Exercise 6(E)

Solution 1:

Let the two numbers be x and y
According to the question,

$$\frac{x}{y}=\frac{2}{3}$$

3x - 2y = 0 ...(1)

Also, $\frac{x-2}{y-8}=\frac{3}{2}$

2x - 3y = -20 ...(2)

Multiplying equation no. (1) by 2 and (2) by 3and substracting

$$\begin{array}{r} 6x-4y=0 \\ 6x-9y=-60 \\ -\quad +\qquad + \\ \hline 5y=60 \end{array}$$

$$y=12$$

From (1), we get

3x - 2(12) = 0

$$x=\frac{24}{3}$$

x = 8

Thus, the numbers are 8 and 12.

Solution 2:

Let the smaller number be x
and the larger number bey.
According to the question,

$$\frac{x}{y}=\frac{4}{7}$$

7x -; 4y = 0...(1)

and,3y + 2x = 59...(2)

Multiplying equation no. (1) by 3 and (2) by 4.and adding them

$$\begin{array}{rl} 21x-12y=0 & \ldots(3) \\ \underline{8x+12y=236} & \ldots(4) \\ 29x=236 & \end{array}$$

$$x=\frac{236}{29}$$

From (1)

$$7\left(\frac{236}{29}\right)=4y$$

$$y=7\left(\frac{59}{29}\right)$$

$$y=\frac{413}{29}$$

Hence, the number are $\frac{236}{29}$ and $\frac{413}{29}$.

Solution 3:

Let x be the greater number and y be the smaller number.

When the greater of the two numbers increased by 1 divides the sum of the numbers, the result is $\frac{3}{2}$.

$\Rightarrow \frac{x+y}{(x+1)} = \frac{3}{2}$

$\Rightarrow 2x + 2y = 3(x+1)$

$\Rightarrow x - 2y = -3 \ldots\ldots\ldots\ldots\ldots\ldots (i)$

When the difference of these number is divided by the smaller, the result is $\frac{1}{2}$.

$\Rightarrow \frac{x-y}{y} = \frac{1}{2}$

$\Rightarrow 2x - 2y = y$

$\Rightarrow 2x - 3y = 0 \ldots\ldots\ldots\ldots\ldots\ldots (ii)$

Multiply equation (i) by 2, we get:

$2x - 4y = -6$

$2x - 3y = 0$ [Equation (ii)]

$- \quad + \quad -$ [Subtracting]

$-y = -6$

$\Rightarrow y = 6$

Substituting $y = 6$ in equation (i), we get

$x - 2(6) = -3$

$\Rightarrow x = 9$

$\therefore$ 9 is the greater number and 6 is the smaller number.

Solution 4:

Let the common multiple between the numbers be x.

So, the numbers are 4x and 5x.

According to the question,

$$\frac{4x-30}{5x-30}=\frac{1}{2}$$

$\Rightarrow 8x-60=5x-30$

$\Rightarrow 3x=30$

$\Rightarrow x=10$

So, $4x=4(10)=40$ and $5x=5(10)=50$

Thus, the numbers are 40 and 50.

Solution 5:

Let the numerator and denominator a fraction be x and y respectively .

According to the question,

$$\frac{x+2}{y-1}=\frac{2}{3}$$

3x - 2y = -8...(1)

And,

$$\frac{x+1}{y+2}=\frac{1}{3}$$

$$3x-y=-1 \qquad \ldots(2)$$

Now subtracting,

$$3x-y=-1 \qquad \ldots(2)$$

$$3x-2y=-8 \qquad \ldots(1)$$

$$\begin{array}{r} - \quad + \quad + \\ \hline y=7 \end{array}$$

From (1) ,

3x - 2 (7) = -8

3x = -8 + 14

x = 2

Required fraction = $\frac{2}{7}$

Solution 6:

Let the numerator and denominator of a fraction be x and y respectively .Then the fraction will be $\frac{x}{y}$

According to the question,

$x + y = 7$...(1)

$5y - 4x = 8$...(2)

Multiplying equation no. (1) by 4 and add with (2),

$$\begin{array}{r} 4x + 4y = 28 \quad ..(3) \\ -4x + 5y = 8 \\ \hline 9y = 36 \end{array}$$

$y = 4$

From (1)

$x + 4 = 7$

$x = 3$

Required fraction = $\frac{3}{4}$

Solution 7:

Let the numerator of the fraction be x and the denominator be y.

So, the fraction is $\frac{x}{y}$.

According to the question,

$\frac{2x}{y+1} = 1 \Rightarrow 2x = y + 1 \Rightarrow 2x - y = 1$...(i)

and $\frac{x+4}{2y} = \frac{1}{2} \Rightarrow 2x + 8 = 2y \Rightarrow 2x - 2y = -8$...(ii)

Solving equations (i) and (ii), we get

$y = 9$

Putting the value of y in (i), we get

$2x - (9) = 1 \Rightarrow 2x = 1 + 9 \Rightarrow x = 5$

So, the fraction is $\frac{5}{9}$.

Solution 8:

Let the numerator of the fraction be x and denominator of the fraction be y.

