EXERCISE 2.3

Classify the following polynomials as polynomials in one variable, two variables etc.

(i) $x^2 + x + 1$
(ii) $y^3 – 5y$
(iii) $xy + yz + zx$
(iv) $x^2 – 2xy + y^2 + 1$

Solution:

(i) $x^2 + x + 1$
Here, the polynomial contains only one variable, i.e., x.
Hence, the given polynomial is a polynomial in **one** variable.

(ii) $y^3 – 5y$
Here, the polynomial contains only one variable, i.e., y.
Hence, the given polynomial is a polynomial in **one** variable.

(iii) $xy + yz + zx$
Here, the polynomial contains three variables, i.e., x, y and z.
Hence, the given polynomial is a polynomial in **three** variable.

(iv) $x^2 – 2xy + y^2 + 1$
Here, the polynomial contains two variables, i.e., x and y.
Hence, the given polynomial is a polynomial in **two** variable.

2. Determine the degree of each of the following polynomials:

(i) $2x – 1$
(ii) $–10$
(iii) $x^3 – 9x + 3x^5$
(iv) $y^3 (1 – y^4)$

Solution:

Degree of a polynomial in one variable = highest power of the variable in algebraic expression

(i) $2x – 1$
Power of x = 1
Highest power of the variable x in the given expression = 1
Hence, degree of the polynomial $2x – 1 = 1$

(ii) $–10$
There is no variable in the given term.
Let us assume that the variable in the given expression is x.
$– 10 = –10x^0$
Power of x = 0
Highest power of the variable x in the given expression = 0
Hence, degree of the polynomial $– 10 = 0$

(iii) $x^3 – 9x + 3x^5$

Powers of x = 3, 1 and 5 respectively.
Highest power of the variable x in the given expression = 5
Hence, degree of the polynomial $x^3 – 9x + 3x^5 = 5$

(iv) $y^3 (1 – y^4)$
The equation can be written as,
$y^3 (1 – y^4) = y^3 – y^7$
Powers of y = 3 and 7 respectively.
Highest power of the variable y in the given expression = 7
Hence, degree of the polynomial $y^3 (1 – y^4) = 7$

3. For the polynomial

$$\frac{x^3+2x+1}{5}-\frac{7}{2}x^2-x^6$$ **, write**

(i) the degree of the polynomial
(ii) the coefficient of x^3
(iii) the coefficient of x^6
(iv) the constant term

Solution:

The given polynomial is

$$\frac{(x^3 + 2x + 1)}{5}-\frac{7}{2}x^2-x^6$$

(i) Powers of x = 3, 1, 2 and 6 respectively.
Highest power of the variable x in the given expression = 6
Hence, degree of the polynomial = 6

(ii) The given equation can be written as,

$$\frac{(x^3 + 2x + 1)}{5}-\frac{7}{2}x^2-x^6=\frac{1}{5}x^3+\frac{2}{5}x+\frac{1}{5}-\frac{7}{2}x^2-x^6$$

Hence, the coefficient of x^3 in the given polynomial is 1/5.

(iii) The coefficient of x^6 in the given polynomial is – 1

(iv) Since the given equation can be written as,

$$\frac{(x^3 + 2x + 1)}{5}-\frac{7}{2}x^2-x^6=\frac{1}{5}x^3+\frac{2}{5}x+\frac{1}{5}-\frac{7}{2}x^2-x^6$$

The constant term in the given polynomial is 1/5 as it has no variable x associated with it.

4. Write the coefficient of x^2 in each of the following:

(i) $(\pi/6)x + x^2 – 1$
(ii) $3x – 5$
(iii) $(x –1) (3x – 4)$
(iv) $(2x – 5) (2x^2 – 3x + 1)$

Solution:

(i) $(\pi/6)\ x + x^2 - 1$

$(\pi/6)\ x + x^2 - 1 = (\pi/6)\ x + (1)\ x^2 - 1$

The coefficient of x^2 in the polynomial $(\pi/6)\ x + x^2 - 1 = 1$.

(ii) $3x - 5$

$3x - 5 = 0x^2 + 3x - 5$

The coefficient of x^2 in the polynomial $3x - 5 = 0$, zero.

