

16. Definite Integrals

Exercise 16A

1. Question

Evaluate:

$$\int_1^3 x^4 dx$$

Answer

$$\frac{242}{5}$$

Evaluation:

$$\int_1^3 x^4 dx = \left[\frac{x^5}{5} \right]$$

$$= \frac{3^5}{5} - \frac{1}{5}$$

$$= \frac{243 - 1}{5}$$

$$= \frac{242}{5}$$

2. Question

Evaluate:

$$\int_1^4 \sqrt{x} dx$$

Answer

$$\frac{14}{3}$$

Evaluation:

$$\int_1^4 \sqrt{x} dx = \left[\frac{2}{3} x^{\frac{3}{2}} \right]$$

$$= \frac{2}{3} \left[4^{\frac{3}{2}} - 1 \right]$$

$$= \frac{14}{3}$$

3. Question

Evaluate:

$$\int_1^2 x^{-5} dx$$

Answer

$$\frac{15}{64}$$

Evaluation:

$$\int_1^2 x^{-5} dx = \left[\frac{x^{-4}}{-4} \right]$$

$$= \frac{2^{-4}}{-4} - \frac{1}{-4}$$



$$= \frac{16 - 1}{64}$$

$$= \frac{15}{64}$$

4. Question

Evaluate:

$$\int_0^{16} x^{\frac{3}{4}} dx$$

Answer

$$\frac{512}{7}$$

Evaluation:

$$\int_0^{16} x^{\frac{3}{4}} dx = \left[\frac{4}{7} x^{\frac{7}{4}} \right]$$

$$= \frac{4}{7} \left[16^{\frac{7}{4}} - 1 \right]$$

$$= \frac{512}{7}$$

5. Question

Evaluate:

$$\int_{-4}^{-1} \frac{dx}{x}$$

Answer

$$-\log 4$$

Evaluation:

$$\int_{-4}^{-1} \frac{dx}{x} = -[\log x]$$

$$= [\log(-1) - \log(-4)]$$

$$= -[\log(-4) - \log(-1)]$$

$$= -\left[\log\left(\frac{-4}{-1}\right) \right]$$

$$= -\log 4$$

6. Question

Evaluate:

$$\int_1^4 \frac{dx}{\sqrt{x}}$$

Answer

$$2$$

Evaluation:

$$\int_1^4 \frac{dx}{\sqrt{x}} = [2\sqrt{x}]$$

$$= [2\sqrt{4} - 2]$$



$$=[4-2]$$

$$=2$$

7. Question

Evaluate:

$$\int_0^1 \frac{dx}{\sqrt[3]{x}}$$

Answer

$$\frac{3}{2}$$

Evaluation:

$$\int_0^1 \frac{dx}{\sqrt[3]{x}} = \left[\frac{3}{2} x^{\frac{2}{3}} \right]$$

$$= \left[\frac{3}{2} 1^{\frac{2}{3}} - 0 \right]$$

$$= \frac{3}{2}$$

8. Question

Evaluate:

$$\int_1^8 \frac{dx}{x^{\frac{2}{3}}}$$

Answer

$$3$$

Evaluation:

$$\int_1^8 \frac{dx}{x^{\frac{2}{3}}} = \left[\frac{3}{1} x^{\frac{1}{3}} \right]$$

$$= \left[3(8)^{\frac{1}{3}} - 3(1)^{\frac{1}{3}} \right]$$

$$=[6-3]$$

$$=3$$

9. Question

Evaluate:

$$\int_2^4 3dx$$

Answer

$$6$$

Evaluation:

$$\int_2^4 3dx = 3[x]$$

$$=3[4-2]$$

$$=6$$



10. Question

Evaluate:

$$\int_0^1 \frac{dx}{(1+x^2)}$$

Answer

$$\frac{\pi}{4}$$

Evaluation:

$$\int_0^1 \frac{dx}{1+x^2} = [\tan^{-1}x]$$

$$=[\tan^{-1} 1 - \tan^{-1} 0]$$

$$=\pi/4$$

11. Question

Evaluate:

$$\int_0^{\infty} \frac{dx}{(1+x^2)}$$

Answer

$$\frac{\pi}{2}$$

Evaluation:

$$\int_0^{\infty} \frac{dx}{1+x^2} = [\tan^{-1}x]$$

$$=[\tan^{-1} \infty - \tan^{-1} 0]$$

$$=\pi/2$$

12. Question

Evaluate:

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

Answer

$$\frac{\pi}{2}$$

Evaluation:

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}} = [\sin^{-1}x]$$

$$=[\sin^{-1} 1 - \sin^{-1} 0]$$



$$= \frac{\pi}{2}$$

13. Question

Evaluate:

$$\int_0^{\pi/6} \sec^2 x \, dx$$

Answer

$$\frac{1}{\sqrt{3}}$$

Evaluation:

$$\begin{aligned} \int_0^{\pi/6} \sec^2 x \, dx &= [\tan x] \\ &= \left[\tan\left(\frac{\pi}{6}\right) - \tan 0 \right] \\ &= \frac{1}{\sqrt{3}} \end{aligned}$$

14. Question

Evaluate:

$$\int_{-\pi/4}^{\pi/4} \operatorname{cosec}^2 x \, dx$$

Answer

$$-2$$

Evaluation:

$$\begin{aligned} \int_{-\pi/4}^{\pi/4} \operatorname{cosec}^2 x \, dx &= [-\cot x] \\ &= \left[-\cot\left(\frac{\pi}{4}\right) + \cot\left(-\frac{\pi}{4}\right) \right] \\ &= \left[-\cot\left(\frac{\pi}{4}\right) - \cot\left(\frac{\pi}{4}\right) \right] \\ &= -2 \end{aligned}$$

15. Question

Evaluate:

$$\int_{\pi/4}^{\pi/2} \cot^2 x \, dx$$

Answer

$$\left(1 - \frac{\pi}{4}\right)$$



Evaluation:

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^2 x dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\operatorname{cosec}^2 x - 1) dx$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\operatorname{cosec}^2 x - 1) dx = [-\cot x - x]$$

$$= \left[-\cot\left(\frac{\pi}{2}\right) - \frac{\pi}{2} + \cot\left(\frac{\pi}{4}\right) + \frac{\pi}{4} \right]$$

$$= \left[0 - \frac{\pi}{4} + 1 \right]$$

$$= \left[1 - \frac{\pi}{4} \right]$$

16. Question

Evaluate:

$$\int_0^{\pi/4} \tan^2 x dx$$

Answer

$$\left(1 - \frac{\pi}{4} \right)$$

Evaluation:

$$\int_0^{\pi/4} \tan^2 x dx = \int_0^{\pi/4} (\sec^2 x - 1) dx$$

$$\int_0^{\pi/4} (\sec^2 x - 1) dx = [\tan x - x]$$

$$= \left[\tan\left(\frac{\pi}{4}\right) - \frac{\pi}{4} - \tan(0) - 0 \right]$$

$$= \left[1 - \frac{\pi}{4} \right]$$

17. Question

Evaluate:

$$\int_0^{\pi/2} \sin^2 x dx$$

Answer

$$\frac{\pi}{4}$$

Evaluation:

$$\int_0^{\pi/2} \sin^2 x dx = \int_0^{\pi/2} \frac{1}{2} (1 - \cos 2x) dx$$



$$\begin{aligned}
&= \frac{1}{2} \left[x - \frac{\sin 2x}{2} \right] \\
&= \frac{1}{2} \left[\frac{\pi}{2} - \frac{\sin \pi}{2} - 0 + \frac{\sin 0}{2} \right] \\
&= \frac{\pi}{4}
\end{aligned}$$

18. Question

Evaluate:

$$\int_0^{\pi/4} \cos^2 x \, dx$$

Answer

$$\left(\frac{\pi}{8} + \frac{1}{4} \right)$$

Evaluation:

$$\int_0^{\pi/4} \cos^2 x \, dx = \int_0^{\pi/4} \frac{1}{2} (1 + \cos 2x) \, dx$$

$$= \frac{1}{2} \left[x + \frac{\sin 2x}{2} \right]$$

$$= \frac{1}{2} \left[\frac{\pi}{4} + \frac{\sin(\frac{\pi}{2})}{2} - 0 - \frac{\sin 0}{2} \right]$$

$$= \frac{\pi}{8} + \frac{1}{4}$$



19. Question

Evaluate:

$$\int_0^{\pi/3} \tan x \, dx$$

Answer

log 2

Evaluation:

$$\int_0^{\pi/3} \tan x \, dx = \log |\sec x|$$

$$= \log \left| \sec \left(\frac{\pi}{3} \right) \right| - \ln |\cos 0|$$

$$= \log |2| - \log |1|$$

$$= \log 2$$

20. Question

Evaluate:

$$\int_{\pi/6}^{\pi/4} \operatorname{cosec} x \, dx$$

Answer

$$\log(\sqrt{2}-1) + \log(2+\sqrt{3})$$

Evaluation:

$$\begin{aligned} \int_{\pi/6}^{\pi/4} \operatorname{cosec} x \, dx &= -\log|\operatorname{cosec} x + \cot x| \\ &= -\log\left|\operatorname{cosec}\left(\frac{\pi}{4}\right) + \cot\left(\frac{\pi}{4}\right)\right| + \log\left|\operatorname{cosec}\left(\frac{\pi}{6}\right) + \cot\left(\frac{\pi}{6}\right)\right| \\ &= -\log|\sqrt{2}+1| + \log|2+\sqrt{3}| \end{aligned}$$