Then, the fraction $= \frac{x}{y}$

According to given condition, we have

$\frac{x-5}{y-3} = \frac{1}{2}$

$\Rightarrow 2x - 10 = y - 3$

$\Rightarrow 2x - y = 7 \quad \ldots.(i)$

And,

$x + 5 = y$

$\Rightarrow x - y = -5 \quad \ldots.(ii)$

Subtracting (ii) from (i), we get

$x = 12$

$\Rightarrow y = x + 5 = 12 + 5 = 17$

hence, the fraction is $\frac{12}{17}$.

Solution 9:

Let the numerator of the fraction be x and denominator of the fraction be y.

Then, the fraction $= \frac{x}{y}$

According to given condition, we have

$\frac{x-5}{y-3} = \frac{1}{2}$

$\Rightarrow 2x - 10 = y - 3$

$\Rightarrow 2x - y = 7 \quad \ldots.(i)$

And,

$x + 5 = y$

$\Rightarrow x - y = -5 \quad \ldots.(ii)$

Subtracting (ii) from (i), we get

$x = 12$

$\Rightarrow y = x + 5 = 12 + 5 = 17$

hence, the fraction is $\frac{12}{17}$.

Solution 10:

Let the digit at unit's place be x and the digit at ten's place be y.
Required no. = 10y + x
If the digit's are reversed
Reversed no. = 10x + y
According to the question,
x + y = 7...(1)
and,
10x + y - 2 = 2(10y + x).
8x - 19y = 2...(2)
Multiplying equation no. (1) by 19.

$$19x + 19y = 133 \quad \ldots(3)$$

Now adding equation(2) and (3)

$$\begin{array}{rl} 19x + 19y = 133 & \ldots(3) \\ 8x - 19y = 2 & \ldots(2) \\ \hline 27x = 135 & \end{array}$$

x = 5
From (1)
5 +y = 7
y = 2
Required number is
10(2) + 5
= 25.

Solution 11:

Let the digit at unit's place be x and the digit at ten's place be y.
Required no. = 10y + x
According to the question
y = 3x ⇒ 3x - y = 0...(1)
and,10y + x + x = 32
10y + 2x = 32...(2)
Multiplying equation no. (1) by 10

$$30x - 10y = 0 \quad \ldots(3)$$

Now adding (3)and(2)

$$\begin{array}{rl} 30x - 10y = 0 & \ldots(3) \\ 2x + 10y = 32 & \ldots(2) \\ \hline 32x = 32 & \end{array}$$

$$x = 1$$

From (1),we get
y = 3(1) = 3
Required no is
10(3) + 1 = 31

Solution 12:

Let the digit a unit's place be x and the digit at ten's place be y.
Required no. = 10y + x.
According to the question,
y - 2x = 2
-2x + y = 2...(1)
and,
(10x + y) -3 (y + x) = 5
7x - 2y = 5...(2)
Multiplying equation no. (1) by 2.

$$-4x + 2y = 4 \quad \ldots(3)$$

Now adding (2)and(3)

$$\begin{array}{r} -4x + 2y = 4 \\ 7x - 2y = 5 \\ \hline 3x = 9 \end{array}$$

$$x = 3$$

From (1) ,we get
-2(3) + y = 2
$\Rightarrow$ y = 8
Required number is
10(8) + 3 = 83.

Solution 13:

Let x be the number at the ten's place
and y be the number at the unit's place.
So, the number is 10x + y.

Four times a certain two-digit number is seven times the number obtained on interchanging its digits.

$$\Rightarrow 4(10x + y) = 7(10y + x)$$
$$\Rightarrow 40x + 4y = 70y + 7x$$
$$\Rightarrow 33x - 66y = 0$$
$$\Rightarrow x - 2y = 0 \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

If the difference between the digits is 4, then

$$\Rightarrow x - y = 4 \ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Subtracting equation (i) from equation (ii), we get:

$$\begin{array}{rl} & x - y = 4 \\ & x - 2y = 0 \qquad \text{[Equation } (i)\text{]} \\ - \quad + & \qquad\qquad\quad \text{[Subtracting]} \\ \hline & y = 4 \end{array}$$

Substituting $y = 4$ in equation (i), we get

$$x - 2(4) = 0$$
$$\Rightarrow x = 8$$

$\therefore$ The number is $10x + y = 10(8) + 4 = 84$.

Solution 14:

Let the tens digit of the number be x and the units digit be y.
So, the number is $10x + y$.
The number obtained by interchanging the digits will be $10y + x$.
According to question, we have
$10x + y + 10y + x = 121$
$\Rightarrow 11x + 11y = 121$
$\Rightarrow 11(x + y) = 121$
$\Rightarrow x + y = 11 \quad \ldots.(i)$
And,
$x - y = 3 \quad \ldots.(ii)$
Adding (i) and (ii), we get
$2x = 14$
$\Rightarrow x = 7$
$\Rightarrow y = 11 - x = 11 - 7 = 4$
Hence, the number is 74.

Solution 15:

Let the tens digit of the number be x and the units digit be y.
So, the number is $10x + y$.
According to the question,
$10x + y = 8(x + y) \Rightarrow 2x = 7y \ldots(i)$
and $10x + y = 14(x - y) + 2$ or $10x + y = 14(y - x) + 2$
$\Rightarrow 4x - 15y = -2 \ldots(ii)$ or $24x - 13y = 2 \ldots(iii)$
Solving (i) and (ii), we get
$y = 2$ and $x = 7$
Solving (i) and (iii), we get
$y = \frac{2}{71}$
This is not possible, since y is a digit and cannot be in fraction form.
So the number is 72.