NCERT Solutions for Class-XI Maths

Chapter-7 Exercise-7.3

NCERT Math Class 11

1. How many 3-digit numbers can be formed by using the digits 1 to 9 if no digit is repeated?

1. 3-digit numbers have to be formed using the digits 1 to 9.
Here, the order of the digits matters.
Therefore, there will be as many 3-digit numbers as there are permutations of 9 different digits taken 3 at a time.

Therefore, required number of 3-digit numbers $= {}^{9}P_{3} = \frac{9!}{(9-3)!} = \frac{9!}{6!}$

$= \frac{9\times 8\times 7\times 6!}{6!} = 9\times 8\times 7 = 504$

2. How many 4-digit numbers are there with no digit repeated?

2. Total number of digits possible at a place (except 4th place)=10
At 4th place, there cannot be a zero(it will become 3 digit number),
No. of digits possible at 4th place=9.
$\Rightarrow$ No. of permutations$= {}^{9}_{3}P = 504$
Hence, total no. of permutations=9× 504=4536.

3. How many 3-digit even numbers can be made using the digits $1,2,3,4,6$, 7, if no digit is repeated?

3. 3-digit even numbers are to be formed using the given six digits, 1, 2, 3, 4, 6, and 7, without repeating the digits.
Then, units digits can be filled in 3 ways by any of the digits, 2, 4, or 6.
Since the digits cannot be repeated in the 3-digit numbers and units place is already occupied with a digit (which is even), the hundreds and tens place is to be filled by the remaining 5 digits.
Therefore, the number of ways in which hundreds and tens place can be filled with the remaining 5 digits is the permutation of 5 different digits taken 2 at a time.

Number of ways of filling hundreds and tens place $= {}^{5}P_{2} = \frac{5!}{(5-2)!} = \frac{5!}{3!}$

$= \frac{5\times 4\times 3!}{3!} = 20$

Thus, by multiplication principle, the required number of 3 -digit numbers is $3 \times 20 = 60$

4. Find the number of 4-digit numbers that can be formed using the digits 1, 2, 3, 4, 5 if no digit is repeated. How many of these will be even?

4. Total no. of digits possible for choosing =5

No. of places for which a digit has to be taken =4

As there is no repetition allowed;

$\Rightarrow$ No. of permutations = ${}^{5}_{4}\mathrm{P} = \frac{5!}{(5-4)!} = \frac{5!}{1!} = 120.$

The number will be even when 2 and 4 are at one's place.

The possibility of (2,4) at one's place = $\frac{2}{5} = 0.4$

Total number of even number = 120 × 0.4 = 48.

5. From a committee of 8 persons, in how many ways can we choose a chairman and a vice chairman assuming one person cannot hold more than one position?

5. From a committee of 8 persons, a chairman and a vice chairman are to be chosen in such a way that one person cannot hold more than one position.

Here, the number of ways of choosing a chairman and a vice chairman is the permutation of 8 different objects taken 2 at a time.

Thus, required number of ways $= {}^{8}\mathrm{P}_{2} = \frac{8!}{(8-2)!} = \frac{8!}{6!} = \frac{8 \times 7 \times 6!}{6!} = 8 \times 7 = 56$

6. Find n if ${}^{n-1}\mathrm{P}_{3} : {}^{n}\mathrm{P}_{4} = 1 : 9.$

6. $\frac{{}^{n-1}_{3}\mathrm{P}}{{}^{n}_{4}\mathrm{P}} = \frac{1}{9}$

$\Rightarrow \frac{\frac{(n-1)!}{(n-4)!}}{\frac{n!}{(n-4)!}} = \frac{1}{9}$

$\Rightarrow \frac{(n-1)!}{n!} = \frac{1}{9}$

$\Rightarrow \frac{1}{n} = \frac{1}{9}$

$\Rightarrow$ n=9.

7. Find r if

(i) ${}^5P_r = 2^6P_{r-1}$

(ii) ${}^5P_r = {}^6P_{r-1}$

7. (i)

${}^5P_r = 2^6P_{r-1}$

$\Rightarrow \frac{5!}{(5-r)!} = 2 \times \frac{6!}{(6-r+1)!}$

$\Rightarrow \frac{5!}{(5-r)!} = \frac{2 \times 6!}{(7-r)!}$

$\Rightarrow \frac{5!}{(5-r)!} = \frac{2 \times 6 \times 5!}{(7-r)(6-r)(5-r)!}$

$\Rightarrow 1 = \frac{2 \times 6}{(7-r)(6-r)}$

$\Rightarrow (7-r)(6-r) = 12$

$\Rightarrow 42 - 6r - 7r + r^2 = 12$

$\Rightarrow r^2 - 13r + 30 = 0$

$\Rightarrow r^2 - 3r - 10r + 30 = 0$

$\Rightarrow r(r-3) - 10(r-3) = 0$

$\Rightarrow (r-3)(r-10) = 0$

$\Rightarrow (r-3) = 0 \text{ or } (r-10) = 0$

$\Rightarrow r = 3 \text{ or } r = 10$

It is known that, ${}^nP_r = \frac{n!}{(n-r)!}$, where $0 \le r \le n$

$\therefore 0 \le r \le 5$

Hence, $r \neq 10$

$\therefore r = 3$

(ii)

