NCERT Solutions for Class-XI Maths

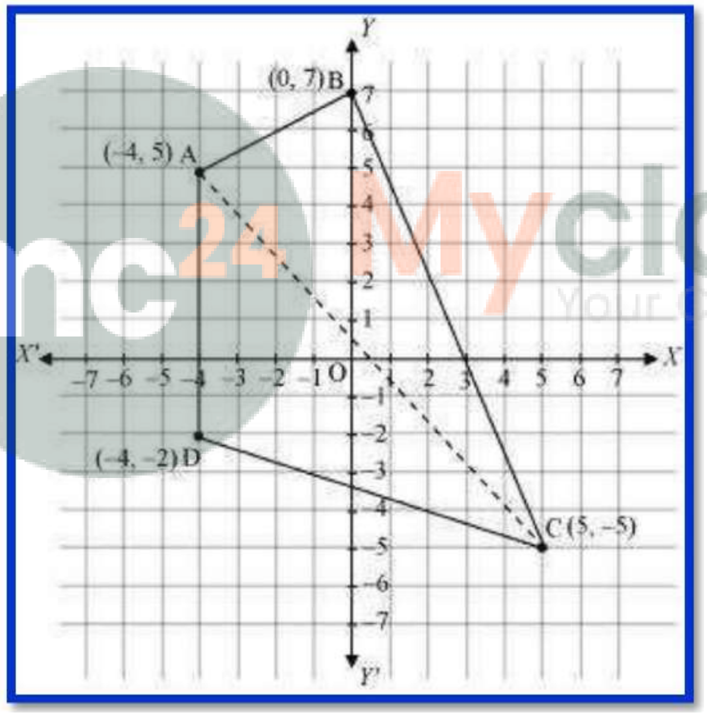
Chapter-10 Exercise-10.1

NCERT Math Class 11

1. Draw a quadrilateral in the Cartesian plane, whose vertices are $(-4,5),(0,7),(5,-5)$ and $(-4,-2)$. Also, find its area.

1. Let $ABCD$ be the given quadrilateral with vertices $A(-4,5), B(0,7), C(5,-5)$, and $D(-4,-2)$

Then, by plotting A, B, C, and D on the Cartesian plane and joining AB, BC, CD, and DA, the given quadrilateral can be drawn as



To find the area of quadrilateral $ABCD$, we draw one diagonal, say AC.

Accordingly, area $(ABCD) = \text{area}(\triangle ABC) +$ area $(\triangle ACD)$

We know that the area of a triangle whose vertices are $(x_1,y_1),(x_2,y_2)$, and (x_3,y_3) is

$$\frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$

Therefore, area of $\triangle ABC$

$$=\frac{1}{2}\left|-4(7+5)+0(-5-5)+5(5-7)\right| \text{ unit}^2$$

$= \frac{1}{2}\left|-4(12)+5(-2)\right|$ unit^2

$= \frac{1}{2}\left|-48-10\right|$ unit^2

$= \frac{1}{2}\left|-58\right|$ unit^2

$= \frac{1}{2}\times 58$ unit^2

$= 29$ unit^2

Area of$\triangle$ACD

$= \frac{1}{2}\left|-4(-5+2)+5(-2-5)+(-4)(5+5)\right|$ unit^2

$= \frac{1}{2}\left|-4(-3)+5(-7)-4(10)\right|$ unit^2

$= \frac{1}{2}\left|12-35-40\right|$ unit^2

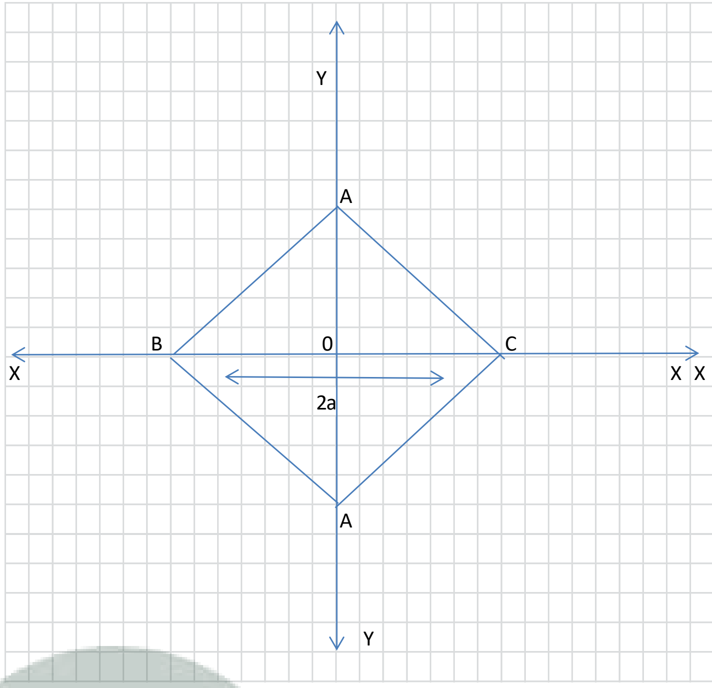
$= \frac{1}{2}\left|-63\right|$ unit^2

$= \frac{63}{2}$ unit^2

Thus, area $(ABCD) = \left(29+\frac{63}{2}\right)$ unit $^2 = \frac{58+63}{2}$ unit $^2 = \frac{121}{2}$ unit ^{2}

