Permutations

Exercise 8A

Q. 1. Compute:

(i) $\frac{9!}{(5!)\times(3!)}$!

(ii) $\frac{32!}{29!}$

(iii) $\frac{(12!)-(10!)}{9!}$

Answer : (i) To Find : Value of $\frac{9!}{(5!)\times(3!)}$

Formulae :

- $n! = n \times (n-1)!$
- $n! = n \times (n-1) \times (n-2) \dots\dots\dots 3 \times 2 \times 1$

Let,

$$x = \frac{9!}{(5!) \times (3!)}$$

By using above formula, we can write,

$$\therefore x = \frac{9 \times 8 \times 7 \times 6 \times (5!)}{(5!) \times (3 \times 2 \times 1)}$$

Cancelling (5!) from numerator and denominator we get,

$$\therefore x = \frac{9 \times 8 \times 7 \times 6}{3 \times 2 \times 1}$$

$\therefore x = 504$

Conclusion : Hence, value of the expression $\frac{9!}{(5!) \times (3!)}$ is 504.

(ii) To Find : Value of $\frac{32!}{29!}$

Formula : $n! = n \times (n-1)!$

Let,

$$x = \frac{32!}{29!}$$

By using the above formula we can write,

$$\therefore x = \frac{32 \times 31 \times 30 \times (29!)}{29!}$$

Cancelling (29!) from numerator and denominator,

∴x = 32 × 31 × 30

∴x = 29760

Conclusion : Hence, the value of the expression $\frac{32!}{29!}$ is 29760.

(iii) To Find : Value of $\frac{(12!)-(10!)}{9!}$

Formula : $n! = n \times (n-1)!$

Let,

$$x = \frac{(12!) - (10!)}{9!}$$

By using the above formula we can write,

$$\therefore x = \frac{[12 \times 11 \times 10 \times (9!)] - [10 \times (9!)]}{9!}$$

Taking (9!) common from numerator,

$$\therefore x = \frac{(9!)[(12 \times 11 \times 10) - 10]}{9!}$$

Cancelling (9!) from numerator and denominator,

∴x = (12 × 11 × 10) - 10

∴x = 1310

Conclusion : Hence, the value of the expression $\frac{(12!) - (10!)}{9!}$ is 1310.

Q. 2. Prove that LCM {6!, 7!, 8!} = 8!

Answer : To Prove : LCM {6!, 7!, 8!} = 8!

Formula : $n! = n \times (n-1)!$

LCM is the smallest possible number that is a multiple of two or more numbers.

Here, we observe that (8!) is the first number which is a multiple of all three given numbers i.e. 6!, 7! and 8!.

$1 \times (8!) = 8!$

$8 \times (7!) = 8!$

$8 \times 7 \times (6!) = 8!$

Therefore, 8! is the LCM of {6!, 7!, 8!}

Conclusion : Hence proved

Q. 3. Prove that $\frac{1}{10!} + \frac{1}{11!} + \frac{1}{12!} = \frac{145!}{12!}$.

Answer : To Prove :

$$\frac{1}{10!} + \frac{1}{11!} + \frac{1}{12!} = \frac{145}{12!}$$

Formula : n! = n × (n - 1)!

$L.H.S. = \frac{1}{10!} + \frac{1}{11!} + \frac{1}{12!}$

$= \frac{12 \times 11}{12 \times 11 \times (10!)} + \frac{12}{12 \times (11!)} + \frac{1}{12!}$

$= \frac{132}{12!} + \frac{12}{12!} + \frac{1}{12!}$

$= \frac{145}{12!}$

= R.H.S.

∴L.H.S. = R.H.S.

Conclusion : $\therefore \frac{1}{10!} + \frac{1}{11!} + \frac{1}{12!} = \frac{145}{12!}$

Q. 4. If $\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$", find the value of x.

Answer : Given Equation :

$$\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$$

To Find : Value of x.

Formula : $n! = n \times (n-1)!$

By given equation,

$$\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$$

$$\therefore \frac{8 \times 7}{8 \times 7 \times 6!} + \frac{8}{8 \times 7!} = \frac{x}{8!}$$

By using the above formula we can write,

$$\therefore \frac{56}{8!} + \frac{8}{8!} = \frac{x}{8!}$$

$\therefore \frac{64}{8!} = \frac{x}{8!}$

Cancelling (8!) from both the sides,

$\therefore x = 64$

Conclusion : Value of x is 64.

