distributions of a random variable.

(iv)

Z	3	2	1	0	-1
P(Z)	0.3	0.2	0.4	0.1	0.05

Solution: Given: A given table with values for X and P(X).

As we know the sum of all the probabilities in a probability distribution of a random variable must be one.

i.e. $\sum_{i=1}^{n} p_i = 1$, where $p_i > 1$ and $i = 0, 1, 2, \ldots, n$

Hence the sum of probabilities of given table = 0.3+0.2+0.4+0.1+0.05

$= 1.05 \neq 1$

Hence, the given table is not the probability distributions of a random variable.

2. An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represent the number of black balls. What are the possible values of X? Is X a random variable?

2. The two balls selected can be represented as BB, BR, RB, RR, where B represents a black ball and R represents a red ball.

X represents the number of black balls.

$\therefore X(BB) = 2$

$X(BR) = 1$

$X(RB) = 1$

$X(RR) = 0$

Therefore, the possible values of X are 0,1 , and 2 .

Yes, X is a random variable.

3. Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X?

3. A coin is tossed 6 times.

X represents the difference between the number of heads and the number of tails.

$\Rightarrow X(6H, 0T) = |6-0| = 6$

$X(5H, 1T) = |5-1| = 4$

$X(4H, 2T) = |4-2| = 2$

X (3H, 3T) = |3-3|=0
X (2H, 4T) = |2-4|=2
X (1H, 5T) = |1-5|=4
X (0H, 6T) = |0-6|=6
Therefore, X is a function on sample space whose range is {0, 2, 4, 6}.
Thus, X is a random variable which can take the values 0, 2, 4 or 6.

4. Find the probability distribution of
(i) number of heads in two tosses of a coin.
(ii) number of tails in the simultaneous tosses of three coins.
(iii) number of heads in four tosses of a coin.

4. (i) When one coin is tossed twice, the sample space is
$\{HH, HT, TH, TT\}$
Let X represent the number of heads.
$\therefore X(HH)=2, X(HT)=1, X(TH)=1, X(TT)=0$
Therefore, X can take the value of 0,1 , or 2 .
It is known that,

$$P(HH)=P(HT)=P(TH)=P(TT)=\frac{1}{4}$$

$$P(X=0)=P(TT)=\frac{1}{4}$$

$$P(X=1)=P(HT)+P(TH)=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$$

$$P(X=2)=P(HH)=\frac{1}{4}$$

Thus, the required probability distribution is as follows.

X	0	1	2
P (X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(ii) When three coins are tossed simultaneously, the sample space is
{HHH. HHT. HTH. HTT. THH, THT, TTH. TIT}
Let X represent the number of tails.
It can be seen that X can take the value of $0,1,2$, or 3.

$$P(X=0)=P(HHH)=\frac{1}{8}$$

$$P(x=1)=P(HHT)+P(\ HIH)+P(\ THI)=\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{3}{8}$$

$P(X=2)=P(HTT)+P(THT)+P(TTH)=\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{3}{8}$

$P(x=3)=P(TT)=\frac{1}{8}$

Thus, the probability distribution is as follows.

X	0	1	2	3
P (X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(iii) When a coin is tossed four times, the sample space is

$$S=\left\{\begin{matrix}\text{HHHH, HHHT, HHTH, HHTT, HTHT, HTHH, HTTH, HTTT,}\\ \text{THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT}\end{matrix}\right\}$$

Let X be the random variable, which represents the number of heads.
It can be seen that X can take the value of 0,1,2,3, or 4 .

$P(X=0)=P(TTT)=\frac{1}{16}$

$P(X=1)=P(TTTH)+P(TTHT)+P(THTT)+P(HTTT)$

$=\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}=\frac{4}{16}=\frac{1}{4}$

$P(X=2)=P(HHTT)+P(THHT)+P(TTHH)+P(HTTH)+P(HTHT)+P(THTH)$

$=\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}=\frac{6}{16}=\frac{3}{8}$

$P(X=3)=P(HHHT)+P(HHTH)+P(HTHH)P(THHH)$

$=\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}=\frac{4}{16}=\frac{1}{4}$

$P(X=4)=P(HHHH)=\frac{1}{16}$

Thus, the probability distribution is as follows.

