## EXERCISE 24.3

**Find the equation of the circle, the end points of whose diameter are (2, -3) and (- 2, 4). Find its centre and radius.**

**Solution:**

Given:

The diameters (2, -3) and (-2, 4).

By using the formula,

Centre = (-a, -b)

= [-(2-2)/2, -(-3+4)/2]

= (0, -½)

By using the distance formula,

$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

So, r = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

= $\sqrt{[(2-0)^2 + (-3-½)^2]}$

= $\sqrt{[(2)^2 + (-7/2)^2]}$

= √[4 + 49/4]

= √[65/4]

= [√65]/2



We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by: $(x - p)^2 + (y - q)^2 = r^2$

Now by substituting the values in the above equation, we get

$(x - 0)^2 + (y – ½)^2 = [[\sqrt{65}]/2]^2$

$x^2 + y^2 – y + ¼ = 65/4$

$4x^2 + 4y^2 - 4y + 1 = 65$

∴The equation of the circle is $4x^2 + 4y^2 - 4y - 64 = 0$ or $x^2 + y^2 - y - 16 = 0$

**1. Find the equation of the circle the end points of whose diameter are the centres of the circles $x^2 + y^2 + 6x – 14y – 1 = 0$ and $x^2 + y^2 – 4x + 10y – 2 = 0$.**

**Solution:**

Given:

$x^2 + y^2 + 6x – 14y – 1 = 0$…. (1)

So the centre = [(-6/2), -(-14/2)]

= [-3, 7]

$x^2 + y^2 - 4x + 10y - 2 = 0$… (2)
So the centre = [-(-4/2), (-10/2)]
= [2, -5]

We know that the equation of the circle is given by,
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
$(x + 3)(x - 2) + (y - 7)(y + 5) = 0$
Upon simplification we get
$x^2 + 3x - 2x - 6 + y^2 - 7y + 5y - 35 = 0$
$x^2 + y^2 + x - 2y - 41 = 0$
∴The equation of the circle is $x^2 + y^2 + x - 2y - 41 = 0$

**2. The sides of a squares are x = 6, x = 9, y = 3 and y = 6. Find the equation of a circle drawn on the diagonal of the square as its diameter.**
**Solution:**
Given:
The sides of a squares are x = 6, x = 9, y = 3 and y = 6.
Let us assume A, B, C, D be the vertices of the square. On solving the lines, we get the coordinates as: A = (6, 3)
B = (9, 3)
C = (9, 6)
D = (6, 6)



We know that the equation of the circle with diagonal AC is given by
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
$(x - 6)(x - 9) + (y - 3)(y - 6) = 0$
Upon simplifying, we get
$x^2 - 6x - 9x + 54 + y^2 - 3y - 6y + 18 = 0$
$x^2 + y^2 - 15x - 9y + 72 = 0$

We know that the equation of the circle with diagonal BD as diameter is given by
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
$(x - 9)(x - 6) + (y - 3)(y - 6) = 0$
Upon simplifying, we get
$x^2 - 9x - 6x + 54 + y^2 - 3y - 6y + 18 = 0$
$x^2 + y^2 - 15x - 9y + 72 = 0$
∴The equation of the circle is $x^2 + y^2 - 15x - 9y + 72 = 0$

**3. Find the equation of the circle circumscribing the rectangle whose sides are x – 3y**

**= 4, 3x + y = 22, x – 3y = 14 and 3x + y = 62.**
**Solution:**
Given:
The sides x – 3y = 4 …. (1)
3x + y = 22 … (2)
x – 3y = 14 …. (3)
3x + y = 62 … (4)
Let us assume A, B, C, D be the vertices of the square. On solving the lines, we get the coordinates as: A = (7, 1)
B = (8, - 2)
C = (20, 2)
D = (19, 5)
We know that the equation of the circle with diagonal AC as diameter is given by
$(x – x_1) (x – x_2) + (y – y_1) (y – y_2) = 0$
(x - 7) (x - 20) + (y - 1) (y - 2) = 0
Upon simplification we get
$x^2 + y^2 – 27x – 3y + 142 = 0$
∴The equation of the circle is $x^2 + y^2 – 27x – 3y + 142 = 0$

**4. Find the equation of the circle passing through the origin and the points where the line 3x + 4y = 12 meets the axes of coordinates.**
**Solution:**
Given:
The line 3x + 4y = 12
The value of x is 0 on meeting the y - axis. So,
3(0) + 4y = 12
4y = 12
y = 3
The point is A(0, 3)

The value of y is 0 on meeting the x - axis. So,
3x + 4(0) = 12
3x = 12
x = 4
The point is B(4, 0)

Since the circle passes through origin and A and B
So, AB is the diameter
We know that the equation of the circle with AB as diameter is given by

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
$(x - 0)(x - 4) + (y - 3)(y - 0) = 0$
$x^2 + y^2 - 4x - 3y = 0$
∴The equation of the circle is $x^2 + y^2 - 4x - 3y = 0$

**5. Find the equation of the circle which passes through the origin and cuts off intercepts a and b respectively from x and y - axes.**
**Solution:**
Since the circle has intercept 'a' from x – axis, the circle must pass through (a, 0) and (-a, 0) as it already passes through the origin.

Since the circle has intercept 'b' from x - axis, the circle must pass through (0, b) and (0, -b) as it already passes through the origin.
Let us assume the circle passing through the points A(a,0) and B(0,b).
We know that the equation of the circle with AB as diameter is given by
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
$(x - a)(x - 0) + (y - 0)(y - b) = 0$
$x^2 + y^2 + ax + by = 0$ or $x^2 + y^2 - ax - by = 0$
∴The equation of the circle is $x^2 + y^2 + ax + by = 0$ or $x^2 + y^2 - ax - by = 0$