Q. 5. Write the following products in factorial notation:

(i) 6 × 7 × 8 × 9 × 10 × 11 × 12
(ii) 3 × 6 × 9 × 12 × 15

Answer : (i) Formula : $n! = n \times (n-1) \times (n-2) \ldots \ldots \ldots \ldots 3 \times 2 \times 1$

Let,

$x = 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6$

Multiplying and dividing by $(5 \times 4 \times 3 \times 2 \times 1)$

$\therefore x = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1}$

From the above formula,

$x = \frac{12!}{5!}$

Conclusion :

$\therefore (12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6) = \frac{12!}{5!}$

(ii) Formula : $n! = n \times (n-1) \times (n-2) \ldots \ldots \ldots \ldots 3 \times 2 \times 1$

Let,

$x = 3 \times 6 \times 9 \times 12 \times 15$

Above equation can be written as

$$x = 3(1) \times 3(2) \times 3(3) \times 3(4) \times 3(5)$$

$$\therefore x = 3^5 \times (5 \times 4 \times 3 \times 2 \times 1)$$

By using above formula,

$$\therefore x = 3^5 \times (5!)$$

Conclusion :

$$\therefore (3 \times 6 \times 9 \times 12 \times 15) = 3^5 \times (5!)$$

Q. 6. Which of the following are true of false?

(i) (2 + 3) ! = 2 ! + 3!
(ii) (2 × 3)! = (2!) × (3!)

Answer : Option (i) and (ii) both are false

Proofs :

For option (i),

L.H.S. = (2 + 3)! = (5!) = 120

R.H.S. = (2!) + (3!) = 2 + 6 = 8

∴ L.H.S. ≠R.H.S.

For option (ii),

L.H.S. = (2 × 3)! = (6!) = 720

R.H.S = (2!) × (3!) = 4 × 6 = 24

∴L.H.S. ≠R.H.S.

Important Notes : for any two whole numbers a and b,

- $(a + b)! \neq (a!) + (b!)$
- $(a \times b)! \neq (a!) \times (b!)$

Q. 7. If (n + 1) ! = 12 × (n – 1) !, find the value of n.

Answer : Given Equation :

(n + 1)! = 12 × (n - 1)!

To Find : Value of n

Formula : $n! = n \times (n-1)!$

By given equation,

(n + 1)! = 12 × (n - 1)!

By using above formula we can write,

∴(n + 1) × (n) × (n - 1)! = 12 × (n - 1)!

Cancelling the term (n - 1)! from both the sides,

∴(n + 1) × (n) = 12 .... eq(1)

∴(n + 1) × (n) = (4) × (3)

Comparing both the sides, we get,

∴n = 3

Conclusion : Value of n is 3.

Note : Instead of taking product of two brackets in eq(1), it is easy to convert the constant term that is 12 into product of two consecutive numbers and then by observing two sides of equation we can get value of n.

Q. 8. If (n + 2) ! = 2550 × n!, find the value of n.

Answer : Given Equation :

$$(n + 2)! = 2550 \times n!$$

To Find : Value of n

Formula : $n! = n \times (n-1)!$

By given equation,

(n + 2)! = 2550 × n!

By using above formula we can write,

∴ (n + 2) × (n + 1) × (n!) = 2550 × n!

Cancelling the term (n)! from both the sides,

∴ (n + 2) × (n + 1) = 2550

∴ (n + 2) × (n + 1) = (51) × (50)

Comparing both the sides, we get,

∴n = 49

Conclusion : Value of n is 49.

Note : Instead of taking product of two brackets in eq(1), it is easy to convert the constant term that is 2550 into product of two consecutive numbers and then by observing two sides of equation we can get value of n.

Q. 9. If (n + 3) ! = 56 × (n + 1) !, find the value of n.

Answer : Given Equation :

$$(n + 3)! = 56 \times (n + 1)!$$

To Find : Value of n

Formula : $n! = n \times (n-1)!$

By given equation,

(n + 3)! = 56 × (n + 1)!

By using above formula we can write,

∴ (n + 3) × (n + 2) × (n + 1)! = 56 × (n + 1)!

Cancelling the term (n + 1)! from both the sides,

∴ (n + 3) × (n + 2) = 56

∴ (n + 3) × (n + 2) = (8) × (7)

Comparing both the sides, we get,

∴ n = 5

Conclusion : Value of n is 5.

Note : Instead of taking product of two brackets in eq(1), it is easy to convert the constant term that is 56 into product of two consecutive numbers and then by observing two sides of equation we can get value of n.

Q. 10. If $\frac{n!}{(2!)\times(n-2)!}:\frac{n!}{(4!)\times(n-4)!}$ **= 2 : 1, find the value of n.**

Answer : Given Equation :

$$\frac{n!}{(2!)\times(n-2)!}:\frac{n!}{(4!)\times(n-4)!} = 2:1$$

To Find : Value of n

Formula : $n! = n \times (n-1)!$

By given equation,

$$\frac{n!}{(2!)\times(n-2)!}:\frac{n!}{(4!)\times(n-4)!} = 2:1$$

$$\therefore \frac{\frac{n!}{(2!)\times(n-2)!}}{\frac{n!}{(4!)\times(n-4)!}} = \frac{2}{1}$$

$$\therefore \frac{n!}{(2!)\times(n-2)!} \times \frac{(4!)\times(n-4)!}{n!} = 2$$

By using above formula,

$$\therefore \frac{(4\times 3\times 2!)\times(n-4)!}{(2!)\times[(n-2)\times(n-3)\times(n-4)!]} = 2$$

Cancelling terms (n - 4)! And (2!),

$$\therefore \frac{(4 \times 3)}{[(n-2) \times (n-3)]} = 2$$

∴ (n - 2) × (n - 3) = 6

∴ (n - 2) × (n - 3) = (3) × (2)

By comparing both the sides,

$$\therefore n = 5$$

Conclusion : Value of n is 5.