21. Question


Evaluate:

$$\int_0^{\pi/3} \cos^3 x \, dx$$

Answer

$$\frac{3\sqrt{3}}{8}$$

Evaluation:



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$$\begin{aligned} \int_0^{\pi/3} \cos^3 x \, dx &= \frac{1}{4} \int_0^{\pi/3} (3\cos x + \cos 3x) \, dx \\ \frac{1}{4} \int_0^{\pi/3} (3\cos x - \cos 3x) \, dx &= \frac{1}{4} \left[3\sin x + \frac{\sin 3x}{3} \right] \\ &= \frac{1}{4} \left[3\sin\left(\frac{\pi}{3}\right) + \frac{\sin \pi}{3} \right] - \frac{1}{4} \left[3\sin 0 + \frac{\sin 0}{3} \right] \\ &= \frac{1}{4} \left[\frac{3\sqrt{3}}{2} \right] \\ &= \frac{3\sqrt{3}}{8} \end{aligned}$$

22. Question

Evaluate:

$$\int_0^{\pi/2} \sin^3 x \, dx$$

Answer

$$\frac{2}{3}$$

Evaluation:

$$\int_0^{\frac{\pi}{2}} \sin^3 x \, dx = \frac{1}{4} \int_0^{\frac{\pi}{2}} (3\sin x - \sin 3x) \, dx$$

$$\frac{1}{4} \int_0^{\frac{\pi}{2}} (3\sin x - \sin 3x) \, dx = \frac{1}{4} \left[-3\cos x + \frac{\cos 3x}{3} \right]$$

$$= \frac{1}{4} \left[-3\cos\left(\frac{\pi}{2}\right) + \frac{\cos\left(\frac{3\pi}{2}\right)}{3} \right] - \frac{1}{4} \left[-3\cos 0 + \frac{\cos 0}{3} \right]$$

$$= \frac{1}{4} \left[\frac{9-1}{3} \right]$$

$$= \frac{2}{3}$$

23. Question

Evaluate:

$$\int_{\pi/4}^{\pi/2} \frac{(1-3\cos x)}{\sin^2 x} \, dx$$

Answer

$$(4-3\sqrt{2})$$

Evaluation:

$$\int_{\pi/4}^{\pi/2} \frac{(1-3\cos x)}{\sin^2 x} \, dx = \int_{\pi/4}^{\pi/2} (\operatorname{cosec}^2(x) - 3\operatorname{cosec}(x)\cot(x)) \, dx$$

$$= \int_{\pi/4}^{\pi/2} (\operatorname{cosec}^2(x) - 3\operatorname{cosec}(x)\cot(x)) \, dx$$

24. Question

Evaluate:

$$\int_0^{\pi/4} \sqrt{1+\cos 2x} \, dx$$

Answer

1

Evaluation:

$$\int_0^{\pi/4} \sqrt{1+\cos 2x} \, dx = \int_0^{\pi/4} \sqrt{2\cos^2 x} \, dx$$

$$= \sqrt{2}[\sin x]$$

$$= \sqrt{2} \left[\sin\left(\frac{\pi}{4}\right) - \sin 0 \right]$$

$$= \sqrt{2} \left[\frac{1}{\sqrt{2}} \right]$$

=1

25. Question

Evaluate:

$$\int_0^{\pi/4} \sqrt{1 - \sin 2x} \, dx$$

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Answer

$$(\sqrt{2} - 1)$$

Evaluation:

$$\int_0^{\pi/4} \sqrt{1 - \sin 2x} \, dx = \int_0^{\pi/4} \sqrt{\sin^2 x + \cos^2 x - 2 \sin x \cos x} \, dx$$

$$= \int_0^{\pi/4} (\cos x - \sin x) \, dx$$

$$= [\sin x + \cos x]$$

$$= \left[\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right) - \cos 0 - \sin 0 \right]$$

$$= \left[+\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 \right]$$

$$= [\sqrt{2} - 1]$$

**26. Question**

Evaluate:

$$\int_{-\pi/4}^{\pi/4} \frac{dx}{(1 + \sin x)}$$

Answer

2

Evaluation:

$$\int_{-\pi/4}^{\pi/4} \frac{dx}{1 + \sin x} = \int_{-\pi/4}^{\pi/4} \frac{\sec^2\left(\frac{x}{2}\right)}{\left(\tan^2\left(\frac{x}{2}\right) + 1\right)^2} dx$$

$$\text{Let } u = \left(\tan\left(\frac{x}{2}\right) + 1\right)$$

$$dx = \frac{2}{\sec^2\left(\frac{x}{2}\right)} du$$

$$\int_{-\pi/4}^{\pi/4} \frac{dx}{1 + \sin x} = 2 \int_{-\pi/4}^{\pi/4} \frac{1}{u^2} du$$

$$= -\frac{2}{u}$$

$$= -\frac{2}{\tan\left(\frac{x}{2}\right) + 1}$$

$$= 2$$

27. Question

Evaluate:

$$\int_0^{\pi/4} \frac{dx}{(1 + \cos 2x)}$$

Answer

$$\frac{1}{2}$$

Evaluation:

$$\int_0^{\pi/4} \frac{dx}{1 + \cos 2x} = \int_0^{\pi/4} \frac{dx}{2\cos^2 x}$$

$$\int_0^{\pi/4} \frac{dx}{2\cos^2 x} = \int_0^{\pi/4} \frac{1}{2} \sec^2 x dx$$

$$\int_0^{\pi/4} \frac{1}{2} \sec^2 x dx = \frac{1}{2} [\tan x]$$

$$= \frac{1}{2} \left[\tan\left(\frac{\pi}{4}\right) - \tan 0 \right]$$

$$= \frac{1}{2} [1]$$

$$= \frac{1}{2}$$



28. Question

Evaluate:

$$\int_{\pi/4}^{\pi/2} \frac{dx}{1 - \cos 2x}$$

Answer

$$\frac{1}{2}$$

Evaluation:

$$\int_{\pi/4}^{\pi/2} \frac{dx}{1 - \cos 2x} = \int_{\pi/4}^{\pi/2} \frac{dx}{2\sin^2 x}$$

$$\int_{\pi/4}^{\pi/2} \frac{dx}{2\sin^2 x} = \int_{\pi/4}^{\pi/2} \frac{1}{2} \operatorname{cosec}^2 x dx$$

$$\int_{\pi/4}^{\pi/2} \frac{1}{2} \operatorname{cosec}^2 x dx = \frac{1}{2} [\cot x]$$

$$= \frac{1}{2} \left[\cot\left(\frac{\pi}{4}\right) - \cot 0 \right]$$

$$= \frac{1}{2} [1]$$

$$= \frac{1}{2}$$

29. Question

Evaluate:

$$\int_0^{\pi/4} \sin 2x \sin 3x \, dx$$

Answer

$$\frac{3}{5\sqrt{2}}$$

Evaluation:

$$\int_0^{\pi/4} \sin 2x \sin 3x \, dx = \frac{1}{2} \int_0^{\pi/4} (\cos x - \cos 5x) \, dx$$

$$= \frac{1}{2} \int_0^{\pi/4} (\cos x - \cos 5x) \, dx$$

$$= \frac{1}{2} \left[\sin x - \frac{\sin 5x}{5} \right]$$

$$= \frac{1}{2} \left[\sin\left(\frac{\pi}{4}\right) - \frac{\sin\left(\frac{5\pi}{4}\right)}{5} \right] - \frac{1}{2} \left[\sin(0) - \frac{\sin(0)}{5} \right]$$

$$= \frac{1}{2} \left[\frac{1}{\sqrt{2}} + \frac{1}{5\sqrt{2}} \right]$$

$$= \frac{3}{5\sqrt{2}}$$

30. Question

Evaluate:

$$\int_0^{\pi/6} \cos x \cos 2x \, dx$$

Answer

$$\frac{5}{12}$$

Evaluation:

$$\int_0^{\pi/6} \cos x \cos 2x \, dx = \frac{1}{2} \int_0^{\pi/6} (\cos 3x + \cos x) \, dx$$

$$= \frac{1}{2} \left[\frac{\sin 3x}{3} + \sin x \right]$$



$$\begin{aligned}
&= \frac{1}{2} \left[\frac{\sin\left(\frac{\pi}{2}\right)}{3} + \sin\left(\frac{\pi}{6}\right) \right] - 0 \\
&= \frac{1}{2} \left[\frac{1}{3} + \frac{1}{2} \right] \\
&= \frac{5}{12}
\end{aligned}$$