2. The base of an equilateral triangle with side 2a lies along the y-axis such that the mid-point of the base is at the origin. Find vertices of the triangle.

2.



Let ABC be the given equilateral triangle with side 2a

⇒ AB= BC = AC = 2a

Assuming that the base BC lies on the y axis such that the mid-point of BC is at the origin i.e.BO =OC = a and O is the origin

⇒ Co-ordinates of point 0 are (0,a) and that of B are (0,-a)

Since the line joining a vertex of an equilateral Δ with the mid-point of its opposite side is perpendicular

⇒ Vertex of A lies on the y –axis

On applying Pythagoras theorem

$(AC)^2 = OA^2 + OC^2$

$\Rightarrow (2a)^2 = OA^2 + a^2$

$\Rightarrow 4a^2 - a^2 = OA^2$

$\Rightarrow 3a^2 = OA^2$

$\Rightarrow OA = \sqrt{3}a$

∴ Co-ordinates of point A = $\pm\sqrt{3}a, 0$

Thus, the vertices of the given equilateral triangle are (0, a) , (0, -a) , ($\sqrt{3}$a, 0)

Or (0, a), (0, -a) and ($-\sqrt{3}a$,0)

3. Find the distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$ when:

(i) PQ is parallel to the y-axis,

(ii) PQ is parallel to the x-axis.

3. The given points are $P(x_1, y_1)$ and $Q(x_2, y_2)$.

(i) When PQ is parallel to the y-axis, $x_1 = x_2$.

In this case, distance between P and $Q = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(y_2 - y_1)^2}$

$= |y_2 - y_1|$

(ii)When PQ is parallel to the x-axis, $y_1 = y_2$.

In this case, distance between P and $Q = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(x_2 - x_1)^2}$

$= |x_2 - x_1|$

4. Find a point on the x-axis, which is equidistant from the points (7, 6) and (3, 4).

4. Let (a, 0) be the point on the x-axis that is equidistant from the point (7, 6) and (3, 4).

Accordingly, $\sqrt{(7-a)^2 + (6-0)^2} = \sqrt{(3-a)^2 + (4-0)^2}$

$\Rightarrow \sqrt{49 + a^2 - 14a + 36} = \sqrt{9 + a^2 - 6a + 16}$

$\Rightarrow \sqrt{a^2 - 14a + 85} = \sqrt{a^2 - 6a + 25}$

Squaring both the sides we get,

$\Rightarrow a^2 - 14a + 85 = a^2 - 6a + 25$

$\Rightarrow$ - 8a = - 60

$\Rightarrow a = \frac{15}{2}$

The required point is $\left(\frac{15}{2}, 0\right)$

5. Find the slope of a line, which passes through the origin, and the mid-point of the line segment joining the points $P(0,-4)$ and $B(8,0)$.

5. The coordinates of the mid-point of the line segment joining the points

$P(0,-4)$ and $B(8,0)$ are $\left(\frac{0+8}{2}, \frac{-4+0}{2}\right) = (4,-2)$

It is known that the slope (m) of a non-vertical line passing through the points (x_1, y_1)

and $\left(x_2, y_2\right)$ is given by $m = \frac{y_2 - y_1}{x_2 - x_1}, x_2 \neq x_1$.

Therefore, the slope of the line passing through $(0,0)$ and $(4,-2)$ is

$$\frac{-2-0}{4-0} = \frac{-2}{4} = -\frac{1}{2}$$

Hence, the required slope of the line is $-\frac{1}{2}$.

6. Without using the Pythagoras theorem, show that the points (4, 4), (3, 5) and (–1, –1) are the vertices of a right angled triangle.