Note : Instead of taking product of two brackets in eq(1), it is easy to convert the constant term that is 6 into product of two consecutive numbers and then by observing two sides of equation we can get value of n.

Q. 11. If $\frac{(2n)!}{(3!)\times(2n-3)!} : \frac{n!}{(2!)\times(n-2)!}$ **= 44 : 3, find the value of n.**

Answer : Given Equation :

$$\frac{(2n)!}{(3!) \times (2n-3)!} : \frac{n!}{(2!) \times (n-2)!} = 44:3$$

To Find : Value of n

Formula : $n! = n \times (n-1)!$

By given equation,

$$\frac{(2n)!}{(3!) \times (2n-3)!} : \frac{n!}{(2!) \times (n-2)!} = 44:3$$

$$\therefore \frac{\frac{(2n)!}{(3!) \times (2n-3)!}}{\frac{n!}{(2!) \times (n-2)!}} = \frac{44}{3}$$

$$\therefore \frac{(2n)!}{(3!) \times (2n-3)!} \times \frac{(2!) \times (n-2)!}{n!} = \frac{44}{3}$$

By using above formula,

$$\therefore \frac{(2n) \times (2n-1) \times (2n-2) \times (2n-3)!}{(3 \times 2!) \times (2n-3)!} \times \frac{(2!) \times (n-2)!}{n \times (n-1) \times (n-2)!}$$
$$= \frac{44}{3}$$

Cancelling terms (n - 2)! , (2!), (2n - 3)! & n, we get,

$$\therefore \frac{2 \times (2n-1) \times 2(n-1)}{3} \times \frac{1}{(n-1)} = \frac{44}{3}$$

..... taking 2 common from the term (2n - 2)

$$\therefore (2n-1) = \frac{44 \times 3}{3 \times 2 \times 2}$$

∴ (2n - 1) = 11

∴n = 6

Conclusion : Value of n is 6.

Q. 12. Evaluate $\frac{n!}{(r!) \times (n-r)!}$, when n = 15 and r = 12.

Answer : Given : n = 15 and r = 12

To Find : Value of $\frac{n!}{(r!) \times (n-r)!}$ at given n and r

Formula :

- $n! = n \times (n-1)!$
- $n! = n \times (n-1) \times (n-2) \ldots \ldots \ldots 3 \times 2 \times 1$

Let ,

$$x = \frac{n!}{(r!) \times (n-r)!}$$

Substituting n = 15 and r = 12 in above equation,

$$\therefore x = \frac{(15!)}{(12!) \times (15 - 12)!}$$

$$\therefore x = \frac{(15!)}{(12!) \times (3)!}$$

By using above formula,

$$\therefore x = \frac{15 \times 14 \times 13 \times 12!}{(12!) \times (3 \times 2 \times 1)}$$

Cancelling (12!) from numerator & denominator,

$$\therefore x = \frac{15 \times 14 \times 13}{3 \times 2 \times 1}$$

$\therefore$x = 455

Conclusion : Value of $\frac{n!}{(r!) \times (n-r)!}$ at n = 15 and r = 12 is 6.

Q. 13. Prove that (n + 2) × (n!) + (n + 1) ! = (n!). (2n + 3)

Answer : To Prove : $(n + 2) \times (n!) + (n + 1)! = (n!) \times (2n + 3)$

Formula : $n! = n \times (n - 1)!$

L.H.S. = (n + 2) × (n!) + (n + 1)!

= (n + 2) × (n!) + (n + 1) × (n!)

= (n!) × [(n + 2) + (n + 1)]

= (n!) × (2n + 3)

= R.H.S.

$\therefore$L.H.S. = R.H.S.

Conclusion : (n + 2) × (n!) + (n + 1)! = (n!) × (2n + 3)

Q. 14. Prove that

(i) $\frac{n!}{r!}$ **= n(n – 1) (n – 2) …. (r + 1)**

(ii) (n – r + 1). $\frac{n!}{(n-r+1)!}=\frac{n!}{(n-r)!}$

(iii) $\frac{n!}{r!(n-r)!}+\frac{n!}{(r-1)!(n-r+1)!}=\frac{(n+1)!}{r!(n-r+1)!}$

Answer :

(i) To Prove : $\frac{n!}{r!} = n(n-1)(n-2)\ldots.(r + 1)$

Formula : $n! = n \times (n-1)!$

$L.H.S. = \frac{n!}{r!}$

Writing (n!) in terms of (r!) by using above formula,

$= \frac{n(n-1)(n-2)\ldots\ldots(r+1)(r!)}{r!}$

Cancelling (r!),

= n(n - 1)(n - 2)…. (r + 1)

= R.H.S.