(iii) $(x - 1)(3x - 4)$

$(x - 1)(3x - 4) = 3x^2 - 4x - 3x + 4$

$= 3x^2 - 7x + 4$

The coefficient of x^2 in the polynomial $3x^2 - 7x + 4 = 3$.

(iv) $(2x - 5)(2x^2 - 3x + 1)$

$(2x - 5)(2x^2 - 3x + 1)$

$= 4x^3 - 6x^2 + 2x - 10x^2 + 15x - 5$

$= 4x^3 - 16x^2 + 17x - 5$

The coefficient of x^2 in the polynomial $(2x - 5)(2x^2 - 3x + 1) = -16$

5. Classify the following as a constant, linear, quadratic and cubic polynomials:

(i) $2 - x^2 + x^3$

(ii) $3x^3$

(iii) $5t - \sqrt{7}$

(iv) $4 - 5y^2$

(v) 3

(vi) $2 + x$

(vii) $y^3 - y$

(viii) $1 + x + x^2$

(ix) t^2

(x) $\sqrt{2}x - 1$

Solution:

Constant polynomials: The polynomial of the degree zero.

Linear polynomials: The polynomial of degree one.

Quadratic polynomials: The polynomial of degree two.

Cubic polynomials: The polynomial of degree three.

(i) $2 - x^2 + x^3$

Powers of x = 2, and 3 respectively.

Highest power of the variable x in the given expression = 3

Hence, degree of the polynomial = 3

Since it is a polynomial of the degree 3, it is a cubic polynomial.

(ii) $3x^3$

Power of x = 3.

Highest power of the variable x in the given expression = 3

Hence, degree of the polynomial = 3

Since it is a polynomial of the degree 3, it is a cubic polynomial.

(iii) $5t - \sqrt{7}$

Power of t = 1.
Highest power of the variable t in the given expression = 1
Hence, degree of the polynomial = 1
Since it is a polynomial of the degree 1, it is a linear polynomial.

(iv) $4 - 5y^2$

Power of y = 2.
Highest power of the variable y in the given expression = 2
Hence, degree of the polynomial = 2
Since it is a polynomial of the degree 2, it is a quadratic polynomial.

(v) 3

There is no variable in the given expression.
Let us assume that x is the variable in the given expression.
3 can be written as $3x^0$.
i.e., $3 = x^0$
Power of x = 0.
Highest power of the variable x in the given expression = 0
Hence, degree of the polynomial = 0
Since it is a polynomial of the degree 0, it is a constant polynomial.

(vi) $2 + x$

Power of x = 1.
Highest power of the variable x in the given expression = 1
Hence, degree of the polynomial = 1
Since it is a polynomial of the degree 1, it is a linear polynomial.

(vii) $y^3 - y$

Powers of y = 3 and 1, respectively.
Highest power of the variable x in the given expression = 3
Hence, degree of the polynomial = 3
Since it is a polynomial of the degree 3, it is a cubic polynomial.

(viii) $1 + x + x^2$

Powers of x = 1 and 2, respectively.
Highest power of the variable x in the given expression = 2
Hence, degree of the polynomial = 2
Since it is a polynomial of the degree 2, it is a quadratic polynomial.

(ix) t^2

Power of t = 2.
Highest power of the variable t in the given expression = 2
Hence, degree of the polynomial = 2

Since it is a polynomial of the degree 2, it is a quadratic polynomial.

(x) $\sqrt{2}x - 1$

Power of x = 1.

Highest power of the variable x in the given expression = 1

Hence, degree of the polynomial = 1

Since it is a polynomial of the degree 1, it is a linear polynomial.