31. Question

Evaluate:

$$\int_0^{\pi} \sin 2x \cos 3x \, dx$$

Answer

$$-\frac{4}{5}$$

Evaluation:

$$\int_0^{\pi} \sin 2x \cos 3x \, dx = \frac{1}{2} \int_0^{\pi} (\sin 5x - \sin x) \, dx$$

$$= \frac{1}{2} \left[-\frac{\cos 5x}{5} + \cos x \right]$$

$$= \frac{1}{2} \left[-\frac{\cos(5\pi)}{5} + \cos(\pi) \right] - \frac{1}{2} \left[-\frac{\cos(0)}{5} + \cos(0) \right]$$

$$= \frac{1}{2} \left[\frac{-(-1)}{5} - 1 \right] - \frac{1}{2} \left[-\frac{1}{5} + 1 \right]$$

$$= \frac{1}{2} \left[\frac{-4}{5} - \frac{4}{5} \right]$$

$$= \frac{1}{2} \cdot 2 \left(-\frac{4}{5} \right)$$

$$= -\frac{4}{5}$$

32. Question

Evaluate:

$$\int_0^{\pi/2} \sqrt{1 + \sin x} \, dx$$

Answer

2

Explanation:

$$\int_0^{\pi/2} \sqrt{1 + \sin(x)} \, dx = \int_0^{\pi/2} \sqrt{2} \cos\left(\frac{2x - \pi}{4}\right) \, dx$$

$$= 2^{\frac{3}{2}} \sin\left(\frac{2x - \pi}{4}\right)$$



$$= 2^{\frac{3}{2}} \left(0 - \sin\left(-\frac{\pi}{4}\right) \right)$$

$$= \frac{2\sqrt{2}}{\sqrt{2}}$$

$$= 2$$

33. Question

Evaluate:

$$\int_0^{\pi/2} \sqrt{1 + \cos x} \, dx$$

Answer

$$2$$

Explanation:

$$\int_0^{\pi/2} \sqrt{1 + \cos(x)} \, dx = \int_0^{\pi/2} \sqrt{2} \cos\left(\frac{x}{2}\right) \, dx$$

$$= 2^{\frac{3}{2}} \sin\left(\frac{x}{2}\right)$$

$$= 2^{\frac{3}{2}} \left(\sin\left(\frac{\pi}{4}\right) - 0 \right)$$

$$= \frac{2\sqrt{2}}{\sqrt{2}}$$

$$= 2$$



34. Question

Evaluate:

$$\int_0^2 \frac{(x^4 + 1)}{(x^2 + 1)} \, dx$$

Answer

$$\left(\frac{2}{3} + 2 \tan^{-1} 2 \right)$$

Explanation:

$$\int_0^2 \left\{ \frac{(x^4 + 1)}{x^2 + 1} \right\} \, dx = \int_0^2 \frac{x^4 + 2 - 1}{x^2 + 1} \, dx$$

$$= \int_0^2 \frac{x^4 - 1}{x^2 + 1} \, dx + \int_0^2 \frac{2}{x^2 + 1} \, dx$$

$$= \int_0^2 \frac{(x^2 - 1)(x^2 + 1)}{x^2 + 1} \, dx + \int_0^2 \frac{2}{x^2 + 1} \, dx$$

$$= \int_0^2 (x^2 - 1) \, dx + 2 \tan^{-1} x$$

$$= \left[\frac{x^3}{3} - x + 2 \tan^{-1} x \right]_0^2$$

$$= \frac{2}{3} + 2 \tan^{-1} 2$$

35. Question

Evaluate:

$$\int_1^2 \frac{dx}{(x+1)(x+2)}$$

Answer

$$(2 \log 3 - 3 \log 2)$$

Explanation:

$$\int_1^2 \frac{dx}{(x+1)(x+2)} = \int_1^2 \frac{(x+2) - (x+1)}{(x+1)(x+2)} dx$$

$$= \int_1^2 \frac{1}{(x+1)} dx - \int_1^2 \frac{1}{(x+2)} dx$$

$$= [\log(x+1) - \log(x+2)]_1^2$$

$$= 2 \log 3 - 3 \log 2$$

36. Question

Evaluate:

$$\int_1^2 \frac{(x+3)}{x(x+2)} dx$$



Answer

$$\frac{1}{2} (\log 2 + \log 3)$$

Explanation:

$$\int_1^2 \frac{x+3}{x(x+2)} dx = \int_1^2 \frac{3}{2x} dx - \int_1^2 \frac{1}{x+2} dx$$

$$= \frac{3}{2} \log x - \log(x+2)$$

$$= \frac{1}{2} (\log 2 + \log 3)$$

37. Question

Evaluate:

$$\int_3^4 \frac{dx}{(x^2-4)}$$

Answer

$$\frac{1}{4}(\log 5 - \log 3)$$

Evaluation:

$$\begin{aligned} \int_3^4 \frac{dx}{x^2 - 4} &= \int_3^4 \frac{1}{(x-2)(x+2)} dx \\ &= \int_3^4 \frac{1}{4(x-2)} dx - \int_3^4 \frac{1}{4(x+2)} dx \\ &= \frac{1}{4} \log(x-2) - \frac{1}{4} \log(x+2) \\ &= \frac{1}{4} \log 3 - \frac{1}{4} \log 1 - \frac{1}{4} \log 6 + \frac{1}{4} \log 5 \\ &= \frac{1}{4} (\log 5 - \log \left(\frac{6}{2}\right)) \\ &= \frac{1}{4} (\log 5 - \log 3) \end{aligned}$$

38. Question

Evaluate:

$$\int_0^4 \frac{dx}{\sqrt{x^2 + 2x + 3}}$$

Answer

$$\log \left(\frac{5 + 3\sqrt{3}}{1 + \sqrt{3}} \right)$$



Evaluation:

$$\int \frac{dx}{\sqrt{x^2 + 2x + 3}} = \int \frac{dx}{\sqrt{(x+1)^2 + 2}}$$

Substitute:

$$\frac{x+1}{\sqrt{2}} = u$$

$$\therefore dx = \sqrt{2} du$$

$$= \int \frac{\sqrt{2} du}{\sqrt{2u^2 + 2}}$$

$$= \log(\sqrt{u^2 + 1} + u)$$

$$\text{Undo substitution: } u = \frac{x+1}{\sqrt{2}}$$

$$\therefore \int_0^4 \frac{dx}{\sqrt{x^2 + 4x + 3}} = \log(\sqrt{(x+1)^2 + 2} + x + 1)$$

$$= \log(\sqrt{(4+1)^2 + 2} + 4 + 1) - \log(\sqrt{(0+1)^2 + 2} + 0 + 1)$$

$$= \log(5 + 3\sqrt{3}) - \log(1 + \sqrt{3})$$

$$= \log\left(\frac{5 + 3\sqrt{3}}{1 + \sqrt{3}}\right)$$

39. Question

Evaluate:

$$\int_1^2 \frac{dx}{\sqrt{x^2 + 4x + 3}}$$

Answer

$$\log(4 + \sqrt{15}) - \log(3 + \sqrt{8})$$

Evaluation:

$$\int \frac{dx}{\sqrt{x^2 + 4x + 3}} = \int \frac{dx}{\sqrt{(x+2)^2 - 1}}$$

Substitute:

$$x+2=u$$

$$\therefore dx=du$$

$$= \int \frac{du}{\sqrt{u^2 - 1}}$$

$$= \log(\sqrt{u^2 - 1} + u)$$

Undo substitution: $u = x + 2$

$$\therefore \int_1^2 \frac{dx}{\sqrt{x^2 + 4x + 3}} = \log(\sqrt{(x+2)^2 - 1} + x + 2)$$

$$= \log(\sqrt{(2+2)^2 - 1} + 2 + 2) - \log(\sqrt{(1+2)^2 - 1} + 1 + 2)$$

$$= \log(4 + \sqrt{15}) - \log(3 + \sqrt{8})$$

40. Question

Evaluate:

$$\int_0^1 \frac{dx}{(1+x+2x^2)}$$

Answer

$$\frac{2}{\sqrt{7}} \left\{ \tan^{-1} \frac{5}{\sqrt{7}} - \tan^{-1} \frac{1}{\sqrt{7}} \right\}$$

Evaluation:

$$\int_0^1 \frac{1}{2x^2 + x + 1} dx = \int_0^1 \frac{1}{\left(\left(\sqrt{2x} + \frac{1}{\sqrt{2}}\right)^2 + \frac{7}{8}\right)} dx$$

Substitute $4x+1=\sqrt{7}=u$



$$\therefore dx = \frac{\sqrt{7}}{4} du$$

Now solving:

$$\int \left(\frac{1}{u^2} + 1 \right) du = \tan^{-1} u$$

$$\frac{2}{\sqrt{7}} \int \frac{1}{u^2 + 1} du = \frac{2}{\sqrt{7}} \tan^{-1} u$$

$$\therefore \int_0^1 \frac{1}{2x^2 + x + 1} dx = \frac{2}{\sqrt{7}} \tan^{-1} \left(\frac{4x + 1}{\sqrt{7}} \right)$$

$$= \frac{2}{\sqrt{7}} \tan^{-1} \left(\frac{4 + 1}{\sqrt{7}} \right) - \frac{2}{\sqrt{7}} \tan^{-1} \left(\frac{1}{\sqrt{7}} \right)$$

$$= \frac{2}{\sqrt{7}} \left\{ \tan^{-1} \left(\frac{5}{\sqrt{7}} \right) - \tan^{-1} \left(\frac{1}{\sqrt{7}} \right) \right\}$$

