EXERCISE 17.3

How many different words, each containing 2 vowels and 3 consonants can be formed with 5 vowels and 17 consonants?

Solution:

Given:

Total number of vowels = 5

Total number of consonants = 17

Number of ways = (No. of ways of choosing 2 vowels from 5 vowels) × (No. of ways of choosing 3 consonants from 17 consonants)

$= ({}^5C_2) \times ({}^{17}C_3)$

By using the formula,

${}^nC_r = n!/r!(n - r)!$

$$({}^5C_2) \times ({}^{17}C_3) = \left(\frac{5!}{(5-2)!2!}\right) \times \left(\frac{17!}{(17-3)!3!}\right)$$

$$= \left(\frac{5!}{3!2!}\right) \times \left(\frac{17!}{14!3!}\right)$$

$$= \left(\frac{5 \times 4}{2 \times 1}\right) \times \left(\frac{17 \times 16 \times 15}{3 \times 2 \times 1}\right)$$

$= 10 \times (17 \times 8 \times 5)$

$= 10 \times 680$

$= 6800$

Now we need to find the no. of words that can be formed by 2 vowels and 3 consonants. The arrangement is similar to that of arranging n people in n places which are n! Ways to arrange. So, the total no. of words that can be formed is 5!

So, $6800 \times 5! = 6800 \times (5 \times 4 \times 3 \times 2 \times 1)$

$= 6800 \times 120$

$= 816000$

∴ The no. of words that can be formed containing 2 vowels and 3 consonants are 816000.

1. There are 10 persons named $P_1, P_2, P_3 \ldots, P_{10}$. Out of 10 persons, 5 persons are to be arranged in a line such that is each arrangement P_1 must occur whereas P_4 and P_5 do not occur. Find the number of such possible arrangements.

Solution:

Given:

Total persons = 10

Number of persons to be selected = 5 from 10 persons ($P_1, P_2, P_3 \ldots P_{10}$)

It is also told that P_1 should be present and P_4 and P_5 should not be present.

We have to choose 4 persons from remaining 7 persons as P_1 is selected and P_4 and P_5 are

already removed.
Number of ways = Selecting 4 persons from remaining 7 persons
= ${}^7C_4$
By using the formula,
${}^nC_r = n!/r!(n - r)!$
${}^7C_4 = 7! / 4!(7 - 4)!$
$= 7! / (4!\ 3!)$
$= [7×6×5×4!] / (4!\ 3!)$
$= [7×6×5] / (3×2×1)$
$= 7×5$
$= 35$
Now we need to arrange the chosen 5 people. Since 1 person differs from other.
$35 × 5! = 35 × (5×4×3×2×1)$
$= 4200$
∴ The total no. of possible arrangement can be done is 4200.

2. How many words, with or without meaning can be formed from the letters of the word 'MONDAY', assuming that no letter is repeated, if
(i) 4 letters are used at a time
(ii) all letters are used at a time
(iii) all letters are used but first letter is a vowel ?
Solution:
Given:
The word 'MONDAY'
Total letters = 6
(i) 4 letters are used at a time
Number of ways = (No. of ways of choosing 4 letters from MONDAY)
= $({}^6C_4)$
By using the formula,
${}^nC_r = n!/r!(n - r)!$
${}^6C_4 = 6! / 4!(6 - 4)!$
$= 6! / (4!\ 2!)$
$= [6×5×4!] / (4!\ 2!)$
$= [6×5] / (2×1)$
$= 3×5$
$= 15$
Now we need to find the no. of words that can be formed by 4 letters.
$15 × 4! = 15 × (4×3×2×1)$
$= 15 × 24$

= 360

∴ The no. of words that can be formed by 4 letters of MONDAY is 360.

(ii) all letters are used at a time

Total number of letters in the word 'MONDAY' is 6

So, the total no. of words that can be formed is 6! = 360

∴ The no. of words that can be formed by 6 letters of MONDAY is 360.

(iii) all letters are used but first letter is a vowel ?

In the word 'MONDAY' the vowels are O and A. We need to choose one vowel from these 2 vowels for the first place of the word.

So,

Number of ways = (No. of ways of choosing a vowel from 2 vowels)

$= ({}^{2}C_{1})$

By using the formula,

${}^{n}C_{r} = n!/r!(n - r)!$

${}^{2}C_{1} = 2! / 1!(2 - 1)!$

$= 2! / (1!\ 1!)$

$= (2\times1)$

$= 2$

Now we need to find the no. of words that can be formed by remaining 5 letters.

$2 \times 5! = 2 \times (5\times4\times3\times2\times1)$

$= 2 \times 120$

$= 240$

∴ The no. of words that can be formed by all letters of MONDAY in which the first letter is a vowel is 240.

3. Find the number of permutations of n distinct things taken r together, in which 3 particular things must occur together.

Solution:

Here, it is clear that 3 things are already selected and we need to choose (r – 3) things from the remaining (n – 3) things.

Let us find the no. of ways of choosing (r – 3) things.

Number of ways = (No. of ways of choosing (r – 3) things from remaining (n – 3) things)

$= {}^{n-3}C_{r-3}$

Now we need to find the no. of permutations than can be formed using 3 things which are together. So, the total no. of words that can be formed is 3!

Now let us assume the together things as a single thing this gives us total (r – 2) things which were present now. So, the total no. of words that can be formed is (r – 2)!

Total number of words formed is:
$^{n-3}C_{r-3} \times 3! \times (r - 2)!$

∴ The no. of permutations that can be formed by r things which are chosen from n things in which 3 things are always together is $^{n-3}C_{r-3} \times 3! \times (r - 2)!$

4. How many words each of 3 vowels and 2 consonants can be formed from the letters of the word INVOLUTE?
Solution:
Given:
The word 'INVOLUTE'
Total number of letters = 8
Total vowels are = I, O, U, E
Total consonants = N, V, L, T
So number of ways to select 3 vowels is $^4C_3$
And numbre of ways to select 2 consonants is $^4C_2$
Then, number of ways to arrange these 5 letters = $^4C_3 \times {}^4C_2 \times 5!$
By using the formula,
$^nC_r = n!/r!(n - r)!$
$^4C_3 = 4!/3!(4-3)!$
$= 4!/(3!\ 1!)$
$= [4\times3!] / 3!$
$= 4$

$^4C_2 = 4!/2!(4-2)!$
$= 4!/(2!\ 2!)$
$= [4\times3\times2!] / (2!\ 2!)$
$= [4\times3] / (2\times1)$
$= 2 \times 3$
$= 6$
So, by substituting the values we get
$^4C_3 \times {}^4C_2 \times 5! = 4 \times 6 \times 5!$
$= 4 \times 6 \times (5\times4\times3\times2\times1)$
$= 2880$
∴ The no. of words that can be formed containing 3 vowels and 2 consonants chosen from 'INVOLUTE' is 2880.