X	0	1	2	3	4
P (X)	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

5. Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as
(i) number greater than 4
(ii) six appears on at least one die

5. A die is tossed two times.

When a die is tossed two times then the number of observations will be (6 × 6) = 36.

Now, let X is a random variable which represents the success.

(i) Here success is given as the number greater than 4.

Now

P(X = 0) = P(number ≤ 4 in both tosses) $=\frac{4}{6}\times\frac{4}{6}=\frac{4}{9}$

P(X = 1) = P(number ≤ 4 in first toss and number ≥ 4 in second case) + P(number ≥ 4 in first toss and number ≤ 4 in second case) is:

$=\left(\frac{4}{6}\times\frac{2}{6}\right)+\left(\frac{2}{6}\times\frac{4}{6}\right)=\frac{4}{9}$

P(X = 2) = P(number ≥ 4 in both tosses) $=\frac{2}{6}\times\frac{2}{6}=\frac{1}{9}$

Hence, the required probability distribution is,

X	0	1	2
P(X)	4/9	4/9	1/9

(ii) Here success is given as six appears on at least one die.

Now

P(X = 0) = P(six does not appear on any of die) $=\frac{5}{6}\times\frac{5}{6}=\frac{25}{36}$

P(X = 1) = P(six appears atleast once of the die) $=\left(\frac{1}{6}\times\frac{5}{6}\right)+\left(\frac{5}{6}\times\frac{1}{6}\right)=\frac{10}{36}=\frac{5}{18}$

Hence, the required probability distribution is,

X	0	1
P(X)	25/36	5/18

6. From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

6. It is given that out of 30 bulbs, 6 are defective.

⇒ Number of non-defective bulbs $=30-6=24$

4 bulbs are drawn from the lot with replacement.

Let X be the random variable that denotes the number of defective bulbs in the selected bulbs.

$\therefore P(X=0)=P(4\text{ non-defective and 0 defective })={}^4C_0\cdot\frac{4}{5}\cdot\frac{4}{5}\cdot\frac{4}{5}\frac{4}{5}=\frac{256}{625}$

$P(X=1)=P(3\text{ non-defective and 1 defective })={}^4C_1\cdot\left(\frac{1}{5}\right)\cdot\left(\frac{4}{5}\right)^3=\frac{256}{625}$

$P(X=2) = P(2 \text{ non-defective and 2 defective }) = {}^4C_2 \cdot \left(\frac{1}{5}\right)^2 \cdot \left(\frac{4}{5}\right)^2 = \frac{96}{625}$

$P(X=3) = P(1 \text{ non-defective and 3 defective }) = {}^4C_3 \cdot \left(\frac{1}{5}\right)^3 \cdot \left(\frac{4}{5}\right) = \frac{16}{625}$

$P(X=4) = P(0 \text{ non-defective and 4 defective }) = {}^4C_4 \cdot \left(\frac{1}{5}\right)^4 \cdot \left(\frac{4}{5}\right)^0 = \frac{1}{625}$

Therefore, the required probability distribution is as follows.

X	0	1	2	3	4
P (X)	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

7. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

7. head is 3 times as likely to occur as tail.

Now, let the probability of getting a tail in the biased coin be x.

⇒ P(T) = x

And P(H) = 3x

For a biased coin, P(T) + P(H) = 1

⇒ x + 3x = 1

⇒ 4x = 1

⇒ x = ¼

Hence, P(T) = ¼ and P(H) = ¾

As the coin is tossed twice, so the sample space is {HH, HT, TH, TT}

Let X be a random variable representing the number of tails.

Clearly, X can take the value of 0, 1 or 2.

P(X = 0) = P(no tail) = P(H) × P(H) $= \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

P(X = 1) = P(one tail) = P(HT) × P(TH) $= \frac{3}{4}.\frac{1}{4} \times \frac{1}{4}.\frac{3}{4} = \frac{3}{8}$

P(X = 2) = P(two tail) = P(T) × P(T) $= \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

Hence, the required probability distribution is,

X	0	1	2
P(X)	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{16}$

8. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	K^2	2 K^2	7 K^2+k

Determine

(i) k (ii) $P(X < 3)$

(iii) $P(X > 6)$ (iv) $P(0 < X < 3)$

8. (i) It is known that the sum of probabilities of a probability distribution of random variables is one.