6. Give an example of a polynomial, which is:

(i) monomial of degree 1

(ii) binomial of degree 20

(iii) trinomial of degree 2

Solution:

(i) Monomial = an algebraic expression that contains one term

An example of a polynomial, which is a monomial of degree 1 = 2t

(ii) Binomial = an algebraic expression that contains two terms

An example of a polynomial, which is a binomial of degree 20 = $x^{20} + 5$

(iii) Trinomial = an algebraic expression that contains three terms

An example of a polynomial, which is a trinomial of degree 2 = $y^2 + 3y + 11$

7. Find the value of the polynomial $3x^3 - 4x^2 + 7x - 5$, when x = 3 and also when x = –3

Solution:

Given that,

$p(x) = 3x^3 - 4x^2 + 7x - 5$

According to the question,

<u>When x = 3,</u>

p(x) = p(3)

$p(x) = 3x^3 - 4x^2 + 7x - 5$

Substituting x = 3,

$p(3) = 3(3)^3 - 4(3)^2 + 7(3) - 5$

$p(3) = 3(3)^3 - 4(3)^2 + 7(3) - 5$

$= 3(27) - 4(9) + 21 - 5$

$= 81 - 36 + 21 - 5$

$= 102 - 41$

$= 61$

<u>When x = – 3,</u>

p(x) = p(– 3)

$p(x) = 3x^3 - 4x^2 + 7x - 5$

Substituting x = – 3,

$p(-3) = 3(-3)^3 - 4(-3)^2 + 7(-3) - 5$

$p(-3) = 3(-3)^3 - 4(-3)^2 + 7(-3) - 5$

$= 3(-27) - 4(9) - 21 - 5$

$= -81 - 36 - 21 - 5$

$= -143$

8. If $p(x) = x^2 - 4x + 3$, evaluate: $p(2) - p(-1) + p(½)$.

Given that,

$p(x) = x^2 - 4x + 3$

According to the question,

<u>When x = 2,</u>

p(x) = p(2)

$p(x) = x^2 - 4x + 3$

Substituting x = 2,

$p(2) = (2)^2 - 4(2) + 3$
$= 4 - 8 + 3$
$= -4 + 3$
$= -1$

<u>When x = – 1,</u>

p(x) = p(– 1)

$p(x) = x^2 - 4x + 3$

Substituting x = – 1,

$p(-1) = (-1)^2 - 4(-1) + 3$
$= 1 + 4 + 3$
$= 8$

<u>When x = ½ ,</u>

p(x) = p(½)

$p(x) = x^2 - 4x + 3$

Substituting x = ½,

$p(½) = (½)^2 - 4(½) + 3$
$= ¼ - 2 + 3$
$= ¼ + 1$
$= 5/4$

Now,

$p(2) - p(-1) + p(½) = -1 - 8 + (5/4)$
$= -9 + (5/4)$
$= (-36 + 5)/4$
$= -31/4$

9. Find p(0), p(1), p(−2) for the following polynomials:

(i) $(x) = 10x - 4x^2 - 3$

(ii) $(y) = (y + 2)(y - 2)$

Solution:

(i) According to the question,

$p(x) = 10x - 4x^2 - 3$

<u>When x = 0,</u>

p(x) = p(0)

Substituting x = 0,

$p(0) = 10(0) - 4(0)^2 - 3$
$= 0 - 0 - 3$

$= -3$

<u>When x = 1,</u>
p(x) = p(1)
Substituting x = 1,
$p(1) = 10(1) - 4(1)^2 - 3$
$= 10 - 4 - 3$
$= 6 - 3$
$= 3$

<u>When x = – 2,</u>
p(x) = p(– 2)
Substituting x = – 2,
$p(-2) = 10(-2) - 4(-2)^2 - 3$
$= -20 - 16 - 3$
$= -36 - 3$
$= -39$

(ii) According to the question,
$p(y) = (y + 2)(y - 2)$

<u>When y = 0,</u>
p(y) = p(0)
Substituting y = 0,
$p(0) = (0 + 2)(0 - 2)$
$= (2)(-2)$
$= -4$

<u>When y = 1,</u>
p(y) = p(1)
Substituting y = 1,
$p(1) = (1 + 2)(1 - 2)$
$= (3)(-1)$
$= -3$

<u>When y = – 2,</u>
p(y) = p(– 2)
Substituting y = – 2,
$p(-2) = (-2 + 2)(-2 - 2)$
$= (0)(-4)$
$= 0$

10. Verify whether the following are true or false:

(i) –3 is a zero of x – 3
(ii) – 1/3 is a zero of 3x + 1
(iii) – 4/5 is a zero of 4 –5y