${}^5P_r = {}^6P_{r-1}$

$\Rightarrow \frac{5!}{(5-r)!} = \frac{6!}{(6-r+1)!}$

$\Rightarrow \frac{5!}{(5-r)!} = \frac{6 \times 5!}{(7-r)!}$

$\Rightarrow \frac{1}{(5-r)!} = \frac{6}{(7-r)(6-r)(5-r)!}$

$$\Rightarrow 1 = \frac{6}{(7-r)(6-r)}$$

$$\Rightarrow (7-r)(6-r) = 6$$

$$\Rightarrow 42 - 7r - 6r + r^2 - 6 = 0$$

$$\Rightarrow r^2 - 13r + 36 = 0$$

$$\Rightarrow r^2 - 4r - 9r + 36 = 0$$

$$\Rightarrow r(r-4) - 9(r-4) = 0$$

$$\Rightarrow (r-4)(r-9) = 0$$

$$\Rightarrow (r-4) = 0 \text{ or } (r-9) = 0$$

$$\Rightarrow r = 4 \text{ or } r = 9$$

It is known that, $"P_r = \frac{n!}{(n-r)!}$, where $0 \le r \le n$

$\therefore 0 \le r \le 5$

Hence, $r \neq 9$

$\therefore r = 4$

8. How many words, with or without meaning, can be formed using all the letters of the word EQUATION, using each letter exactly once?

8. Total number of different letters in EQUATION = 8

Number of letters to be used to form a word =8

$\Rightarrow$ No. of permutations = $^8_8P = \frac{8!}{(8-8)!} = \frac{8!}{0!} = 40320.$

9. How many words, with or without meaning can be made from the letters of the word MONDAY, assuming that no letter is repeated, if.

(i) 4 letters are used at a time,

(ii) all letters are used at a time,

(iii) all letters are used but first letter is a vowel?

9. Total number of letters in MONDAY =6

(i) No. of letters to be used =4

$\Rightarrow$ No. of permutations = $^6_4P = \frac{6!}{(6-4)!} = \frac{6!}{2!} = 360.$

(ii) No. of letters to be used =6

$\Rightarrow$ No. of permutations = $^6_6P = \frac{6!}{(6-6)!} = \frac{6!}{0!} = 720.$

(iii) No. of vowels in MONDAY = 2 (O and A)

$\Rightarrow$ No. of permutations in vowel = ${}^{2}_{1}\mathrm{P} = 2$

Now, remaining places =5

Remaining letters to be used =5

$\Rightarrow$ No. of permutations = ${}^{5}_{5}\mathrm{P} = \frac{5!}{(5-5)!} = \frac{5!}{0!} = 120.$

Therefore, total number of permutations = 2 × 120 =240.

10. In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

10. In the given word MISSISSIPPI, I appears 4 times, S appears 4 times, P appears 2 times, and M appears just once.

Therefore, number of distinct permutations of the letters in the given word

$= \frac{11!}{4!4!2!}$

$= \frac{11\times10\times9\times8\times7\times6\times5\times4!}{4!\times4\times3\times2\times1\times2\times1}$

$= \frac{11\times10\times9\times8\times7\times6\times5}{4\times3\times2\times1\times2\times1}$

$= 34650$

There are 4 Is in the given word. When they occur together, they are treated as a single object III I for the time being. This single object together with the remaining 7 objects will account for 8 objects.

These 8 objects in which there are 4 Ss and 2 Ps can be arranged in $\frac{8!}{4!2!}$ ways i.e., 840 ways.

Number of arrangements where all Is occur together $= 840$

Thus, number of distinct permutations of the letters in MISSISSIPPI in which four Is do not come together $= 34650 - 840 = 33810$

11. In how many ways can the letters of the word PERMUTATIONS be arranged if the
(i) words start with P and end with S, (ii) vowels are all together,
(iii) there are always 4 letters between P and S?

11. Total number of letters in PERMUTATIONS =12

Only repeated letter is T ; 2times

(i) First and last letter of the word are fixed as P and S respectively.

Number of letters remaining =12-2=10

$\Rightarrow$ No. of permutations = $\frac{{}^{10}_{10}P}{2!} = \frac{10!}{2(10-10)!} = \frac{10!}{2} = 1814400$

(ii) No. of vowels in PERMUTATIONS = 5 (E,U,A,I,O)

Now, we consider all the vowels together as one.

Number of permutations of vowels $= {}^{5}_{5}P = 120.$

Now total number of letters = 12-5+1=8

$\Rightarrow$ No. of permutations = $\frac{{}^{8}_{8}P}{2!} = \frac{8!}{2(8-8)!} = \frac{8!}{2} = 20160.$

Therefore total number of permutations = 120× 20160=2419200

(iii) Number of places are as 1 2 3 4 5 6 7 8 9 10 11 12

There should always be 4 letters between P and S.

Possible places of P and S are 1 and 6, 2and 7, 3 and 8, 4 and 9 , 5 and 10, 6 and 11 , 7 and 12

Possible ways =7 ,

Also P and S can be interchanged,

No. of permutations =2× 7 =14

Remaining 10 places can be filled with 10 remaining letters,

$\therefore$ No. of permutations = $\frac{{}^{10}_{10}P}{2!} = \frac{10!}{2(10-10)!} = \frac{10!}{2} = 1814400$

Therefore, total number of permutations = 14 × 1814400 =25401600.