41. Question

Evaluate:

$$\int_0^{\pi/2} (a \cos^2 x + b \sin^2 x) dx$$

Answer

$$\frac{\pi}{4} (a + b)$$



Evaluation:

$$\int_0^{\pi/2} (a \cos^2 x + b \sin^2 x) dx = \int_0^{\pi/2} \left[\frac{a}{2} (\cos 2x + 1) + \frac{b}{2} (1 - \cos 2x) \right] dx$$

$$= \left[\frac{a}{2} \left(\frac{\sin 2x}{2} + x \right) + \frac{b}{2} \left(x - \frac{\sin 2x}{2} \right) \right]$$

$$= \left[\frac{a}{2} \left(\frac{\sin \pi}{2} + \frac{\pi}{2} \right) + \frac{b}{2} \left(\frac{\pi}{2} - \frac{\sin \pi}{2} \right) - \frac{a}{2} \left(\frac{\sin 0}{2} + 0 \right) - \frac{b}{2} \left(0 - \frac{\sin 0}{2} \right) \right]$$

$$= \left[\frac{a}{2} \left(0 + \frac{\pi}{2} \right) + \frac{b}{2} \left(\frac{\pi}{2} - 0 \right) - \frac{a}{2} (0 + 0) - \frac{b}{2} (0 - 0) \right]$$

$$= \frac{\pi}{4} (a + b)$$

42. Question

Evaluate:

$$\int_{\pi/3}^{\pi/4} (\tan x + \cot x)^2 dx$$

Answer

$$\frac{-2}{\sqrt{3}}$$

Evaluation:

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} (\tan x + \cot x)^2 dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{\tan^2 x + 1}{\tan x} \right)^2 dx$$

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{\tan^2 x + 1}{\tan x} \right)^2 dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \frac{\sec^2 x (\tan^2 x + 1)}{\tan^2 x} dx$$

Substitute:

$$\tan(x) = u$$

$$\therefore dx = \frac{1}{\sec^2(x)} du$$

$$\therefore = \int \frac{(u^2 + 1)}{u^2} du$$

$$\therefore = u - \frac{1}{u}$$

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{\tan^2 x + 1}{\tan x} \right)^2 dx = [\tan(x) - \cot(x)]$$

$$= \left[\tan\left(\frac{\pi}{4}\right) - \cot\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{3}\right) + \cot\left(\frac{\pi}{3}\right) \right]$$

$$= \left[1 - 1 - \sqrt{3} + \frac{1}{\sqrt{3}} \right]$$

$$= -\frac{2}{\sqrt{3}}$$

43. Question

Evaluate:

$$\int_0^{\pi/2} \cos^4 x dx$$

Answer

$$\frac{3\pi}{16}$$

Evaluation:

By reduction formula:

$$\int_0^{\pi/2} \cos^4 x dx = \frac{\cos^3(x)\sin(x)}{4} + \frac{3}{4} \int \cos^2 x dx$$

We know that,

$$\int \cos^2 x dx = \frac{1}{2} \left[\frac{\sin 2x}{2} + x \right]$$

$$\int_0^{\pi/2} \cos^4 x dx = \frac{\cos^3(x)\sin(x)}{4} + \frac{3}{8} \left[\frac{\sin 2x}{2} + x \right]$$

$$= \frac{\cos^3\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right)}{4} + \frac{3}{8} \left[\frac{\sin \pi}{2} + \frac{\pi}{2} \right] - \frac{\cos^3(0)\sin(0)}{4} - \frac{3}{8} \left[\frac{\sin 0}{2} + 0 \right]$$



$$= 0 + \frac{3}{8} \left[0 + \frac{\pi}{2} \right] - 0 - \frac{3}{8} [0 + 0]$$

$$= \frac{3\pi}{16}$$

44. Question

Evaluate:

$$\int_0^a \frac{dx}{(ax + a^2 - x^2)}$$

Answer

$$\frac{1}{\sqrt{5}a} \log \left\{ \frac{7 + 3\sqrt{5}}{2} \right\}$$

Evaluation:

Assume that $a \neq 0$.

$$\int_0^2 \frac{1}{-x^2 + ax + a^2} dx = - \int_0^2 \frac{1}{x^2 - ax - a^2} dx$$

$$= \int_0^2 \frac{4}{(2x + (-\sqrt{5} - 1)a)(2x + (\sqrt{5} - 1)a)} dx$$

$$= \int_0^2 \left(\frac{2}{\sqrt{5}a(2x + (-\sqrt{5} - 1)a)} - \frac{2}{\sqrt{5}a(2x + (\sqrt{5} - 1)a)} \right) dx$$

Now,

$$\int \frac{1}{2x + (-\sqrt{5} - 1)a} dx$$

Substitute:

$$u = 2x + (-\sqrt{5} - 1)a$$

$$\therefore dx = \frac{1}{2} du$$

$$= \frac{1}{2} \int \frac{1}{u} du$$

$$= \frac{1}{2} \log u$$

Undo substitution:

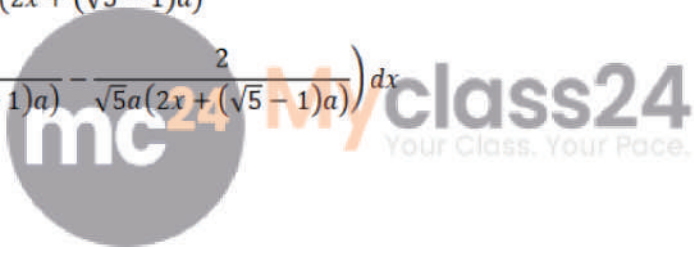
$$u = 2x + (-\sqrt{5} - 1)a$$

$$\therefore \int \frac{1}{2x + (-\sqrt{5} - 1)a} dx = \frac{1}{2} \log(2x + (-\sqrt{5} - 1)a)$$

Now,

$$\int \frac{1}{2x + (\sqrt{5} - 1)a} dx$$

Substitute:



$$u = 2x + (\sqrt{5} - 1)a$$

$$\therefore dx = \frac{1}{2} du$$

$$= \frac{1}{2} \int \frac{1}{u} du$$

$$= \frac{1}{2} \log u$$

Undo substitution:

$$u = 2x + (\sqrt{5} - 1)a$$

$$\therefore \int \frac{1}{2x + (\sqrt{5} - 1)a} dx = \frac{1}{2} \log(2x + (\sqrt{5} - 1)a)$$

$$\frac{2}{\sqrt{5}a} \int_0^2 \frac{1}{(2x + (-\sqrt{5} - 1)a)} dx - \frac{2}{\sqrt{5}a} \int_0^2 \frac{1}{2x + (\sqrt{5} - 1)a} dx$$

$$= \frac{\log(2x + (-\sqrt{5} - 1)a)}{\sqrt{5}a} - \frac{\log(2x + (\sqrt{5} - 1)a)}{\sqrt{5}a}$$

$$- \int_0^2 \frac{1}{x^2 - ax - a^2} dx = \frac{\log(2x + (\sqrt{5} - 1)a)}{\sqrt{5}a} - \frac{\log(2x + (-\sqrt{5} - 1)a)}{\sqrt{5}a}$$

$$= \frac{\log(4 + (\sqrt{5} - 1)a)}{\sqrt{5}a} - \frac{\log(4 + (-\sqrt{5} - 1)a)}{\sqrt{5}a} - \frac{\log(0 + (\sqrt{5} - 1)a)}{\sqrt{5}a}$$

$$+ \frac{\log(0 + (-\sqrt{5} - 1)a)}{\sqrt{5}a}$$

$$= \frac{1}{\sqrt{5}a} \log\left(\frac{7 + 3\sqrt{5}}{2}\right)$$



45. Question

Evaluate:

$$\int_{1/4}^{1/2} \frac{dx}{\sqrt{x - x^2}}$$

Answer

$$\frac{\pi}{6}$$

Evaluation:

$$\int_{1/4}^{1/2} \frac{dx}{\sqrt{x - x^2}} = \int_{1/4}^{1/2} \frac{1}{\sqrt{\frac{1}{4} - (x - \frac{1}{2})^2}}$$

Substitute:

$$2x - 1 = u$$

$$\therefore dx = \frac{1}{2} du$$

$$\int \frac{1}{\sqrt{1 - u^2}} du = \sin^{-1}(u)$$

Undo Substitution:

$$u=2x-1$$

$$\therefore \sin^{-1}(2x-1)$$

$$\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{dx}{\sqrt{x-x^2}} = \sin^{-1}(2x-1)$$
$$= \sin^{-1}(1-1) - \sin^{-1}\left(\frac{1}{2}-1\right)$$
$$= \frac{\pi}{6}$$

46. Question

Evaluate:

$$\int_0^1 \sqrt{x(1-x)} dx$$

Answer

$$\frac{\pi}{8}$$

Evaluation:

$$\int_0^1 \sqrt{x-x^2} dx = \int_0^1 \sqrt{\frac{1}{4} - \left(x - \frac{1}{2}\right)^2} dx$$
$$= \frac{1}{2} \int_0^1 \sqrt{1 - (2x-1)^2} dx$$