(iv) 0 and 2 are the zeroes of $t^2 – 2t$
(v) –3 is a zero of $y^2 + y – 6$

Solution:

(i) –3 is a zero of x – 3
False
Zero of x – 3 is given by,
$x – 3 = 0$
$\Rightarrow x=3$

(ii) – 1/3 is a zero of 3x + 1
True
Zero of 3x + 1 is given by,
$3x + 1 = 0$
$\Rightarrow 3x = – 1$
$\Rightarrow x = – 1/3$

(iii) – 4/5 is a zero of 4 –5y
False
Zero of 4 – 5y is given by,
$4 – 5y =0$
$\Rightarrow – 5y = – 4$
$\Rightarrow y = 4/5$

(iv) 0 and 2 are the zeroes of $t^2 – 2t$
True
Zeros of $t^2 – 2t$ is given by,
$t^2 – 2t = t(t – 2) = 0$
$\Rightarrow t = 0$ or 2

(v) –3 is a zero of $y^2 + y – 6$
True
Zero of $y^2 + y – 6$ is given by,
$y^2 + y – 6 = 0$
$\Rightarrow y^2 + 3x – 2x – 6 = 0$
$\Rightarrow y (y + 3) – 2(x + 3) = 0$
$\Rightarrow (y – 2) (y + 3) =0$
$\Rightarrow y = 2$ or $– 3$

11. Find the zeroes of the polynomial in each of the following:

(i) p(x) = x – 4
(ii) g(x) = 3 – 6x
(iii) q(x) = 2x –7
(iv) h(y) = 2y

Solution:

(i) $p(x) = x – 4$
Zero of the polynomial p(x) $\Rightarrow p(x) = 0$

P(x) = 0
⇒ x – 4= 0
⇒ x = 4
Therefore, the zero of the polynomial is 4.

(ii) g(x) = 3 – 6x
Zero of the polynomial g(x) ⇒ g(x) = 0
g(x) = 0
⇒3 – 6x = 0
⇒ x = 3/6 = ½
Therefore, the zero of the polynomial is ½

(iii) q(x) = 2x –7
Zero of the polynomial q(x) ⇒ q(x) = 0
q(x) = 0
⇒2x – 7 = 0
⇒ x = 7/2
Therefore, the zero of the polynomial is 7/2

(iv) h(y) = 2y
Zero of the polynomial h(y) ⇒ h(y) = 0
h(y) = 0
⇒2y =0
⇒ y = 0
Therefore, the zero of the polynomial is 0

12. Find the zeroes of the polynomial:
$p(x)= (x-2)^2-(x+2)^2$
Solution:
$p(x) = (x-2)^2-(x+2)^2$
We know that,
Zero of the polynomial p(x) = 0
Hence, we get,
⇒ $(x-2)^2-(x+2)^2 = 0$
Expanding using the identity, $a^2 - b^2 = (a - b)(a + b)$
⇒ (x – 2 + x + 2) (x – 2 –x – 2) = 0
⇒ 2x (– 4) = 0
⇒ – 8 x= 0
Therefore, the zero of the polynomial = 0

13. By actual division, find the quotient and the remainder when the first polynomial is divided by the second polynomial: $x^4 + 1$; x –1
Solution:
Performing the long division method, we get,

```
x-1) x⁴ + 1 (x³ + x² + x + 1
     x⁴ -x³
    -   +
    ---------
     x³ + 1
     x³ - x²
    -   +
    ---------
       x² + 1
       x² -x
      -   +
         --------
          X + 1
          x - 1
         -  +
         --------
               2
```

Hence, from the above long division method, we get,
Quotient = $x^3 + x^2 + x + 1$
Remainder = 2.