∴ LHS = RHS

Note : In permutation and combination r is always less than n, so we can write n! in terms of r! by using given formula.

(ii) To Prove : $(n-r+1).\frac{n!}{(n-r+1)!} = \frac{n!}{(n-r)!}$

Formula : $n! = n \times (n-1)!$

$$L.H.S. = (n-r+1)\frac{n!}{(n-r+1)!}$$

By using above formula,

$$= (n-r+1)\frac{n!}{(n-r+1)(n-r)!}$$

Cancelling (n - r + 1),

$$= \frac{n!}{(n-r)!}$$

= R.H.S.

∴LHS = RHS

(iii) To Prove : $\frac{n!}{(r!)\times(n-r)!} + \frac{n!}{(r-1)!\times(n-r+1)!} = \frac{(n+1)!}{(r!)\times(n-r+1)!}$

Formula : $n! = n \times (n-1)!$

$$L.H.S. = \frac{n!}{(r!)\times(n-r)!} + \frac{n!}{(r-1)!\times(n-r+1)!}$$

By using above formula,

$$= \frac{(n-r+1)n!}{(r!)\times(n-r+1)(n-r)!} + \frac{(r)\times n!}{(r)(r-1)!\times(n-r+1)!}$$

$$= \frac{(n-r+1)n!}{(r!)\times(n-r+1)!} + \frac{(r)\times n!}{(r)!\times(n-r+1)!}$$

Taking $\left(\frac{(n!)}{(r!)\times(n-r+1)!}\right)$ common,

$$= \frac{n!}{(r!)\times(n-r+1)!}(n-r+1+r)$$

$$= \frac{(n+1)\times n!}{(r!)\times(n-r+1)!}$$

$$= \frac{(n+1)!}{(r!) \times (n-r+1)!}$$

= R.H.S.

∴ LHS = RHS

Exercise 8B

Q. 1. There are 10 buses running between Delhi and Agra. In how many ways can a man go from Delhi to Agra and return by a different bus?

Answer : Given: 10 buses running between Delhi and Agra.

To Find: Number of ways a man can go from Delhi to Agra and return by a different bus.

There are 10 buses running between Delhi and Agra so there are 10 different ways to go from Delhi to Agra. The man cannot return from the same bus he went so number of ways are reduced to 9.

These second event occur in completion of first event so there are: $10 \times 9 = 90$ ways in which a man can go from Delhi to Agra and return by a different bus.

Q. 2. A, B and C are three cities. There are 5 routes from A to B and 3 routes from B to C. How many different routes are there from A to C via B?

Answer : Given: 5 routes from A to B and 3 routes from B to C.

To find: number of different routes from A to C via B.

Let E_1 be the event : 5 routes from A to B

Let E_2 be the event : 3 routes from B to C

Since going from A to C via B is only possible if both the events E_1 and E_2 occur simultaneously.

So there are $5 \times 3 = 15$ different routes from A to C via B.

Q. 3. There are 12 steamers plying between A and B. In how many ways could the round trip from A be made if the return was made on (i) the same steamer? (ii) a different steamer?

Answer : Given: 12 steamers plying between A and B.

To find: number of ways the round trip from A can be made.

(i) The steamer which will go from A to B will be returning back, since the given condition is that same steamer should return.

There are 12 steamers available so there are 12 different ways to make around trip between A & B if done on same steamer.

(ii) If the return trip is done on different steamer than the once used in trip on going from A to B then the possible number of ways are:12 × 11=132.

(11 because the once used in going from A to B cannot be used in returning hence, reduced by 1.)

Q. 4. In How many ways can 4 people be seated in a row containing 5 seats?

Answer : To find : Number of ways in which 4 people can be seated in a row containing 5 seats.

The possible number of ways in which 4 people be seated in a row containing 5 seats $={}^7P_4$ (There are 5 places to be filled with 4 persons where arrangement doesn't matter.)

${}^7P_4 = \frac{7!}{(7-4)!}$ …($^nP_r = \frac{n!}{(n-r)!}$)

$= \frac{7!}{3!}$

=7 × 6 × 5 × 4

= 840

Q. 5. In How many ways can 5 ladies draw water from 5 taps, assuming the no tap remains unused?

Answer : To find: number of ways in which 5 ladies draw water from 5 taps.

Condition: no tap remains unused

The condition given is that no well should remain unused.

So possible number of ways are: 5 × 4 × 3 × 2 × 1=120.

Q. 6. In a textbook on mathematics there are three exercises A, B and C consisting of 12, 18 and 10 questions respectively. In how many ways can three questions be selected choosing one from each exercise?