$k = -1$ is not possible as the probability of an event is never negative.

$$\therefore 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + \left(7k^2 + k\right) = 1$$

$$\Rightarrow 10k^2 + 9k - 1 = 0$$

$$\Rightarrow (10k - 1)(k + 1) = 0$$

$$\Rightarrow k = -1, \frac{1}{10}$$

$$\therefore k = \frac{1}{10}$$

(ii) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$$= 0 + k + 2k$$
$$= 3k$$
$$= 3 \times \frac{1}{10}$$
$$= \frac{3}{10}$$

(iii) $P(X > 6) = P(X = 7)$

$$= 7k^2 + k$$
$$= 7 \times \left(\frac{1}{10}\right)^2 + \frac{1}{10}$$
$$= \frac{7}{100} + \frac{1}{10}$$
$$= \frac{17}{100}$$

(iv) $P(0 < X < 3) = P(X = 1) + P(X = 2)$

$$= k + 2k$$
$$= 3k$$
$$= 3 \times \frac{1}{10}$$
$$= \frac{3}{10}$$

9. The random variable X has a probability distribution P(X) of the following form, where k is some number:

$$P(X) = \begin{cases} k, & \text{If } x = 0 \\ 2k, & \text{If } x = 1 \\ 3k, & \text{If } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

(a) Determine the value of k.

(b) Find P $(X < 2)$, P $(X \leq 2)$, $P(X \geq 2)$.

9. A random variable X with its probability distribution.

(a) As we know the sum of all the probabilities in a probability distribution of a random variable must be one.

i.e. $\sum_{i=1}^{n} p_i = 1, \text{ where } p_i > 1 \text{ and } i = 0, 1, 2, \ldots, n$

Hence the sum of probabilities of given table:

$\Rightarrow k + 2k + 3k + 0 = 1$

$\Rightarrow 6k = 1$

$\Rightarrow k = \frac{1}{6}$

(b)

(i) $P(X < 2) = ?$

$P(X < 2) = P(X = 0) + P(X = 1)$

$= k + 2k$

$= 3k$

$P(X < 2) = 3 \times \frac{1}{6} = \frac{1}{2}$

(ii)

$P(X \leq 2) = ?$

$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$= k + 2k + 3k$

$= 6k$

$P(X \leq 2) = 6 \times \frac{1}{6} = 1$

(iii)

$P(X \geq 2) = ?$

$P(X \geq 2) = P(X = 2) + P(X > 2)$

$= 3k + 0$

$= 3k$

$P(X \geq 2) = 3 \times \frac{1}{6} = \frac{1}{2}$

10. Find the mean number of heads in three tosses of a fair coin.

10. Let X denote the success of getting heads.

Therefore, the sample space is

$S = \{HHH, HHT, HTH, HTT, THH, THT, TH, TTT\}$

It can be seen that X can take the value of $0,1,2$, or 3.

$\therefore P(X=0) = P(TTT)$

$= P(T) \cdot P(T) \cdot P(T)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{8}$

$\therefore P(X=1) = P(HHT) + P(HTH) + P(THH)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{8}$

$\therefore P(X=2) = P(HHT) + P(HTH) + P(TH)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{8}$

$\therefore P(X=3)$

$= P(HHH)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{8}$

Therefore, the required probability distribution is as follows.

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Mean of XE $(X), \mu = \sum X_i P(X_i)$

$= 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$

$= \frac{3}{8} + \frac{3}{4} + \frac{3}{8}$

$= \frac{3}{2}$

$= 1.5$

11. Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.

11. A die is thrown two times.

When a die is tossed two times then the number of observations will be (6 × 6) = 36.

Now, let X is a random variable which represents the success and is given as six appears on at least one die.

