

EXERCISE 1.8

1. Find a rational number between -3 and 1.

Solution:

Let us consider two rational numbers x and y

We know that between two rational numbers x and y where $x < y$ there is a rational number $(x+y)/2$

$$x < (x+y)/2 < y$$

$$(-3+1)/2 = -2/2 = -1$$

So, the rational number between -3 and 1 is -1

$$\therefore -3 < -1 < 1$$

2. Find any five rational numbers less than 2.

Solution:

Five rational numbers less than 2 are 0, $1/5$, $2/5$, $3/5$, $4/5$

3. Find two rational numbers between $-2/9$ and $5/9$

Solution:

The rational numbers between $-2/9$ and $5/9$ is

$$(-2/9 + 5/9)/2$$

$$(1/3)/2$$

$$1/6$$

The rational numbers between $-2/9$ and $1/6$ is

$$(-2/9 + 1/6)/2$$

$$((-2 \times 2 + 1 \times 3)/18)/2$$

$$(-4+3)/36$$

$$-1/36$$

\therefore the rational numbers between $-2/9$ and $5/9$ are $-1/36$, $1/6$

4. Find two rational numbers between $1/5$ and $1/2$

Solution:

The rational numbers between $1/5$ and $1/2$ is

$$(1/5 + 1/2)/2$$

$$((1 \times 2 + 1 \times 5)/10)/2$$

$$(2+5)/20 = 7/20$$

The rational numbers between $1/5$ and $7/20$ is

$$(1/5 + 7/20)/2$$

$$((1 \times 4 + 7 \times 1)/20)/2$$

$$(4+7)/40$$

$11/40$

\therefore the rational numbers between $1/5$ and $1/2$ are $7/20, 11/40$

5. Find ten rational numbers between $1/4$ and $1/2$.

Solution:

Firstly convert the given rational numbers into equivalent rational numbers with same denominators.

The LCM for 4 and 2 is 4.

$$1/4 = 1/4$$

$$1/2 = (1 \times 2) / 4 = 2/4$$

$$1/4 = (1 \times 20 / 4 \times 20) = 20/80$$

$$1/2 = (2 \times 20 / 4 \times 20) = 40/80$$

So, we now know that 21, 22, 23, ... 39 are integers between numerators 20 and 40.

\therefore the rational numbers between $1/4$ and $1/2$ are $21/80, 22/80, 23/80, \dots, 39/80$

6. Find ten rational numbers between $-2/5$ and $1/2$.

Solution:

Firstly convert the given rational numbers into equivalent rational numbers with same denominators.

The LCM for 5 and 2 is 10.

$$-2/5 = (-2 \times 2) / 10 = -4/10$$

$$1/2 = (1 \times 5) / 10 = 5/10$$

$$-2/5 = (-4 \times 2 / 10 \times 2) = -8/20$$

$$1/2 = (5 \times 2 / 10 \times 2) = 10/20$$

So, we now know that -7, -6, -5, ... 10 are integers between numerators -8 and 10.

\therefore the rational numbers between $-2/5$ and $1/2$ are $-7/20, -6/20, -5/20, \dots, 9/20$

7. Find ten rational numbers between $3/5$ and $3/4$.

Solution:

Firstly convert the given rational numbers into equivalent rational numbers with same denominators.

The LCM for 5 and 4 is 20.

$$3/5 = 3 \times 4 / 5 \times 4 = 12/20$$

$$3/4 = 3 \times 5 / 4 \times 5 = 15/20$$

So, we now know that 13, 14, 15, ... 19 are integers between numerators 12 and 15.

\therefore the rational numbers between $3/5$ and $3/4$ are $13/20, 14/20, 15/20, \dots, 19/20$