

EXERCISE 3.3

1. Find the identity element in the set I^+ of all positive integers defined by $a * b = a + b$ for all $a, b \in I^+$.

Solution:

Let e be the identity element in I^+ with respect to $*$ such that

$$a * e = a = e * a, \forall a \in I^+$$

$$a * e = a \text{ and } e * a = a, \forall a \in I^+$$

$$a + e = a \text{ and } e + a = a, \forall a \in I^+$$

$$e = 0, \forall a \in I^+$$

Thus, 0 is the identity element in I^+ with respect to $*$.

2. Find the identity element in the set of all rational numbers except -1 with respect to $*$ defined by $a * b = a + b + ab$

Solution:

Let e be the identity element in I^+ with respect to $*$ such that

$$a * e = a = e * a, \forall a \in Q - \{-1\}$$

$$a * e = a \text{ and } e * a = a, \forall a \in Q - \{-1\}$$

$$a + e + ae = a \text{ and } e + a + ea = a, \forall a \in Q - \{-1\}$$

$$e + ae = 0 \text{ and } e + ea = 0, \forall a \in Q - \{-1\}$$

$$e(1 + a) = 0 \text{ and } e(1 + a) = 0, \forall a \in Q - \{-1\}$$

$$e = 0, \forall a \in Q - \{-1\} \text{ [because } a \text{ not equal to } -1]$$

Thus, 0 is the identity element in $Q - \{-1\}$ with respect to $*$.