Substitute:

$$2x-1=u$$

$$\therefore dx = \frac{1}{2} du$$

$$\therefore \frac{1}{2} \int \sqrt{1-u^2} du$$

Substitute:

$$u=\sin(v)$$

$$\therefore \sin^{-1}(u)=v$$

$$\therefore du=\cos(v)dv$$

$$= \int \cos(v)\sqrt{1-\sin^2(v)} dv$$

$$= \int \cos^2(v) dv$$

We know that,

$$\int \cos^2(v) dv = \frac{1}{2} \left[\frac{\sin(2v)}{2} + v \right]$$

Undo Substitution:

$$v=\sin^{-1}(u) \sin(\sin^{-1}(u))=u \cos(\sin^{-1}(u)) = \sqrt{1-u^2}$$

$$= \frac{\sin^{-1}(u)}{2} + \frac{u\sqrt{1-u^2}}{2}$$

Undo Substitution:

$$u=2x-1$$

$$\therefore = \frac{\sin^{-1}(2x-1)}{4} + \frac{(2x-1)\sqrt{1-(2x-1)^2}}{4}$$

$$\frac{1}{2} \int_0^1 \sqrt{1-(2x-1)^2} dx = \frac{\sin^{-1}(2x-1)}{8} + \frac{(2x-1)\sqrt{1-(2x-1)^2}}{8}$$

$$= \frac{\sin^{-1}(2-1)}{8} + \frac{(2-1)\sqrt{1-(2-1)^2}}{8} - \frac{\sin^{-1}(0-1)}{8} - \frac{(0-1)\sqrt{1-(0-1)^2}}{8}$$

$$= \frac{\pi}{16} + 0 - \frac{\pi}{8} - 0$$

$$= \frac{\pi}{8}$$

47. Question

Evaluate:

$$\int_1^3 \frac{dx}{x^2(x+1)}$$

Answer

$$\log 2 - \log 3 + \frac{2}{3}$$



Evaluation:

$$\int_1^3 \frac{1}{x^2(x+1)} dx$$

Perform partial fraction decomposition:

$$\int_1^3 \frac{1}{x^2(x+1)} dx = \int_1^3 \left(\frac{1}{x+1} - \frac{1}{x} + \frac{1}{x^2} \right) dx$$

$$= \left[\log(x+1) - \log(x) - \frac{1}{x} \right]$$

$$= \left[\log(4) - \log(3) - \frac{1}{3} - \log(2) + \log(1) + \frac{1}{1} \right]$$

$$= \log(2) - \log(3) + \frac{2}{3}$$

48. Question

Evaluate:

$$\int_1^2 \frac{dx}{x(1+2x)^2}$$

Answer

$$\log 6 - \log 5 - \frac{2}{15}$$

Evaluation:

$$\begin{aligned} \int_1^2 \frac{1}{x(2x+1)^2} dx &= \int_1^2 \left(-\frac{2}{2x+1} - \frac{2}{(2x+1)^2} + \frac{1}{x} \right) dx \\ &= -2 \int_1^2 \frac{1}{2x+1} dx - 2 \int_1^2 \frac{1}{(2x+1)^2} dx + \int_1^2 \frac{1}{x} dx \\ &= -2 \left[\frac{1}{2} \log(2x+1) \right] - 2 \left[\frac{-1}{2(2x+1)} \right] + [\log(x)] \\ &= -[\log(5)] + \left[\frac{1}{(5)} \right] + [\log(2)] + [\log(3)] - \left[\frac{1}{(3)} \right] + [\log(1)] \\ &= \log(6) - \log(5) - \frac{2}{15} \end{aligned}$$

49. Question

Evaluate:

$$\int_0^1 x e^x dx$$

Answer

1

Evaluation:

$$\begin{aligned} \int_0^1 x e^x dx &= \int_0^1 (x-1+1) e^x dx \\ &= [(x-1)e^x] \\ &= [(1-1) e^1 - (0-1) e^0] \\ &= 1 \end{aligned}$$

50. Question

Evaluate:

$$\int_0^{\pi/2} x^2 \cos x dx$$

Answer

$$\left(\frac{\pi^2}{4} - 2 \right)$$

Evaluation:

$$\begin{aligned} \int_0^{\pi/2} x^2 \cos(x) dx &= x^2 \sin(x) - \int 2x \sin(x) dx \\ \int_0^{\pi/2} x^2 \cos(x) dx &= [x^2 \sin(x) - 2 \sin(x) - 2x \cos(x)] \end{aligned}$$



$$\begin{aligned}
&= \left[\left(\frac{\pi}{2}\right)^2 \sin\left(\frac{\pi}{2}\right) - 2\sin\left(\frac{\pi}{2}\right) - \pi\cos\left(\frac{\pi}{2}\right) - (0)^2\sin(0) + 2\sin(0) + 0 \right] \\
&= \left[\frac{\pi^2}{4} - 2 - 0 - 0 + 0 + 0 \right] \\
&= \left(\frac{\pi^2}{4} - 2 \right)
\end{aligned}$$

51. Question

Evaluate:

$$\int_0^{\pi/4} x^2 \sin x \, dx$$

Answer

$$\left(\sqrt{2} + \frac{\pi}{2\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} - 2 \right)$$

Evaluation:

From integrate by parts:

$$\int_0^{\pi/4} x^2 \sin(x) \, dx = -x^2 \cos(x) - \int -2x \cos(x) \, dx$$

From integrate by parts:

$$\int_0^{\pi/4} x^2 \cos(x) \, dx = [-x^2 \cos(x) + 2x \sin(x) + 2 \cos(x)]$$

$$= [2x \sin(x) + (2 - x^2) \cos(x)]$$

$$= \left[\frac{\pi}{2} \sin\left(\frac{\pi}{4}\right) + \left(2 - \frac{\pi^2}{16}\right) \cos\left(\frac{\pi}{4}\right) - 2(0) \sin(0) - (2 - 0) \cos(0) \right]$$

$$= \left[\frac{\pi}{2\sqrt{2}} + \frac{2}{\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} + 0 - 0 - 2 \right]$$

$$= \sqrt{2} + \frac{\pi}{2\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} - 2$$

52. Question

Evaluate:

$$\int_0^{\pi/2} x^2 \cos 2x \, dx$$

Answer

$$\frac{-\pi}{4}$$

Evaluation:

$$\int_0^{\pi/2} x^2 \cos(2x) \, dx = \frac{x^2 \sin(2x)}{2} - \int x \sin(x) \, dx$$

$$\int_0^{\frac{\pi}{2}} x^2 \cos(x) dx = \left[\frac{x^2 \sin(2x)}{2} - \frac{\sin(2x)}{4} + \frac{x \cos(2x)}{2} \right]$$

$$= \left[\frac{\left(\frac{\pi}{2}\right)^2 \sin(\pi)}{2} - \frac{\sin(\pi)}{4} + \frac{\left(\frac{\pi}{2}\right) \cos(\pi)}{2} - \frac{(0)^2 \sin(0)}{2} + \frac{\sin(0)}{4} - \frac{(0) \cos(0)}{2} \right]$$

$$= \left[0 - 0 - \frac{\pi}{4} - 0 + 0 - 0 \right]$$

$$= -\frac{\pi}{4}$$

53. Question

Evaluate:

$$\int_0^{\pi/2} x^3 \sin 3x \, dx$$

Answer

$$\left(\frac{2}{27} - \frac{\pi^2}{12} \right)$$

Evaluation:

$$\int_0^{\frac{\pi}{2}} x^3 \sin(3x) dx = -\frac{x^3 \cos(3x)}{3} - \int -x^2 \cos(3x) dx$$

$$= -\frac{x^3 \cos(3x)}{3} + \frac{x^2 \sin(3x)}{3} - \int \frac{2x \sin(3x)}{3} dx$$

$$= -\frac{x^3 \cos(3x)}{3} + \frac{x^2 \sin(3x)}{3} + \frac{2x \cos(3x)}{9} + \frac{2}{3} \int -\frac{\cos(3x)}{3} dx$$

$$= -\frac{x^3 \cos(3x)}{3} + \frac{x^2 \sin(3x)}{3} + \frac{2x \cos(3x)}{9} - \frac{2 \sin(3x)}{27}$$

$$= -0 + \frac{\left(\frac{\pi}{2}\right)^2 \sin\left(\frac{3\pi}{2}\right)}{3} + 0 - \frac{2 \sin\left(\frac{3\pi}{2}\right)}{27} + 0 - 0 - 0 + 0$$

$$= \left(\frac{2}{27} - \frac{\pi^2}{12} \right)$$

54. Question

Evaluate:

$$\int_0^{\pi/2} x^2 \cos^2 x \, dx$$

Answer

$$\left(\frac{\pi^3}{48} - \frac{\pi}{8} \right)$$

Evaluation:

$$\begin{aligned}
\int_0^{\frac{\pi}{2}} x^2 \cos^2 x dx &= \int_0^{\frac{\pi}{2}} \frac{x^2}{2} (\cos(2x) + 1) dx \\
&= \int_0^{\frac{\pi}{2}} \left(\frac{x^2}{2} \cos(2x) + \frac{x^2}{2} \right) dx \\
\int_0^{\frac{\pi}{2}} \left(\frac{x^2}{2} \cos(2x) + \frac{x^2}{2} \right) dx &= \frac{x^2 \sin(2x)}{2} - \int x \sin(2x) dx + \frac{x^3}{6} \\
&= \frac{x^2 \sin(2x)}{2} + \frac{x \cos(2x)}{4} + \int -\frac{\cos(2x)}{2} dx + \frac{x^3}{6} \\
&= \frac{x^2 \sin(2x)}{2} + \frac{x \cos(2x)}{4} - \frac{\sin(2x)}{4} + \frac{x^3}{6} \\
&= \frac{x^2 \sin(2x)}{2} + \frac{x \cos(2x)}{4} - \frac{\sin(2x)}{4} + \frac{x^3}{6} \\
&= 0 + \frac{\frac{\pi}{2} \cos(\pi)}{4} - 0 + \frac{\left(\frac{\pi}{2}\right)^3}{6} - 0 - 0 + 0 - 0 \\
&= \left(\frac{\pi^3}{48} - \frac{\pi}{8} \right)
\end{aligned}$$