14. By Remainder Theorem find the remainder, when p(x) is divided by g(x), where
(i) $p(x) = x^3 – 2x^2 – 4x – 1, g(x) = x + 1$
(ii) $p(x) = x^3 – 3x^2 + 4x + 50, g(x) = x – 3$
(iii) $p(x) = 4x^3 – 12x^2 + 14x – 3, g(x) = 2x – 1$
(iv) $p(x) = x^3 – 6x^2 + 2x – 4, g(x) = 1 – 3/2\, x$
Solution:

(i) Given $p(x) = x^3 – 2x^2 – 4x – 1$ and $g(x) = x + 1$
Here zero of g(x) = – 1
By using the remainder theorem
P(x) divided by g(x) = p(– 1)
$P(–1) = (–1)^3 – 2(–1)^2 – 4(–1) – 1 = 0$
Therefore, the remainder = 0

(ii) given $p(x) = x^3 – 3x^2 + 4x + 50$, $g(x) = x – 3$
Here zero of g(x) = 3
By using the remainder theorem p(x) divided by g(x) = p(3)
$p(3) = 3^3 – 3 \times (3)^2 + 4 \times 3 + 50 = 62$
Therefore, the remainder = 62

(iii) $p(x) = 4x^3 – 12x^2 + 14x – 3$, $g(x) = 2x – 1$
Here zero of g(x) = ½
By using the remainder theorem p(x) divided by g(x) = p (½)
$P(½) = 4(½)^3 – 12(½)^2 + 14(½) – 3$
$= 4/8 – 12/4 + 14/2 – 3$
$= ½ + 1$
$= 3/2$
Therefore, the remainder = 3/2

(iv) $p(x) = x^3 – 6x^2 + 2x – 4$, $g(x) = 1 – 3/2\ x$
Here zero of g(x) = 2/3
By using the remainder theorem p(x) divided by g(x) = p(2/3)
$p(2/3) = (2/3)^3 – 6(2/3)^2 + 2(2/3) – 4$
$= – 136/27$
Therefore, the remainder = – 136/27

15. Check whether p(x) is a multiple of g(x) or not:

(i) $p(x) = x^3 – 5x^2 + 4x – 3$, $g(x) = x – 2$

(ii) $p(x)= 2x^3 – 11x^2 – 4x + 5$, $g(x)= 2x+1$

Solution:

(i)
According to the question,
g(x)=x – 2,
Then, zero of g(x),
g(x) = 0
x – 2 = 0
x = 2
Therefore, zero of g(x) = 2
So, substituting the value of x in p(x), we get,
$p(2) =(2)^3 – 5(2)^2 + 4(2) – 3$
$= 8 – 20 + 8 – 3$
$= – 7 \neq 0$
Hence, p(x) is not the multiple of g(x) since the remainder ≠ 0.

(ii)
According to the question,
g(x)= 2x + 1
Then, zero of g(x),
g(x) = 0
2x + 1 = 0
2x = – 1
x = – ½
Therefore, zero of g(x) = – ½
So, substituting the value of x in p(x), we get,
$p(–½) = 2 \times (– ½)^3 – 11 \times (– ½)^2 – 4 \times (– 1/2) + 5$
$= – ¼ - 11/4 + 7$
$= 16/4$
$= 4 \neq 0$
Hence, p(x) is not the multiple of g(x) since the remainder ≠ 0.

16. Show that:

(i) $x + 3$ is a factor of $69 + 11x − x^2 + x^3$.

(ii) $2x − 3$ is a factor of $x + 2x^3 – 9x^2 + 12$

Solution:

(i) According to the question,

Let $p(x) = 69 + 11x - x^2 + x^3$ and $g(x) = x + 3$
$g(x) = x + 3$
zero of $g(x) \Rightarrow g(x) = 0$
$x + 3 = 0$
$x = -3$
Therefore, zero of $g(x) = -3$
So, substituting the value of x in p(x), we get,
$p(-3) = 69 + 11(-3) - (-3)^2 + (-3)^3$
$= 69 - 69$
$= 0$
Since, the remainder = zero,
We can say that,
$g(x) = x + 3$ is factor of $p(x) = 69 + 11x - x^2 + x^3$

(ii) According to the question,
Let $p(x) = x + 2x^3 - 9x^2 + 12$ and $g(x) = 2x - 3$
$g(x) = 2x - 3$
zero of $g(x) \Rightarrow g(x) = 0$
$2x - 3 = 0$
$x = 3/2$
Therefore, zero of $g(x) = 3/2$
So, substituting the value of x in p(x), we get,
$P(3/2) = 3/2 + 2(3/2)^3 - 9(3/2)^2 + 12$
$= (81 - 81) / 4$
$= 0$
Since, the remainder = zero,
We can say that,
$g(x) = 2x - 3$ is factor of $p(x) = x + 2x^3 - 9x^2 + 12$