Now

P(X = 0) = P(six does not appear on any of die) $= \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

P(X = 1) = P(six appears atleast once of the die) $= \left(\frac{1}{6} \times \frac{5}{6}\right) + \left(\frac{5}{6} \times \frac{1}{6}\right) = \frac{10}{36} = \frac{5}{18}$

P(X = 2) = P(six does appear on both of die) $= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

Hence, the required probability distribution is,

X	0	1	2
P(X)	$\frac{25}{36}$	$\frac{5}{18}$	$\frac{1}{36}$

Therefore, Expectation of X E(X):

$E(X) = \sum_{i=1}^{n} x_i p_i$

$= 0 \times \frac{25}{36} + 1 \times \frac{5}{18} + 2 \times \frac{1}{36}$

$= \frac{5}{18} + \frac{1}{18} = \frac{6}{18} = \frac{1}{3}$

$\Rightarrow E(X) = \frac{1}{3}$

12. Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find E(X).

12. The two positive integers can be selected from the first six positive integers without replacement in $6 \times 5 = 30$ ways

X represents the larger of the two numbers obtained. Therefore, X can take the value of $2,3,4,5$, or 6 .

For $X = 2$, the possible observations are $(1,2)$ and $(2,1)$

$\therefore P(X = 2) = \frac{2}{30} = \frac{1}{15}$

For $X = 3$, the possible observations are $(1,3),(2,3),(3,1)$, and $(3,2)$.

$\therefore P(X = 3) = \frac{4}{30} = \frac{2}{15}$

For $X = 4$, the possible observations are $(1,4),(2,4),(3,4),(4,3),(4,2)$, and $(4,1)$.

$\therefore P(X = 4) = \frac{6}{30} = \frac{1}{5}$

For $X = 5$, the possible observations are $(1,5),(2,5),(3,5),(4,5),(5,4),(5,3),(5,2)$, and $(5,1)$

$\therefore P(X = 5) = \frac{8}{30} = \frac{4}{15}$

For $X = 6$, the possible observations are $(1,6),(2,6),(3,6),(4,6),(5,6),(6,4),(6,3),(6,2)$, and $(6,1)$

$\therefore P(X = 6) = \frac{10}{30} = \frac{1}{3}$

Therefore, the required probability distribution is as follows.

X	2	3	4	5	6
P(X)	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{4}{15}$	$\frac{1}{3}$

Then, $E(X) = \sum X_i P(X_i)$

$= 2 \cdot \frac{1}{15} + 3 \cdot \frac{2}{15} + 4 \cdot \frac{1}{5} + 5 \cdot \frac{4}{15} + 6 \cdot \frac{1}{3}$

$= \frac{2}{15} + \frac{2}{5} + \frac{4}{5} + \frac{4}{3} + 2$

$= \frac{70}{15}$

$= \frac{14}{3}$

13. Let X denote the sum of the numbers obtained when two fair dice are rolled. Find the variance and standard deviation of X.

13. Two fair dice are rolled

When two fair dice are rolled then number of observations will be $6 \times 6 = 36$.

X denote the sum of the numbers obtained when two fair dice are rolled. Hence, X can take any value of 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 or 12.

For X = 2, the possible observations are (1, 1).

$$\Rightarrow P(X) = \frac{1}{36}$$

For X = 3, the possible observations are (1,2) and (2,1)

$$\Rightarrow P(X) = \frac{2}{36} = \frac{1}{18}$$

For X = 4, the possible observations are (1,3), (2,2) and (3,1).

$$\Rightarrow P(X) = \frac{3}{36} = \frac{1}{12}$$

For X = 5, the possible observations are (1, 4), (4, 1), (2,3) and (3,2)

$$\Rightarrow P(X) = \frac{4}{36} = \frac{1}{9}$$

For X = 6, the possible observations are (1, 5), (5, 1), (2,4), (4,2) and (3,3).

$$\Rightarrow P(X) = \frac{5}{36}$$

For X = 7, the possible observations are (1, 6), (6, 1), (2,5), (5,2),(3,4) and (4,3).

$$\Rightarrow P(X) = \frac{6}{36} = \frac{1}{6}$$

For X = 8, the possible observations are (2,6), (6,2),(3,5), (5,3) and (4,4).