55. Question

Evaluate:

$$\int_1^2 \log x \, dx$$



Answer

$$(2 \log 2 - 1)$$

Evaluation:

$$\begin{aligned}
\int_1^2 \log(x) dx &= x \log(x) - (x) \\
&= 2 \log(2) - (2) - 1 \log(1) + (1) \\
&= 2 \log(2) - 1
\end{aligned}$$

56. Question

Evaluate:

$$\int_1^3 \frac{\log x}{(1+x)^2} dx$$

Answer

$$\frac{3}{4} \log 3 - \log 2$$

Evaluation:

$$\int_1^3 \frac{\log(x)}{(1+x)^2} dx = -\frac{\log(x)}{1+x} - \int \left(-\frac{1}{x(1+x)} \right) dx$$

Now,

$$\int \left(-\frac{1}{x(1+x)} \right) dx = - \int \left(\frac{1}{x^2 \left(\frac{1}{x} + 1 \right)} \right) dx$$

Let,

$$\frac{1}{x} + 1 = u$$

$$\therefore dx = -x^2 du$$

$$\therefore - \int \left(\frac{1}{x^2 \left(\frac{1}{x} + 1 \right)} \right) dx = \int \frac{1}{u} du$$

$$= \log(u)$$

Undo substitution:

$$u = \frac{1}{x} + 1$$

$$\int_1^3 \frac{\log(x)}{(1+x)^2} dx = -\frac{\log(x)}{1+x} + \log\left(\frac{1}{x} + 1\right)$$

$$= -\frac{\log(3)}{4} + \log\left(\frac{4}{3}\right) + \frac{\log(1)}{2} - \log(2)$$

$$= -\frac{\log(3)}{4} + \log(4) + \log(3) - \log 2$$

$$= \frac{3}{4} \log 3 - \log 2$$

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57. Question

Evaluate:

$$\int_0^2 \left\{ \frac{1}{(\log x)} - \frac{1}{(\log x)^2} \right\} dx$$

Answer

$$\left(\frac{e^2}{2} - e \right)$$

Correct answer is $\frac{e^2}{2}$

Evaluation:

Let,

$$\log(x) = u$$

$$\rightarrow x = e^u$$

$$\rightarrow dx = e^u du$$

$$\int \left(\frac{1}{u} - \frac{1}{u^2} \right) e^u du = \frac{e^u}{u}$$

Undo substitution:

$$u = \log(x)$$

$$\begin{aligned} \int_0^{e^2} \left\{ \frac{1}{\log(x)} - \frac{1}{\log(x)^2} \right\} dx &= \frac{x}{\log(x)} \\ &= \frac{e^2}{\log(e^2)} - 0 \\ &= \frac{e^2}{2} \end{aligned}$$

58. Question

Evaluate:

$$\int_1^e e^x \left(\frac{1+x \log x}{x} \right) dx$$

Answer

$$e^e$$

Evaluation:

$$\begin{aligned} \int_1^e e^x \left(\frac{1+x \log(x)}{x} \right) dx &= \int_1^e e^x \left(\frac{1}{x} + \log(x) \right) dx \\ &= \log(x) e^x \\ &= \log(e) e^e - \log(1) e^1 \\ &= e^e \end{aligned}$$



59. Question

Evaluate:

$$\int_0^1 \frac{x e^x}{(1+x)^2} dx$$

Answer

$$\left(\frac{e}{2} - 1 \right)$$

Evaluation:

$$\int_0^1 \frac{x e^x}{(1+x)^2} dx$$

From Integrates by parts:

$$\begin{aligned} &= -\frac{x e^x}{x+1} - \int \frac{-x e^x - e^x}{x+1} dx \\ \therefore \int \frac{-x e^x - e^x}{x+1} dx &= \int -e^x dx \\ &= -e^x \end{aligned}$$

$$\int_0^1 \frac{x e^x}{(1+x)^2} dx = \left[-\frac{x e^x}{x+1} - e^x \right]$$

$$= \left[-\frac{1e^1}{1+1} - e^1 - \frac{0}{1+0} + e^0 \right]$$

$$= \left[-\frac{e}{2} + e + 0 - 1 \right]$$

$$= \left[\frac{e}{2} - 1 \right]$$

60. Question

Evaluate:

$$\int_0^{\pi/2} 2 \tan^3 x \, dx$$

[CBSE 2004]

Answer

(1 - log 2)

Evaluation:

$$\int_0^{\pi/2} 2 \tan^3 x \, dx = 2 \int_0^{\pi/2} \tan^2 x \tan x \, dx$$

$$= 2 \int_0^{\pi/2} \tan^2 x \tan x \, dx$$

$$= 2 \int_0^{\pi/2} (\sec^2 x - 1) \tan x \, dx$$

Substitute:

$$\sec(x) = u$$

$$\therefore dx = \frac{1}{\sec(x)\tan(x)} du$$

$$= 2 \int_0^{\pi/2} \frac{(u^2 - 1)}{u} du$$

$$= 2 \int_0^{\pi/2} \left(u - \frac{1}{u} \right) du$$

$$= 2 \int_0^{\pi/2} \left(u - \frac{1}{u} \right) du$$

$$= 2 \left[\frac{u^2}{2} - \log u \right]$$

Undo substitution:

$$u = \sec(x)$$

$$\therefore \int_0^{\pi/2} 2 \tan^3 x \, dx = 2 \left[\frac{\sec^2 x}{2} - \log(\sec x) \right]$$

$$= 2 \left[\frac{\sec^2\left(\frac{\pi}{2}\right)}{2} - \log\left(\sec\left(\frac{\pi}{2}\right)\right) - \frac{\sec^2(0)}{2} + \log(\sec(0)) \right]$$



$$= 2 \left[\frac{1}{2} - \log(1) \right]$$

$$= 1 - \log 2$$

61. Question

Evaluate:

$$\int_1^2 \frac{5x^2}{(x^2 + 4x + 3)} dx$$

Answer

$$5 - \frac{5}{2} \left(9 \log \frac{5}{4} - \log \frac{3}{2} \right)$$

Explanation:

$$\int_1^2 \frac{5x^2}{(x^2 + 4x + 3)} dx = 5 \left[\int_1^2 \frac{x^2}{(x+3)(x+1)} dx \right]$$

$$= 5 \left[\int_1^2 \left(1 - \frac{9}{2(x+3)} + \frac{1}{2(x+1)} \right) dx \right]$$

$$= 5 \left[x - \frac{9}{2} \log(x+3) + \frac{1}{2} \log(x+1) \right]_1^2$$

$$= 5 \left[2 - \frac{9}{2} \log 5 + \frac{1}{2} \log 3 - 1 + \frac{9}{2} \log 4 - \frac{1}{2} \log 2 \right]$$

$$= 5 \left[1 - \frac{9}{2} \log \left(\frac{5}{4} \right) + \frac{1}{2} \log \left(\frac{3}{2} \right) \right]$$

$$= 5 - \frac{5}{2} \left(9 \log \left(\frac{5}{4} \right) - \log \left(\frac{3}{2} \right) \right)$$



Exercise 16B

1. Question

Evaluate the following integrals

$$\int_0^1 \frac{dx}{(2x-3)}$$

Answer

$$\text{Let } I = \int_0^1 \frac{1}{2x-3} dx$$

$$\text{Let } 2x-3=t$$

$$\Rightarrow 2dx=dt.$$

Hence,

$$I = \frac{1}{2} \int_0^1 \frac{1}{t} dt = \frac{1}{2} \log_e |t|$$

$$= \frac{1}{2} \log_e |2x-3| \Big|_0^1$$

$$\Rightarrow I = \frac{1}{2} \log_e 1 - \frac{1}{2} \log_e 3 = \frac{1}{2} \log_e \frac{1}{3}$$

$$= -\frac{1}{2} \log_e 3$$

(Since $\log_a \frac{1}{b} = -\log_a b$)

2. Question

Evaluate the following integrals

$$\int_0^1 \frac{2x}{(1+x^2)} dx$$

Answer

$$\text{Let } I = \int_0^1 \frac{2x}{1+x^2} dx$$

$$\text{Let } 1+x^2=t$$

$$\Rightarrow 2x dx = dt.$$

Also,

$$\text{when } x=0, t=1$$

and

$$\text{when } x=1, t=2$$

$$\text{Hence, } I = \int_1^2 \frac{1}{t} dt = \log_e |t| \Big|_1^2$$

$$= \log_e 2 - \log_e 1$$

$$= \log_e 2$$



3. Question

Evaluate the following integrals

$$\int_1^2 \frac{3x}{(9x^2-1)} dx$$

Answer

$$\text{Let } I = \int_1^2 \frac{3x}{9x^2-1} dx$$

$$\text{Let } 9x^2-1=t$$

$$\Rightarrow 18x dx = dt.$$

Also,

$$\text{when } x=1, t=8$$

and

$$\text{when } x=2, t=35.$$

Hence,

$$I = \frac{1}{6} \int_8^{35} \frac{1}{t} dt = \frac{1}{6} \log_e t \Big|_8^{35} = \frac{1}{6} (\log_e 35 - \log_e 8)$$

4. Question

Evaluate the following integrals

$$\int_0^1 \frac{\tan^{-1} x}{(1+x^2)} dx$$

Answer

$$\text{Let } I = \int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$$

$$\text{Let } \tan^{-1} x = t$$

$$\Rightarrow \frac{1}{1+x^2} dx = dt.$$

Also, when $x=0$, $t=0$

and when $x=1$, $t = \frac{\pi}{4}$

Hence,

$$I = \int_0^{\frac{\pi}{4}} t dt = \frac{1}{2} t^2 \Big|_0^{\frac{\pi}{4}} = \frac{\pi^2}{32}$$

5. Question

Evaluate the following integrals

$$\int_0^1 \frac{e^x}{1+e^{2x}} dx$$

Answer

$$\text{Let } I = \int_0^1 \frac{e^x}{1+e^{2x}} dx$$

$$\text{Let } e^x = t$$

$$\Rightarrow e^x dx = dt.$$

Also,

when $x=0$, $t=1$

and

when $x=1$, $t=e$.

