

### EXERCISE 19.5

$$1. \int \frac{x+1}{\sqrt{2x+3}} dx$$

**Solution:**

Given

$$\int \frac{x+1}{\sqrt{2x+3}} dx$$

In this type of questions, little manipulation makes the questions easier to solve

Here we have multiply and divide by 2 to given equation

$$\Rightarrow \frac{1}{2} \int \frac{2x+2}{\sqrt{2x+3}} dx$$

Add and subtract 1 from the numerator

$$\Rightarrow \frac{1}{2} \int \frac{2x+2+1-1}{\sqrt{2x+3}} dx$$

$$\Rightarrow \frac{1}{2} \int \frac{2x+3-1}{\sqrt{2x+3}} dx$$

Splitting the above equation we get

$$\Rightarrow \frac{1}{2} \int \frac{2x+3}{\sqrt{2x+3}} dx - \frac{1}{2} \int \frac{1}{\sqrt{2x+3}} dx$$

Taking  $\frac{1}{2}$  common from the above equation

$$\Rightarrow \frac{1}{2} \left( \int \sqrt{2x+3} dx - \int (2x+3)^{-\frac{1}{2}} dx \right)$$

Now by integrating the above equation we get

$$\Rightarrow \frac{1}{2} \times \frac{(2x+3)^{\frac{3}{2}}}{2 \times \frac{3}{2}} - \frac{1}{2} \times \frac{(2x+3)^{\frac{1}{2}}}{2 \times \frac{1}{2}} + c$$

$$\Rightarrow \frac{(2x+3)^{\frac{3}{2}}}{6} - \frac{(2x+3)^{\frac{1}{2}}}{2} + C$$

2.  $\int x\sqrt{x+2} dx$

**Solution:**

Given

$$\int x\sqrt{x+2} dx$$

In this type of questions, little manipulation makes the questions easier to solve

Here add and subtract 2 from x in the given equation

We get

$$\Rightarrow \int (x+2-2)\sqrt{x+2} dx$$

$$\Rightarrow \int (x+2)^{\frac{3}{2}} dx - \int 2\sqrt{x+2} dx$$

On integrating we get

$$\Rightarrow \frac{2(x+2)^{\frac{5}{2}}}{5} - \frac{4(x+2)^{\frac{3}{2}}}{3} + C$$

3.  $\int \frac{x-1}{\sqrt{x+4}} dx$

**Solution:**

$$4. \int (x + 2) \sqrt{3x + 5} \, dx$$

Given

$$\int \frac{x - 1}{\sqrt{x + 4}} \, dx$$

In this type of questions, little manipulation makes the questions easier to solve

Add and subtract 5 from the numerator

$$\Rightarrow \int \frac{x + 5 - 5 - 1}{\sqrt{x + 4}} \, dx$$

$$\Rightarrow \int \frac{x + 4 - 5}{\sqrt{x + 4}} \, dx$$

By splitting the above equation

$$\Rightarrow \int \frac{x + 4}{\sqrt{x + 4}} \, dx - \int \frac{5}{\sqrt{x + 4}} \, dx$$

$$\Rightarrow \left( \int \sqrt{x + 4} \, dx - 5 \int (x + 4)^{-\frac{1}{2}} \, dx \right)$$

Now by integrating, we get

$$\Rightarrow \frac{(x + 4)^{\frac{3}{2}}}{\frac{3}{2}} - 5 \times \frac{(x + 4)^{\frac{1}{2}}}{\frac{1}{2}} + c$$

By computing

$$\Rightarrow \frac{2(x + 4)^{\frac{3}{2}}}{3} - 10(x + 4)^{\frac{1}{2}} + c$$

**Solution:**

Let

$$I = \int (x + 2) \sqrt{3x + 5} dx$$

Substitute  $3x + 5 = t$

$$\Rightarrow x = \frac{t - 5}{3}$$

$$\Rightarrow 3dx = dt$$

$$\Rightarrow dx = \frac{dt}{3}$$

$$\begin{aligned} \therefore I &= \int \left( \frac{t - 5}{3} + 2 \right) \sqrt{t} \frac{dt}{3} \\ &= \frac{1}{3} \int \left( \frac{t - 5 + 6}{3} \right) \sqrt{t} dt \end{aligned}$$