Answer : Given: three exercises A, B and C consisting of 12, 18 and 10 questions respectively.

To find: number of ways in which three questions be selected choosing one from each exercise.

Ways of selecting one question from exercise A: $^{12}C_1$ (way of selecting one element from n number of elements.)

Ways of selecting one question from exercise B: $^{18}C_1$

Ways of selecting one question from exercise C: $^{10}C_1$

So number of ways of choosing one question from each exercise A ,B,C
$= {}^{12}C_1 \times {}^{18}C_1 \times {}^{10}C_1$

$= 12 \times 18 \times 10$

$= 2160$

Q. 7. In a school, there are four sections of 40 students each in XI standard. In how many ways can a set of 4 student representatives be chosen, one from each section?

Answer : Given: there are four sections of 40 students each in XI standard.

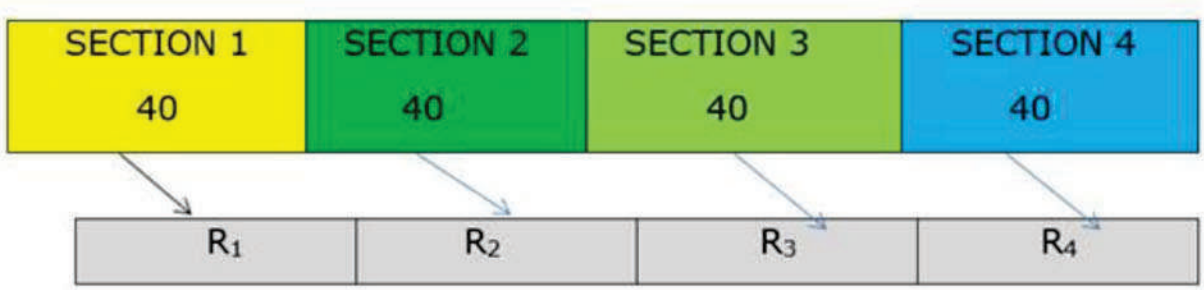
To find : number of ways in which a set of 4 student representatives be chosen, one from each section.

Ways of selecting one student from section 1: $^{40}C_1$

Ways of selecting one student from section 2: $^{40}C_1$

Ways of selecting one student from section 3: $^{40}C_1$

Ways of selecting one student from section 4: $^{40}C_1$



So number of ways of choosing a set of 4 student representatives one from each section=$^{40}C_1 \times {}^{40}C_1 \times {}^{40}C_1 \times {}^{40}C_1$

= 40 × 40 × 40 × 40

= 2560000

Q. 8. In how many ways can a vowel, a consonant and a digit be chosen out of the 26 letters of the English alphabet and the 10 digits?

Answer : To find: number of ways in which a vowel, a consonant and a digit be chosen out of the 26 letters of the English alphabet and the 10 digits.

e.g.

O	Z	9

Way of selecting a vowel from 5 vowels=$^{5}C_1$

Way of selecting a consonant from 26 consonants=$^{21}C_1$

Way of selecting a digit from 10 digits=$^{10}C_1$

So ways of choosing a vowel, a consonant, a digit=$^{5}C_1 \times {}^{21}C_1 \times {}^{10}C_1$

= 5 × 21 × 10

= 1050

Q. 9. How many 8-digit telephone numbers can be constructed using the digits 0 to 9 if each number starts with 270 and n digit appears more than once?

Answer : Given:8 digit telephone number starts with 270 .

To find: How many 8-digit telephone numbers can be constructed?

2	7	0					

There are 10 digits between 0 to 9,and three of them are utilized in filling up the first three digits i.e.270 of the 8 digit phone number, so remaining number of digits=10-

3=7,and this need to be used in filling up the remaining 8-3=5 places of the telephone number.

i.e. the remaining 5 places need to filled up with any one of:1,3,4,5,6,8,9

So, number of ways=$7 \times 6 \times 5 \times 4 \times 3=2520$.

Q. 10. (ac and the outcomes are recorded. How many possible outcomes are there?

(b) How many possible outcomes if the coin is tossed.
(i) four times? (ii) five times? (iii) n times?

Answer : (a) A coin is tossed three times

So possible number of outcomes=$2^3=8$

(HHH,HHT,HTH,HTT,THH,THT,TTH,TTT)

(b) i) A coin is tossed four times

So possible number of outcomes=$2^4=16$

(HHHH,HHHT,HHTH,HHTT,HTHH,HTHT,HTTH,HTTT,THHH,THHT,THTH,THTT,TTHH,TTHT,TTTH,TTTT)

(ii) A coin is tossed n times

So possible number of outcomes=2^n

Q. 11. Find the number of different signals that can be generated by arranging at least 2 flags in order (one below the other) on a vertical staff, if five different flags are available.

Answer : Given:5 Flags

Way of generating signal using 2 different flags =${}^5P_2$ (way of selecting 2 things out of 5 things with considering arrangement.)