Hence,

$$\begin{aligned} I &= \int_1^e \frac{1}{1+t^2} dt = \tan^{-1} t \Big|_1^e \\ &= \tan^{-1} e - \frac{\pi}{4} \end{aligned}$$

6. Question

Evaluate the following integrals

$$\int_0^1 \frac{2x}{(1+x^4)} dx$$

Answer

$$\text{Let } I = \int_0^1 \frac{2x}{1+x^4} dx$$

$$\text{Let } x^2 = t$$

$$\Rightarrow 2x dx = dt.$$



Also,

when $x=0$, $t=0$

and

when $x=1$, $t=1$.

Hence,

$$\begin{aligned} I &= \int_0^1 \frac{1}{1+t^2} dt \\ &= \tan^{-1} t \Big|_0^1 \\ &= \frac{\pi}{4} \end{aligned}$$

7. Question

Evaluate the following integrals

$$\int_0^1 x e^{x^2} dx$$

Answer

$$\text{Let } I = \int_0^1 x e^{x^2} dx$$

$$\text{Let } x^2 = t$$

$$\Rightarrow 2x dx = dt.$$

Also,

when $x=0$, $t=0$

and

when $x=1$, $t=1$.

Hence,

$$\begin{aligned} I &= \frac{1}{2} \int_0^1 e^t dt \\ &= \frac{1}{2} e^t \Big|_0^1 \\ &= \frac{1}{2} (e - 1) \end{aligned}$$

8. Question

Evaluate the following integrals

$$\int_1^2 \frac{e^{1/x}}{x^2} dx$$

Answer

$$\text{Let } I = \int_1^2 \frac{e^{\frac{1}{x}}}{x^2} dx$$

$$\text{Let } \frac{1}{x} = t$$



$$\Rightarrow \frac{-1}{x^2} dx = dt.$$

Also,

when $x=1$, $t=1$

and

when $x=2$, $t = \frac{1}{2}$.

Hence,

$$I = - \int_1^{\frac{1}{2}} e^t dt$$

$$= -e^t \Big|_1^{\frac{1}{2}}$$

$$= e - \sqrt{e}$$

9. Question

Evaluate the following integrals

$$\int_0^{\pi/6} \frac{\cos x}{(3+4\sin x)} dx$$

Answer

$$\text{Let } I = \int_0^{\pi/6} \frac{\cos x}{3+4\sin x} dx$$

Let $3+4\sin x=t$

$$\Rightarrow 4\cos x dx = dt.$$

Also,

when $x=0$, $t=3$

and

when $x = \frac{\pi}{6}$, $t=5$.

Hence,

$$I = \frac{1}{4} \int_3^5 \frac{1}{t} dt$$

$$= \frac{1}{4} \log_e t \Big|_3^5$$

$$= \frac{1}{4} (\log_e 5 - \log_e 3)$$

10. Question

Evaluate the following integrals

$$\int_0^{\pi/2} \frac{\sin x}{(1+\cos^2 x)} dx$$



Answer

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx$$

Let $\cos x = t$

$$\Rightarrow -\sin x dx = dt.$$

Also,

when $x=0$, $t=1$

and

when $x = \frac{\pi}{2}$, $t=0$.

Hence,

$$\begin{aligned} I &= -\int_1^0 \frac{1}{1+t^2} dt \\ &= -\tan^{-1}t \Big|_1^0 \\ &= \frac{\pi}{4} \end{aligned}$$

11. Question

Evaluate the following integrals

$$\int_0^1 \frac{dx}{(e^x + e^{-x})}$$

**Answer**

$$\text{Let } I = \int_0^1 \frac{1}{e^x + e^{-x}} dx = \int_0^1 \frac{e^x}{1+e^{2x}} dx$$

Let $e^x = t$

$$\Rightarrow e^x dx = dt.$$

Also,

when $x=0$, $t=1$

and

when $x=1$, $t=e$.

Hence,

$$\begin{aligned} I &= \int_1^e \frac{1}{1+t^2} dt \\ &= \tan^{-1}t \Big|_1^e \\ &= \tan^{-1}e - \frac{\pi}{4} \end{aligned}$$

12. Question

Evaluate the following integrals

$$\int_{1/e}^e \frac{dx}{x(\log x)^{1/3}}$$

Answer

$$\text{Let } I = \int_{1/e}^e \frac{1}{x(\log_e x)^{1/3}} dx$$

$$\text{Let } \log_e x = t$$

$$\Rightarrow \frac{1}{x} dx = dt.$$

Also,

$$\text{when } x = \frac{1}{e}, t = -1$$

and

$$\text{when } x = e, t = 1.$$

Hence,

$$I = \int_{-1}^1 \frac{1}{t^{1/3}} dt$$

$$= \frac{3}{2} t^{2/3} \Big|_{-1}^1$$

$$= \frac{3}{2} (1 - 1)$$

$$= 0$$



13. Question

Evaluate the following integrals

$$\int_0^1 \frac{\sqrt{\tan^{-1} x}}{(1+x^2)} dx$$

Answer

$$\text{Let } I = \int_0^1 \frac{\sqrt{\tan^{-1} x}}{1+x^2} dx$$

$$\text{Let } \tan^{-1} x = t$$

$$\Rightarrow \frac{1}{1+x^2} dx = dt.$$

Also,

$$\text{when } x = 0, t = 0$$

and

$$\text{when } x = 1, t = \frac{\pi}{4}$$

Hence,

$$I = \int_0^{\pi/4} \sqrt{t} dt$$

$$= \frac{2}{3} t^{\frac{3}{2}} \Big|_0^{\frac{\pi}{4}}$$

$$= \frac{\pi^{\frac{3}{2}}}{12}$$

14. Question

Evaluate the following integrals

$$\int_0^{\pi/2} \frac{\sin x}{\sqrt{1+\cos x}} dx$$

Answer

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sqrt{1+\cos x}} dx$$

$$\text{Let } 1+\cos x=t$$

$$\Rightarrow -\sin x dx=dt.$$

$$\text{Also, when } x=0, t=2$$

and

$$\text{when } x = \frac{\pi}{2}, t=1$$

Hence,

$$I = -\int_2^1 \frac{1}{\sqrt{t}} dt$$

$$= -2\sqrt{t} \Big|_2^1$$

$$= 2(\sqrt{2}-1)$$



15. Question

Evaluate the following integrals

$$\int_0^{\pi/2} \sqrt{\sin x} \cdot \cos^5 x dx$$

Answer

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \sqrt{\sin x} \cos^5 x dx$$

$$\text{Let } \sin x=t$$

$$\Rightarrow \cos x dx=dt.$$

Also,

$$\text{when } x=0, t=0$$

and

$$\text{when } x = \frac{\pi}{2}, t=1.$$

$$\text{Consider } \cos^5 x = \cos^4 x \times \cos x = (1-\sin^2 x)^2 \times \cos x \text{ (Using } \sin^2 x + \cos^2 x = 1)$$

Hence,

$$\begin{aligned}
 I &= \int_0^1 \sqrt{x} (1-x^2)^2 dx \\
 &= \int_0^1 \sqrt{x} dx + \int_0^1 x^{\frac{9}{2}} dx - 2 \int_0^1 x^{\frac{5}{2}} dx \\
 \Rightarrow I &= \frac{2}{3} t^{\frac{3}{2}} \Big|_0^1 + \frac{2}{11} t^{\frac{11}{2}} \Big|_0^1 - \frac{4}{7} t^{\frac{7}{2}} \Big|_0^1 \\
 &= \frac{2}{3} + \frac{2}{11} - \frac{4}{7} \\
 &= \frac{64}{231}
 \end{aligned}$$

16. Question

Evaluate the following integrals

$$\int_0^{\pi/2} \frac{\sin x \cos x}{(1 + \sin^4 x)} dx$$

Answer

$$\text{Let } I = \int_0^{\pi/2} \frac{\sin x \cos x}{1 + \sin^4 x} dx$$

$$\text{Let } \sin^2 x = t$$

$$\Rightarrow 2 \sin x \cos x = dt.$$

Also,

$$\text{when } x=0, t=0$$

and

$$\text{when } x = \frac{\pi}{2}, t=1.$$

Hence,

$$\begin{aligned}
 I &= \frac{1}{2} \int_0^{\pi/2} \frac{1}{1+t^2} dt \\
 &= \frac{1}{2} \tan^{-1} t \Big|_0^1 \\
 &= \frac{\pi}{8}
 \end{aligned}$$

17. Question

Evaluate the following integrals

$$\int_0^a \sqrt{a^2 - x^2} dx$$

Answer

$$\text{Let } I = \int_0^a \sqrt{a^2 - x^2} dx$$

$$\text{Let } x = a \sin t$$

$$\Rightarrow a \cos t dt = dx.$$

Also,



when $x=0$, $t=0$

and

when $x=a$, $t = \frac{\pi}{2}$.