17. Determine which of the following polynomials has x – 2 a factor:
(i) $3x^2 + 6x - 24$.
(ii) $4x^2 + x - 2$.

Solution:

(i) According to the question,
Let $p(x) = 3x^2 + 6x - 24$ and $g(x) = x - 2$
$g(x) = x - 2$
zero of $g(x) \Rightarrow g(x) = 0$
$x - 2 = 0$
$x = 2$
Therefore, zero of $g(x) = 2$
So, substituting the value of x in p(x), we get,
$p(2) = 3(2)^2 + 6(2) - 24$
$= 12 + 12 - 24$
$= 0$
Since, the remainder = zero,
We can say that,

g(x) = x – 2 is factor of p(x) = $3x^2 + 6x - 24$

(ii) According to the question,
Let p(x) = $4x^2 + x - 2$ and g(x) = x – 2
g(x) = x – 2
zero of g(x) ⇒ g(x) = 0
x – 2 = 0
x = 2
Therefore, zero of g(x) = 2
So, substituting the value of x in p(x), we get,
p(2) $= 4(2)^2 + 2 - 2$
$= 16 \neq 0$
Since, the remainder ≠ zero,
We can say that,
g(x) = x – 2 is not a factor of p(x) = $4x^2 + x - 2$

18. Show that p – 1 is a factor of $p^{10} - 1$ and also of $p^{11} - 1$.
Solution:
According to the question,
Let h(p) = $p^{10} - 1$,and g(p) = $p - 1$
zero of g(p) ⇒ g(p) = 0
p – 1 = 0
p = 1
Therefore, zero of g(x) = 1
We know that,
According to factor theorem if g(p) is a factor of h(p) , then h(1) should be zero
So,
$h(1) = (1)^{10} - 1 = 1 - 1 = 0$
⟹ g (p) is a factor of h(p).
Now, we have h(p) = $p^{11} - 1$, g (p) = $p - 1$
Putting g (p) = 0 ⟹ $p - 1 = 0$ ⟹ $p = 1$
According to factor theorem if g (p) is a factor of h(p) ,
Then h(1) = 0
⟹ $(1)^{11} - 1 = 0$
Therefore, g(p) = $p - 1$ is the factor of h(p) = $p^{10} - 1$

19. For what value of m is $x^3 - 2mx^2 + 16$ divisible by x + 2
Solution:
According to the question,
Let p(x) = $x^3 - 2mx^2 + 16$, and g(x) = x + 2
g(x) = 0
⟹ x + 2 = 0
⟹ x = – 2
Therefore, zero of g(x) = – 2
We know that,
According to factor theorem,

if p(x) is divisible by g(x), then the remainder p(−2) should be zero.
So, substituting the value of x in p(x), we get,
$p(-2) = 0$
$\Rightarrow (-2)^3 - 2m(-2)^2 + 16 = 0$
$\Rightarrow 0 - 8 - 8m + 16 = 0$
$\Rightarrow 8m = 8$
$\Rightarrow m = 1$

20. If $x + 2a$ is a factor of $x^5 - 4a^2x^3 + 2x + 2a + 3$, find a.
Solution:

According to the question,
Let $p(x) = x^5 - 4a^2x^3 + 2x + 2a + 3$ and $g(x) = x + 2a$
$g(x) = 0$
$\Rightarrow x + 2a = 0$
$\Rightarrow x = -2a$
Therefore, zero of $g(x) = -2a$
We know that,
According to the factor theorem,
If g(x) is a factor of p(x), then $p(-2a) = 0$
So, substituting the value of x in p(x), we get,
$p(-2a) = (-2a)^5 - 4a^2(-2a)^3 + 2(-2a) + 2a + 3 = 0$
$\Rightarrow -32a^5 + 32a^5 - 2a + 3 = 0$
$\Rightarrow -2a = -3$
$\Rightarrow a = 3/2$