Way of generating signal using 3 different flags =${}^5P_3$

Way of generating signal using 4 different flags $= {}^5P_4$

Way of generating signal using 5 different flags $= {}^5P_5$

So total number of ways $= {}^5P_2 + {}^5P_3 + {}^5P_4 + {}^5P_5$

$=20 + 60 + 120 + 120$

$=320$

Q. 12. How many 4-letter codes can be formed using the first 10 letters of the English alphabet, if no letter can be repeated?

Answer : Given: first 10 letters of the English alphabet.

In 4 letter code for first position there are 10 possibilities for second position there are 9 possibilities, for third position there are 8 possibilities and for fourth position there are 7 possibilities since repetition is not allowed.

So total numbers of combination$=10 \times 9 \times 8 \times 7 = 5040$

Q. 13. Given, A = {2, 3, 5} and B = {0, 1}. Find the number of different ordered pairs in which the first entry is an element of A and the second is an element of B.

Answer : This is the example of Cartesian product of two sets.

The pairs in which the first entry is an element of A and the second is an element of B are :

(2,0),(2,1),(3,0),(3,1),(5,0),(5,1)

$\Rightarrow 3 \times 2 = 6$

Q. 14. How many arithmetic progressions with 10 terms are there whose first term in the set {1, 2, 3} and whose common difference is in the set {2, 3, 4}?

Answer : Given: Two sets: {1, 2, 3} & {2, 3, 4}

To find: number of A.P. with 10n terms whose first term is in the set {1, 2, 3} and whose common difference is in the set {2, 3, 4}

Number of arithmetic progressions with 10 terms whose first term are in the set {1, 2, 3} and whose common difference is in the set {2, 3, 4} are: $3 \times 3=9$

(3 because there are three elements in the set {1, 2, 3} and another 3 because there are three elements in the set {2, 3, 4})

Q. 15. There are 6 items in column A and 6 items in column B. A student is asked to match each item in column A with an item in column B. How many possible (correct or incorrect) answers are there to this question?

Answer :

COLUMN A	COLUMNB
ITEM1	MATCH1
ITEM2	MATCH2
ITEM3	MATCH3
ITEM4	MATCH4
ITEM5	MATCH5
ITEM6	MATCH6

As we can see that For Item2 there can be any of the match

So, For each item in column A there are 6 different options in column B since we don't have to think about correct or incorrect matching.

So possible number of combinations possible to answer:

6 × 5 × 4 × 3 × 2 × 1=720

Q. 16. A mint prepares metallic calendars specifying months, dates and days in the form of monthly sheets (one plate for each month). How many types of February calendars should it prepare to serve for all the possibilities in the future years?

Answer : To find: types of February calendars that can be prepared.

There are two factors to develop FEBRUARY metallic calendars

(1) The day on the start of the year of which possibility=7

(2) Whether the year is leap year or not of which possibility is =2

So, number of FEBRUARY calendars possibilities to serve in future years=7 × 2=14

Q. 17. From among the 36 teachers in a school, one principal and one vice-principal are to be appointed. In how many ways can this be done?

Answer : Given: 36 teachers are there in a school.

To find: Number of ways in which one principal and one vice-principal can be appointed.

There are 36 options of appointing principal and 35 option of appointing vice-principal since same teacher cannot be appointed as principal and vice-principal.

Total number of ways=36 × 35 = 1260

Q. 18. A sample of 3 bulbs is tested. A bulb is labeled as G if it is good and D if it is defective. Find the number of all possible outcomes.

Answer : A bulb can be good or defective, so there are 2 different possibilities of a bulb.

So number of all possible outcomes (of all bulbs)=2 × 2 × 2=8

Q. 19. For a set of five true or false questions, no student has written the all correct answer and no two students have given the same sequence of answers. What is the maximum number of students in the class for this to be possible?

Answer : Given: a set of five true – false questions.

To find: the maximum number of students in the class.

Condition: no student has written the all correct answer and no two students have given the same sequence of answers.

The total number of answering a set of 5 true or false question=2^5=32

Since, no two students have given the same sequence of answers and no student has written the all correct answer.

Therefore total possibilities reduces by 1(of no student has written the all correct answer)

⇒2^5-1=32-1=31

Q. 20. In how many ways can the following prizes be given away to a class of 20 students : first and second in mathematics; first and second in chemistry; first in physics and first in English?

Answer : Given: 20 students.

The number of ways of giving first and second prizes in mathematics to a class of 20 students=20 × 19.

(First prize can be given to any one of the 20 students but the second prize cannot be given to the student that received the first prize so the number of candidates for the second prize is 19.)

The number of ways of giving first and second prizes in chemistry

To a class of 20 students=20 × 19.

The number of ways of giving first prize in physics to a class of 20 students=20

The number of ways of giving first prize in English to a class of 20 students=20

So total number of ways=20 × 19 × 20 × 19 × 20 × 20 = 57760000

Q. 21. Find the total number of ways of answering 5 objective-type question, each question having 4 choices.

Answer : Given: 5 objective-type question, each question having 4 choices.