$$\Rightarrow P(X) = \frac{5}{36}$$

For X = 9, the possible observations are (5, 4), (4, 5), (3,6) and (6,3)

$$\Rightarrow P(X) = \frac{4}{36} = \frac{1}{9}$$

For X = 10, the possible observations are (5,5), (4,6) and (6,4).

$$\Rightarrow P(X) = \frac{3}{36} = \frac{1}{12}$$

For X = 11, the possible observations are (6,5) and (5,6)

$$\Rightarrow P(X) = \frac{2}{36} = \frac{1}{18}$$

For X = 12, the possible observations are (6, 6).

$$\Rightarrow P(X) = \frac{1}{36}$$

Hence, the required probability distribution is,

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

Therefore E(X) is:

$$E(X)=\sum_{i=1}^{n}x_i p_i$$

$$=2\times\frac{1}{36}+3\times\frac{1}{18}+4\times\frac{1}{12}+5\times\frac{1}{9}+6\times\frac{5}{36}+7\times\frac{1}{6}+8\times\frac{5}{36}+9\times\frac{1}{9}+10\times\frac{1}{12}+11\times\frac{1}{18}+12\times\frac{1}{36}$$

$$=\frac{1}{18}+\frac{1}{6}+\frac{1}{3}+\frac{5}{9}+\frac{5}{6}+\frac{7}{6}+\frac{10}{9}+1+\frac{5}{6}+\frac{11}{18}+\frac{1}{3}$$

$$=\frac{1+3+6+10+15+21+20+18+15+11+6}{18}=\frac{126}{18}$$

$\Rightarrow$ E(X) = 7

And $E(X^2)$ is:

$$E(X^2)=\sum_{i=1}^{n}x_i^{\,2}.p(x_i)$$

$$=4\times\frac{1}{36}+9\times\frac{1}{18}+16\times\frac{1}{12}+25\times\frac{1}{9}+36\times\frac{5}{36}+49\times\frac{1}{6}+64\times\frac{5}{36}+81\times\frac{1}{9}+100\times\frac{1}{12}+121\times\frac{1}{18}+144\times\frac{1}{3}$$

$$=\frac{1}{9}+\frac{1}{2}+\frac{4}{3}+\frac{25}{9}+5+\frac{49}{6}+\frac{80}{9}+9+\frac{25}{3}+\frac{121}{18}+4$$

$$=\frac{2+9+24+50+90+147+160+162+150+121+72}{18}=\frac{987}{18}$$

$\Rightarrow E(X^2) = 54.833$

Then Variance, $Var(X) = E(X^2) - (E(X))^2$

$= 54.833 - (7)^2$

$= 54.833 - 49 = 5.833$

And Standard deviation $=\sqrt{Var(X)}$

$=\sqrt{5.833}$

$\Rightarrow$ Standard deviation = 2.415

14. A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find mean, variance and standard deviation of X.

14. The class of 15 students with their ages.

Form the given information we can draw a table:

X	14	15	16	17	18	19	20	21

f	2	1	2	3	1	2	3	1

$P(X = 14) = \frac{2}{15}$

$P(X = 15) = \frac{1}{15}$

$P(X = 16) = \frac{2}{15}$

$P(X = 17) = \frac{3}{15}$

$P(X = 18) = \frac{1}{15}$

$P(X = 19) = \frac{2}{15}$

$P(X = 20) = \frac{3}{15}$

$P(X = 21) = \frac{1}{15}$

Hence, the required probability distribution is,

X	14	15	16	17	18	19	20	21
P(X)	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

Therefore E(X) is:

$$E(X) = \sum_{i=1}^{n} x_i p_i$$

$$= 14\times\frac{2}{15} + 15\times\frac{1}{15} + 16\times\frac{2}{15} + 17\times\frac{3}{15} + 18\times\frac{1}{15} + 19\times\frac{2}{15} + 20\times\frac{3}{15} + 21\times\frac{1}{15}$$

$$= \frac{28+15+32+21+18+38+60+21}{15} = \frac{263}{15}$$

$\Rightarrow E(X) = 17.53$

And $E(X^2)$ is:

$$E(X^2) = \sum_{i=1}^{n} x_i^{\,2}.p(x_i)$$

$$= (14)^2\times\frac{2}{15} + (15)^2\times\frac{1}{15} + (16)^2\times\frac{2}{15} + (17)^2\times\frac{3}{15} + (18)^2\times\frac{1}{15} + (19)^2\times\frac{2}{15} + (20)^2\times\frac{3}{15} + (21)^2\times\frac{1}{15}$$

$$= \frac{392+225+512+867+324+722+1200+441}{15} = \frac{4683}{15}$$

$\Rightarrow E(X^2) = 312.2$

Then Variance, $Var(X) = E(X^2) - (E(X))^2$

$= 312.2 - (17.53)^2$

$= 312.2 - 307.417 \approx 4.78$

And Standard deviation $= \sqrt{\text{Var}(X)}$

$= \sqrt{4.78}$

$\Rightarrow$ Standard deviation ≈ 2.19

15. In a meeting, 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take X = 0 if he opposed, and X = 1 if he is in favour. Find E(X) and Var (X).

15. It is given that $P(X=0) = 30\% = \frac{30}{100} = 0.3$

$P(X=1) = 70\% = \frac{70}{100} = 0.7$

Therefore, the probability distribution is as follows.

X	0	1
P(X)	0.3	0.7

Then, $E(X) = \sum X_i P(X_i)$

$= 0 \times 0.3 + 1 \times 0.7$

$= 0.7$

$E(X^2) = \sum X_i^2 P(X_i)$

$= 0^2 \times 0.3 + (1)^2 \times 0.7$

$= 0.7$

It is known that, $\text{Var}(X) = E(X^2) - [E(X)]^2$

$= 0.7 - (0.7)^2$

$= 0.7 - 0.49$

$= 0.21$

16. The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is

A. 1 B.2

C. 5 D. $\frac{8}{3}$

16. A die having written 1 on three faces, 2 on two faces and 5 on one face.

Let X be the random variable representing a number on given die.
Then X can take any value of 1, 2 or 5.
The total numbers is six.
Now

$P(X = 1) = \frac{3}{6} = \frac{1}{2}$

$P(X = 2) = \frac{1}{3}$

$P(X = 5) = \frac{1}{6}$

Hence, the required probability distribution is,

X	1	2	5
P(X)	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

Therefore Expectation of X E(X):

$E(X) = \sum_{i=1}^{n} x_i p_i$

$= 1 \times \frac{1}{2} + 2 \times \frac{1}{3} + 5 \times \frac{1}{6}$

$= \frac{1}{2} + \frac{2}{3} + \frac{5}{6} = \frac{3+4+5}{6} = \frac{12}{6}$

$\Rightarrow E(X) = 2$

Hence, the correct answer is (B).

17. Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. Then the value of E(X) is

A. $\frac{37}{221}$ B. $\frac{5}{13}$

C. $\frac{1}{13}$ D. $\frac{2}{13}$

17. Let X denote the number of aces obtained. Therefore, X can take any of the values of 0,1, or 2.

In a deck of 52 cards, 4 cards are aces. Therefore, there are 48 non-ace cards.

$\therefore P(X=0) = P(0 \text{ ace and 2 non-ace cards}) = \frac{{}^4C_0 \times {}^{48}C_2}{{}^{52}C_2} = \frac{1128}{1326}$

$P(X=1) = P(1 \text{ ace and 1 non-ace cards}) = \frac{{}^4C_1 \times {}^{48}C_1}{{}^{52}C_2} = \frac{192}{1326}$

$$P(X=2)=P(2\text{ ace and 0 non- ace cards})=\frac{{}^{4}C_{2}\times{}^{48}C_{0}}{{}^{52}C_{2}}=\frac{6}{1326}$$

Thus, the probability distribution is as follows.

X	0	1	2
$P(X)$	$\frac{1128}{1326}$	$\frac{192}{1326}$	$\frac{6}{1326}$

Then, $E(X)=\sum p_i x_i$

$=0\times\frac{1128}{1326}+1\times\frac{192}{1326}+2\times\frac{6}{1326}$

$=\frac{204}{1326}$

$=\frac{2}{13}$

Therefore, the correct answer is D.