6. The vertices of the given triangle are (4, 4), (3, 5) and (–1, –1).

The slope (m) of the line non-vertical line passing through the point (x_1, y_1) and (x_2, y_2) is given by $m = \frac{(y_2 - y_1)}{x_2 - x_1}$ and $x_2 \neq x_1$

∴ the slope of the line AB $(m_1) = \frac{5-4}{3-4} = \frac{1}{-1} = -1$

the slope of the line BC $(m_2) = \frac{-1-5}{-1-3} = \frac{-6}{-4} = \frac{3}{2}$

the slope of the line CA $(m_3) = \frac{4+1}{4+1} = \frac{5}{5} = 1$

Now, $m_1 m_3 = -1$

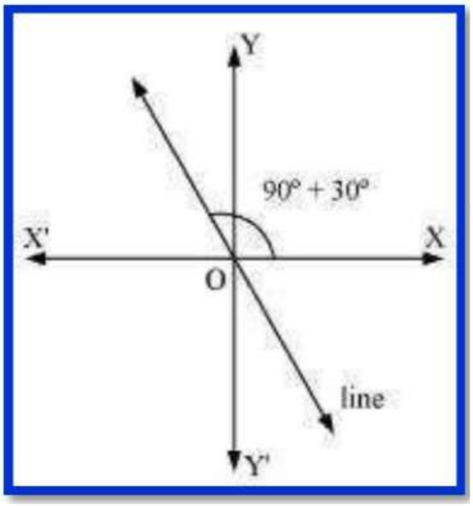
⇒ Lines AB and CA are perpendicular to each other

∴ given triangle is right-angled at A (4, 4)

And the vertices of the right-angled Δ are (4, 4) , (3, 5) and (-1, -1)

7. Find the slope of the line, which makes an angle of 30° with the positive direction of y axis measured anticlockwise.

7. If a line makes an angle of 30° with the positive direction of the y-axis measured anticlockwise, then the angle made by the line with the positive direction of the x-axis measured anticlockwise is $90^\circ + 30^\circ = 120^\circ$.



Thus, the slope of the given line is $\tan 120^\circ = \tan(180^\circ - 60^\circ) = -\tan 60^\circ = -\sqrt{3}$

8. Find the value of x for which the points (x, – 1), (2, 1) and (4, 5) are collinear.

8. If the points (x, – 1), (2, 1) and (4, 5) are collinear, then

Slope of AB = Slope of BC

$\Rightarrow \frac{1+1}{2-x} = \frac{5-1}{4-2}$

$\Rightarrow \frac{2}{2-x} = \frac{4}{2} = 2$

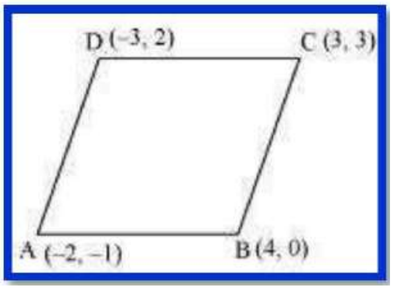
$\Rightarrow$ 2= 4 - 2x

$\Rightarrow$ x= 1

Thus, required value of x is 1.

9. Without using distance formula, show that points $(-2,-1),(4,0),(3,3)$ and $(-3,2)$ are vertices of a parallelogram.

9. Let points $(-2,-1),(4,0),(3,3)$, and $(-3,2)$ be respectively denoted by A, B, C, and D.



Slope of $AB = \frac{0+1}{4+2} = \frac{1}{6}$

Slope of $CD = \frac{2-3}{-3-3} = \frac{-1}{-6} = \frac{1}{6}$

$\Rightarrow$ Slope of $AB =$ Slope of CD

$\Rightarrow AB$ and CD are parallel to each other.

Now, slope of $BC = \frac{3-0}{3-4} = \frac{3}{-1} = -3$

Slope of $AD = \frac{2+1}{-3+2} = \frac{3}{-1} = -3$

$\Rightarrow$ Slope of $BC =$ Slope of AD

$\Rightarrow BC$ and AD are parallel to each other.

Therefore, both pairs of opposite sides of quadrilateral $ABCD$ are parallel. Hence, $ABCD$ is a parallelogram.

Thus, points $(-2,-1),(4,0),(3,3)$, and $(-3,2)$ are the vertices of a parallelogram.

10. Find the angle between the x-axis and the line joining the points (3, –1) and (4, –2).

10. The Slope of the line joining the points (3, -1) and (4, -2) is

$m = \frac{-2+1}{4-3} = -\frac{1}{1} = -1$

The angle of inclination of line joining the points (3, -1) and (4, -2) is given by tan θ = -1 θ = (90° + 45°) = 135°

Thus, the angle between the x-axis and the line joining the points (3, –1) and (4, –2) is 135°.

11. The slope of a line is double of the slope of another line. If tangent of the angle between them is $1/3$, find the slopes of the lines.