By taking 3 as common and multiplying, we get

$$= \frac{1}{9} \int (t^{\frac{3}{2}} + t^{\frac{1}{2}}) dt$$

On integrating we get

$$= \frac{1}{9} \left[ \frac{t^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right] + C$$

On simplifying

$$= \frac{1}{9} \left[ \frac{2}{5} t^{\frac{5}{2}} + \frac{2}{3} t^{\frac{3}{2}} \right] + C$$

By substituting the value of t

$$= \frac{1}{9} \left[ \frac{2}{5} (3x + 5)^{\frac{5}{2}} + \frac{2}{3} (3x + 5)^{\frac{3}{2}} \right] + C$$

$$= \frac{2}{9} \left[ (3x + 5)^{\frac{3}{2}} \left\{ \frac{3x + 5}{5} + \frac{1}{3} \right\} \right] + C$$

$$= \frac{2}{9} \left[ (3x + 5)^{\frac{3}{2}} \left\{ \frac{9x + 15 + 5}{15} \right\} \right] + C$$

$$= \frac{2}{9} \left[ (3x + 5)^{\frac{3}{2}} \left\{ \frac{9x + 20}{15} \right\} \right] + C$$

$$= \frac{2}{135} (3x + 5)^{\frac{3}{2}} (9x + 20) + C$$

5.  $\int \frac{2x + 1}{\sqrt{3x + 2}} dx$

**Solution:**

Given

$$\int \left( \frac{2x + 1}{\sqrt{3x + 2}} \right) dx$$

Multiply and divide by 3 in the above equation we get

$$= \frac{1}{3} \int \left( \frac{6x + 3}{\sqrt{3x + 2}} \right) dx$$



The above equation can be written as

$$= \frac{1}{3} \int \left( \frac{6x + 4 - 1}{\sqrt{3x + 2}} \right) dx$$

Taking 2 as common and subtracting

$$= \frac{1}{3} \int \left( \frac{2(3x + 2)}{\sqrt{3x + 2}} - \frac{1}{\sqrt{3x + 2}} \right) dx$$

On simplifying

$$= \frac{1}{3} \int \left( 2\sqrt{3x + 2} - \frac{1}{\sqrt{3x + 2}} \right) dx$$

By splitting the integral

$$= \frac{1}{3} \left[ \int 2(3x + 2)^{\frac{1}{2}} dx - \int (3x + 2)^{-\frac{1}{2}} dx \right]$$

On integrating we get

$$= \frac{1}{3} \left[ 2 \left\{ \frac{(3x + 2)^{\frac{1}{2} + 1}}{3 \left( \frac{1}{2} + 1 \right)} \right\} - \frac{(3x + 2)^{-\frac{1}{2} + 1}}{\left( -\frac{1}{2} + 1 \right) \times 3} \right] + C$$

$$= \frac{1}{3} \left[ \frac{4}{9} (3x + 2)^{\frac{3}{2}} - \frac{2}{3} (3x + 2)^{\frac{1}{2}} \right] + C$$

On simplifying we get

$$= \frac{4}{27} (3x + 2)^{\frac{3}{2}} - \frac{2}{9} (3x + 2)^{\frac{1}{2}} + C$$

$$= \sqrt{3x + 2} \left( \frac{4}{27} (3x + 2) - \frac{2}{9} \right) + C$$

$$= \sqrt{3x + 2} \left( \frac{4(3x + 2) - 6}{27} \right) + C$$

$$= \sqrt{3x + 2} \left( \frac{12x + 8 - 6}{27} \right) + C$$

$$= \frac{2}{27} (6x + 1) \sqrt{3x + 2} + C$$