To find: the number of ways of answering them.

Each objective-type question has 4 choices.

So the total number of ways of answering 5 objective-type question, each question having 4 choices=4 × 4 × 4 × 4 × 4=4^5

Q. 22. A gentleman has 6 friends to invite. In how many ways can be send invitation cards to them, if he has 3 servants to carry the cards?

Answer : Given: A gentleman has 6 friends to invite. He has 3 servants to carry the cards.

Each friend can be invited by 3 possible number of servants.

So the number of ways of inviting 6 friends using 3 servants=3 × 3 × 3 × 3 × 3 × 3=3^6

Q. 23. In how many ways 6 rings of different types can be worn in 4 fingers?

Answer : Given:6 rings and 4 fingers.

Each ring has 4 different fingers that they can be worn.

So total number of ways in which 6 rings of different types can be worn in 4 fingers =4 × 4 × 4 × 4 × 4 × 4=4^6

Q. 24. In how many ways can 5 letters be posted in 4 letter boxes?

Answer : Each letter has 4 possible letter boxes option.

So the number of ways in which 5 letters can be posted in 4 letter boxes =4 × 4 × 4 × 4 × 4=4^5 (Each 4 for each letter.)

Q. 25. How many 3-letters words can be formed using a, b, c, d, e if

(i) Repetition of letters is not allowed?
(ii) Repetition of letters is allowed

Answer :

any one of 6 letters	any one of the remaining 5 letters	any one of th

(i) if repetition of letters is not allowed then number of many 3-letters words that can be formed using a, b, c, d, e are

5 × 4 × 3=60

any one of 5 letters	any one of 5 letters	any one of 5 letters

(ii) if repetition of letters is allowed then number of many 3-letters words that can be formed using a, b, c, d, e are

$5 \times 5 \times 5 = 125$

Q. 26. How many 4-digit numbers are there, when a digit may be repeated any number of times?

Answer : To find: Number of 4 digit numbers when a digit may be repeated any number of times

The first place has possibilities of any of 9 digits.

(0 not included because 0 in starting would make the number a 3 digit number.)

The second place has possibilities of any of 10 digits.

The third place has possibilities of any of 10 digits.

The fourth place has possibilities of any of 10 digits.

Since repetition is allowed.

So there are $9 \times 10 \times 10 \times 10 = 9000$ 4-digit numbers when a digit may be repeated any number of times.

Q. 27. How many numbers can be formed from the digits 1, 3, 5, 9 if repetition of digits is not allowed?

Answer : To find: number of numbers that can be formed from the digits 1, 3, 5, 9 if repetition of digits is not allowed

Forming a 4 digit number:4!

Forming a 3 digit number: $^4C_3 \times 3!$

Forming a 2 digit number: $^4C_2 \times 2!$

Forming a 1 digit number:4

So total number of ways=4! + (${}^4C_3$ × 3!) + (${}^4C_2$ × 2!) + 4

=24 + 24 + 12 + 4

=64

Q. 28. How many 3-digit numbers are there with no digit repeated?

Answer :

Any 1 of 9 digits Not ZERO	Any 1 of 9 digits	Any 1 of 8 digits

In forming a 3 digit number the 100's place can be occupied by any 9 out of 10 digits (0 not included because it will lead to formation of 2 digit number.)

The 10's place can be occupied by any of the remaining 9 digits (here 0 can or cannot be used.)

In one's place any of the remain 8 digits can be used.

So total 3-digit numbers with no digit repeated are: 9 × 9 × 8=648.

Q. 29. How many 3-digit numbers can be formed by using the digits 0, 1, 3, 5, 7 while each digit may be repeated any number of times?

Answer : 100's place 10's place Unit's place

Any one of 1,3,5,7	Any one of 0,1,3,5,7	Any one of 0,1,3,5,7

There are total 5 digits available, for forming a 3 digit number, in 100's place only 1,3,5,7 can be used(0 not included because it will lead to formation of 2 digit number.)

In 10's place any of the 5 can be used and same is the case with one's place.

So total number of 3 digit numbers formed=4 × 5 × 5=100

Q. 30. How many 6-digit numbers can be formed from the digits 0, 1, 3, 5, 7, 9 when no digit is repeated? How many of them are divisible by 10?

Answer :

Any one of 1,3,5,7,9					For divisibility by 10 this place should contain 0

There are total 6 digits available ,for forming a 6 digit number, in 100000's place only 1,3,5,7,9 can be used(0 not included because it will lead to formation of 2 digit number.)

In 10000's place any of the remaining 5 digits can be used(even 0 can be used.)

In 1000's place any of the remaining 4 digits can be used.

In 100's place any of the remaining 3 digits can be used.

In 10's place any of the remaining 2 digits can be used.