11. Let m_1 and m be the slopes of the two given lines such that $m_1 = 2m$.

We know that if θ is the angle between the lines I_1 and I_2 with slopes m_1 and m_2, then

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|.$$

It is given that the tangent of the angle between the two lines is $\frac{1}{3}$.

$$\therefore \frac{1}{3} = \left|\frac{m-2m}{1+(2m)\cdot m}\right|$$

$$\Rightarrow \frac{1}{3} = \left|\frac{-m}{1+2m^2}\right|$$

$\Rightarrow \frac{1}{3} = \frac{-m}{1+2m^2}$ or $\frac{1}{3} = -\left(\frac{-m}{1+2m^2}\right) = \frac{m}{1+2m^2}$

Case I

$\Rightarrow \frac{1}{3} = \frac{-m}{1+2m^2}$

$\Rightarrow 1+2m^2 = -3m$

$\Rightarrow 2m^2 + 3m + 1 = 0$

$\Rightarrow 2m^2 + 2m + m + 1 = 0$

$\Rightarrow 2m(m+1) + 1(m+1) = 0$

$\Rightarrow (m+1)(2m+1) = 0$

$\Rightarrow m = -1$ or $m = -\frac{1}{2}$

If $m = -1$, then the slopes of the lines are -1 and -2.

If $m = -\frac{1}{2}$, then the slopes of the lines are $-\frac{1}{2}$ and -1.

Case II

$\frac{1}{3} = \frac{m}{1+2m^2}$

$\Rightarrow 2m^2 + 1 = 3m$

$\Rightarrow 2m^2 - 3m + 1 = 0$

$\Rightarrow 2m^2 - 2m - m + 1 = 0$

$\Rightarrow 2m(m-1) - 1(m-1) = 0$

$\Rightarrow (m-1)(2m-1) = 0$

$\Rightarrow m = 1$ or $m = \frac{1}{2}$

If $m = 1$, then the slopes of the lines are 1 and 2 .

If $m = \frac{1}{2}$, then the slopes of the lines are $\frac{1}{2}$ and 1

Hence, the slopes of the lines are -1 and -2 or $-\frac{1}{2}$ and -1 or 1 and 2 or $\frac{1}{2}$ and 1 .

12. A line passes through (x_1, y_1) and (h, k). If slope of the line is m, show that $k - y_1 = m(h - x_1)$.

12. The slope of the line passing through (x_1, y_1) and (h, k) is $\frac{k - y_1}{h - x_1}$

Given that the slope of the line is m

$$\Rightarrow \frac{k-y_1}{h-x_1} = m$$

$$\Rightarrow k - y_1 = m(h - x_1) \text{ hence proved.}$$

13. If three point $(h,0),(a,b)$ and $(0,k)$ lie on a line, show that $\frac{a}{h}+\frac{b}{k}=1$.

13. If the points $A(h,0), B(a,b)$, and $C(0,k)$ lie on a line, then

Slope of $AB =$ Slope of BC

$$\frac{b-0}{a-h} = \frac{k-b}{0-a}$$

$$\Rightarrow \frac{b}{a-h} = \frac{k-b}{-a}$$

$$\Rightarrow -ab = (k-b)(a-h)$$

$$\Rightarrow -ab = ka - kh - ab + bh$$

$$\Rightarrow ka + bh = kh$$

On dividing both sides by kh, we obtain

$$\frac{ka}{kh} + \frac{bh}{kh} = \frac{kh}{kh}$$

$$\Rightarrow \frac{a}{h} + \frac{b}{k} = 1$$

Hence, $\frac{a}{h} + \frac{b}{k} = 1$

14. Consider the following population and year graph (Fig 10.10), find the slope of the line AB and using it, find what will be the population in the year 2010?

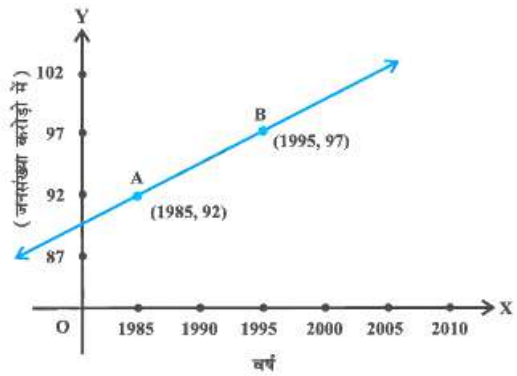


Fig 10.10

14. Since the line AB passes through points A (1985, 92) and B (1995, 97), its slope will be $\frac{97-92}{1995-1985} = \frac{5}{10} = \frac{1}{2}$

Let y be the population in the year 2010. Then, according to the given graph the AB must pass through point C (2010 , y)

Now, slope of AB = Slope of BC

$$\frac{1}{2} = \frac{y-97}{2010-1995}$$

$$\Rightarrow \frac{15}{2} = y-97$$

⇒ y= 7.5 +97 = 104.5

Thus the slope of the line AB is ½, while in the year 2010 the population must be 104.5 crores.