Hence,

$$I = \int_0^{\frac{\pi}{2}} \sqrt{a^2 - a^2 \sin^2 t} \cos t \, dt = a^2 \int_0^{\frac{\pi}{2}} \cos^2 t \, dt$$

Using $\cos^2 t = \frac{1+\cos 2t}{2}$, we get

$$I = \frac{a^2}{2} \int_0^{\frac{\pi}{2}} (1 + \cos 2t) \, dt$$

$$= \frac{a^2}{2} \left(t + \frac{\sin 2t}{2} \right) \Big|_0^{\frac{\pi}{2}}$$

$$= \frac{\pi a^2}{4}$$

18. Question

Evaluate the following integrals

$$\int_0^{\sqrt{2}} \sqrt{2-x^2} \, dx$$

Answer

$$\text{Let } I = \int_0^{\sqrt{2}} \sqrt{2-x^2} \, dx$$

$$\text{Consider, } I = \int_0^a \sqrt{a^2-x^2} \, dx$$

Let $x=a \sin t$

$$\Rightarrow a \cos t \, dt = dx.$$

Also, when $x=0$, $t=0$

and when $x=a$, $t = \frac{\pi}{2}$.

Hence,

$$I = \int_0^{\frac{\pi}{2}} \sqrt{a^2 - a^2 \sin^2 t} \cos t \, dt = a^2 \int_0^{\frac{\pi}{2}} \cos^2 t \, dt$$

Using $\cos^2 t = \frac{1+\cos 2t}{2}$, we get

$$I = \frac{a^2}{2} \int_0^{\frac{\pi}{2}} (1 + \cos 2t) \, dt$$

$$= \frac{a^2}{2} \left(t + \frac{\sin 2t}{2} \right) \Big|_0^{\frac{\pi}{2}}$$

$$= \frac{\pi a^2}{4}$$

Here $a = \sqrt{2}$, hence $I = \frac{\pi}{2}$

19. Question



Evaluate the following integrals

$$\int_0^a \frac{x^4}{\sqrt{a^2 - x^2}} dx$$

Answer

$$\text{Let } I = \int_0^a \frac{x^4}{\sqrt{a^2 - x^2}} dx$$

$$\text{Let } x = a \sin t$$

$$\Rightarrow a \cos t dt = dx.$$

Also, when $x=0$, $t=0$

and when $x=a$, $t = \frac{\pi}{2}$.

Hence,

$$I = \int_0^{\frac{\pi}{2}} \frac{a^4 \sin^4 t}{\sqrt{a^2 - a^2 \sin^2 t}} a \cos t dt$$

$$= a^4 \int_0^{\frac{\pi}{2}} \sin^4 t dt$$

Using $\sin^2 t = \frac{1 - \cos 2t}{2}$, we get

$$I = a^4 \int_0^{\frac{\pi}{2}} \left(\frac{1 - \cos 2t}{2} \right)^2 dt$$

$$= \frac{a^4}{4} \int_0^{\frac{\pi}{2}} (1 + \cos^2 2t - 2\cos 2t) dt$$

$$\Rightarrow I = \frac{a^4}{4} \left(t \Big|_0^{\frac{\pi}{2}} - \sin 2t \Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \left(\frac{1 + \cos 4t}{2} \right) dt \right)$$

$$\left(\text{Using } \cos^2 t = \frac{1 + \cos 2t}{2} \right)$$

Hence,

$$I = \frac{\pi a^4}{8} + \frac{a^4}{4} \times \frac{t}{2} \Big|_0^{\frac{\pi}{2}} + \frac{a^4}{32} \sin 4t \Big|_0^{\frac{\pi}{2}}$$

$$= \frac{3\pi a^4}{16}$$

20. Question

Evaluate the following integrals

$$\int_0^a \frac{x}{\sqrt{a^2 + x^2}} dx$$

Answer

$$\text{Let } I = \int_0^a \frac{x}{\sqrt{a^2 + x^2}} dx$$

$$\text{Let } a^2 + x^2 = t^2$$

$$\Rightarrow x dx = t dt.$$



Also, when $x=0$, $t=a$

and when $x=a$, $t = \sqrt{2}a$.

Hence,

$$I = \int_a^{\sqrt{2}a} \frac{t}{\sqrt{t^2}} dt$$

$$= t \Big|_a^{\sqrt{2}a}$$

$$= a(\sqrt{2}-1)$$

21. Question

Evaluate the following integrals

$$\int_0^2 x\sqrt{2-x} dx$$

Answer

$$\text{Let } I = \int_0^2 x\sqrt{2-x} dx$$

Using the property that $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$, we get

$$I = \int_0^2 (2-x)\sqrt{x} dx$$

$$= \int_0^2 2\sqrt{x} dx - \int_0^2 x^{\frac{3}{2}} dx$$

$$= 2 \times \frac{2}{3} x^{\frac{3}{2}} \Big|_0^2 - \frac{2}{5} x^{\frac{5}{2}} \Big|_0^2$$

Hence,

$$I = 2\sqrt{2} \left(\frac{4}{3} - \frac{4}{5} \right)$$

$$= \frac{16}{15} \sqrt{2}$$

22. Question

Evaluate the following integrals

$$\int_0^1 \sin^{-1} \left(\frac{2x}{1+x^2} \right) dx$$

Answer

$$\text{Let } I = \int_0^1 \sin^{-1} \left(\frac{2x}{1+x^2} \right) dx$$

$$\text{Let } f(x) = \sin^{-1} \left(\frac{2x}{1+x^2} \right)$$

Let $x = \tan \theta$

$$\Rightarrow \theta = \tan^{-1} x$$

$$\Rightarrow f(x) = \sin^{-1} \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right)$$



$$= \sin^{-1} \left(\frac{2 \tan \theta}{\sec^2 \theta} \right)$$

$$= \sin^{-1} (2 \sin \theta \cos \theta)$$

$$= \sin^{-1} (\sin 2\theta)$$

Hence $f(x) = 2\theta$

$$= 2 \tan^{-1} x$$

$$\text{Hence } I = 2 \int_0^1 1 \times \tan^{-1} x dx$$

Using integration by parts, we get

$$I = 2x \tan^{-1} x \Big|_0^1 - \int_0^1 \frac{2x}{1+x^2} dx$$

$$= \frac{\pi}{2} - \int_0^1 \frac{2x}{1+x^2} dx \quad (1)$$

$$\text{Let } I' = \int_0^1 \frac{2x}{1+x^2} dx$$

$$\text{Let } 1+x^2 = t$$

$$\Rightarrow 2x dx = dt.$$

Also, when $x=0$, $t=1$

and when $x=1$, $t=2$

Hence,

$$I' = \int_1^2 \frac{1}{t} dt = \log_e |t| \Big|_1^2$$

$$= \log_e 2 - \log_e 1$$

$$= \log_e 2 \quad (2)$$

Substituting value of (2) in (1), we get

$$I = \frac{\pi}{2} - \log_e 2$$

23. Question

Evaluate the following integrals

$$\int_0^{\pi/2} \sqrt{1 + \cos x} dx$$

Answer

$$\text{Let } I = \int_0^{\pi/2} \sqrt{1 + \cos x} dx$$

Using $1 + \cos x = 2 \cos^2 \frac{x}{2}$, we get

$$I = \sqrt{2} \int_0^{\pi/2} \cos \left(\frac{x}{2} \right) dx$$

$$= 2\sqrt{2} \sin \left(\frac{x}{2} \right) \Big|_0^{\pi/2}$$

$$= 2$$



24. Question

Evaluate the following integrals

$$\int_0^{\pi/2} \sqrt{1 + \sin x} \, dx$$

Answer

$$\text{Let } I = \int_0^{\pi/2} \sqrt{1 + \sin x} \, dx$$

Using $\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} = 1$ and $\sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2}$

$$I = \int_0^{\pi/2} \sqrt{\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)^2} \, dx$$

$$= \int_0^{\pi/2} \left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right) \, dx$$

$$= -2 \cos\left(\frac{x}{2}\right) \Big|_0^{\pi/2} + 2 \sin\left(\frac{x}{2}\right) \Big|_0^{\pi/2}$$

$$= -(\sqrt{2}-2) + (\sqrt{2})$$

$$= 2$$

25. Question

Evaluate the following integrals

$$25. \int_0^{\pi/2} \frac{dx}{(a^2 \cos^2 x + b^2 \sin^2 x)}$$



Answer

$$\text{Let } I = \int_0^{\pi/2} \frac{1}{a^2 \cos^2 x + b^2 \sin^2 x} \, dx$$

Dividing by $\cos^2 x$ in numerator and denominator, we get

$$I = \int_0^{\pi/2} \frac{\sec^2 x}{a^2 + b^2 \tan^2 x} \, dx$$

Let $\tan x = t$

$$\Rightarrow \sec^2 x \, dx = dt$$

$$I = \int_0^{\pi/2} \frac{1}{a^2 + b^2 t^2} \, dt = \frac{1}{b^2} \int_0^{\pi/2} \frac{1}{\frac{a^2}{b^2} + t^2} \, dt$$

Let $t = \frac{a}{b} \tan \theta = \tan \theta$

$$I = \frac{1}{b^2} \int_0^{\pi/2} \frac{\frac{a}{b} \sec^2 \theta}{\frac{a^2}{b^2} + \frac{a^2}{b^2} \tan^2 \theta} \, d\theta$$

$$= \frac{1}{ab} \theta$$