In one's place the remaining digit can be used.

So total number of 6 digit numbers possible=5 × 5 × 4 × 3 × 2 × 1=600

For finding the number of 6 digit numbers divisible by 10 the one's place should contain 0 so possibilities=5 × 4 × 3 × 2 × 1=120

Q. 31. How many natural numbers less than 1000 can be formed from the digits 0, 1, 2, 3, 4, 5 when a digit may be repeated any number of times?

Answer : To find: number of natural numbers less than 1000 that can be formed from the digits 0, 1, 2, 3, 4, 5 when a digit may be repeated any number of times

For forming a 3 digit number less than 1000 possible ways are:

5 × 6 × 6…(in 100's place 5 digits are only possible 0 not included.)

=180

For forming a 2 digit number less than 1000 possible ways are:

5 × 6……(in 10’s place 5 digits are only possible 0 not included.)

= 30

For forming a 1 digit number less than 1000 possible ways are:

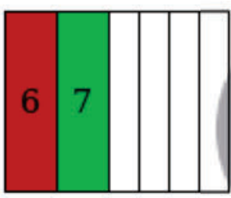
5…(0 not included because it is a whole number and natural number is asked in question.)

So total number of numbers less than 1000 that can be formed from the digits 0, 1, 2, 3, 4, 5 when a digit may be repeated any number of times=180 + 30 + 5=215

Q. 32. How many 6-digit telephone numbers can be constructed using the digits 0 to 9, if each number starts with 67 and no digit appears more than once?

Answer : To find: 6-digit telephone numbers that can be constructed using the digits 0 to 9.

Condition: each number starts with 67 and no digit appears more than once



There are 10 digits between 0 to 9,and two of them are utilized in filling up the first two digits i.e.67 of the 6 digit phone number, so remaining number of digits=10-2=8,and this need to be used in filling up the remaining 6-2=4 places of the telephone number.

So, number of ways=8 × 7 × 6 × 5=1680.

Q. 33. In how many ways can three jobs, I, II and III be assigned to three persons A, B and C if one person is assigned only one job and all are capable of doing each job?

Answer : Given: three jobs, I, II and III to be assigned to three persons A, B and C.

To find: In how many ways this can be done.

Condition: one person is assigned only one job and all are capable of doing each job.

It is given that one person is assigned only one job and all are capable of doing each job.

So if for person one 3 options are available, for person two 2 options and for person three only one option is available.

So total number of ways in which three jobs, I, II and III be assigned to three persons A, B and C if one person is assigned only one job and all are capable of doing each job=3 × 2 × 1 = 6

Q. 34. A number lock on a suitcase has three wheels each labeled with ten digits 0 to 9. if opening of the lock is a particular sequence of three digits with no repeats, how many such sequences will be possible? Also, find the number of unsuccessful attempts to open the lock.

Answer :

e.g

The number of sequences possible = 10 × 9 × 8 = 720(since no repeated digits is the given condition.)

There will be only one successful attempt so the number of unsuccessful attempts to open the lock=720 – 1 = 719.

Q. 35. A customer forgets a four-digit code for an automated teller machine (ATM) in a bank. However, he remembers that this code consists of digits 3, 5, 6, 9. Find the largest possible number of trials necessary to obtain the correct code.

Answer : Given: code consists of digits 3, 5, 6, 9.

To find: the largest possible number of trials necessary to obtain the correct code.

The customer remembers that this 4 digit code consists of digits 3, 5, 6, 9.

So the largest possible number of trials necessary to obtain the correct code = 4! = 4 × 3 × 2 × 1 = 24

Q. 36. In how many ways can 3 prizes be distributed among 4 girls, when

(i) no girl gets more than one prize?
(ii) a girl may get any number of prizes?
(iii) no girl gets all the prizes?

Answer : (i) To distribute 3 prizes among 4 girls where no girl gets more than one prize the possible number of permutation possible are:$^4P_3$=24

(ii) To distribute 3 prizes among 4 girls where a girl may get any number of prizes the number of possibilities are:4 × 4 × 4=64.

(Since a prize can be given to any of the 4 girls.)

(iii) To distribute 3 prizes among 4 girls where no girl gets all the prizes the number of possibilities are: (4 × 4 × 4)-(4)=64-4=60

(The situation where a single girl gets all the prizes has to reduced from the situation where a girl may get any number of prizes.)

Exercise 8C

Q. 1. A. Evaluate:

$^{10}P_4$

Answer : To find: the value of $^{10}P_4$

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^{n}P_r = \frac{n!}{(n-r)!}$

Therefore,

$^{10}P_4 = \frac{10!}{(10-4)!}$

$^{10}P_4 = 10 \times 9 \times 8 \times 7$

$^{10}P_4 = 5040$

Thus, the value of $^{10}P_4$ is 5040.

Q. 1. B. Evaluate:

$^{62}P_3$

Answer : To find: the value of $^{62}P_3$