## Exercise 4(E)

**1. Simplify:**
**(i) (x + 6)(x + 4)(x - 2)**
**(ii) (x - 6)(x - 4)(x + 2)**
**(iii) (x - 6)(x - 4)(x - 2)**
**(iv) (x + 6)(x - 4)(x - 2)**
**Solution:**

Using identity:
$(x + a)(x + b)(x + c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$

(i) We have, $(x + 6)(x + 4)(x - 2)$
$= x^3 + (6 + 4 - 2)x^2 + [6 \times 4 + 4 \times (-2) + (-2) \times 6]x + 6 \times 4 \times (-2)$
$= x^3 + 8x^2 + (24 - 8 - 12)x - 48$
$= x^3 + 8x^2 + 4x - 48$

(ii) We have, $(x - 6)(x - 4)(x + 2)$
$= x^3 + (-6 - 4 + 2)x^2 + [-6 \times (-4) + (-4) \times 2 + 2 \times (-6)]x + (-6) \times (-4) \times 2$
$= x^3 - 8x^2 + (24 - 8 - 12)x + 48$
$= x^3 - 8x^2 + 4x + 48$
(iii) We have, $(x - 6)(x - 4)(x - 2)$
$= x^3 + (-6 - 4 - 2)x^2 + [-6 \times (-4) + (-4) \times (-2) + (-2) \times (-6)]x + (-6) \times (-4) \times (-2)$
$= x^3 - 12x^2 + (24 + 8 + 12)x - 48$
$= x^3 - 12x^2 + 44x - 48$

(iv) We have, $(x + 6)(x - 4)(x - 2)$
$= x^3 + (6 - 4 - 2)x^2 + [6 \times (-4) + (-4) \times (-2) + (-2) \times 6]x + 6 \times (-4) \times (-2)$
$= x^3 - 0x^2 + (-24 + 8 - 12)x + 48$
$= x^3 - 28x + 48$

**2. Simply using following identity:**
**$(a \pm b)(a^2 \mp ab + b^2) = a^3 \pm b^3$**
**(i) $(2x + 3y)(4x^2 – 6xy + 9y^2)$**
**(ii) $(3x – 5/x)(9x^2 + 15 + 25/x^2)$**
**(iii) $(a/3 – 3b)(a^2 + ab + 9b^2)$**
**Solution:**

(i) We have, $(2x + 3y)(4x^2 – 6xy + 9y^2)$
$= (2x + 3y)[(2x)^2 – (2x)(3y) + (3y)^2]$
$= (2x)^3 + (3y)^3$
$= 8x^3 + 27y^3$

(ii) We have, $(3x – 5/x)(9x^2 + 15 + 25/x^2)$
$= (3x – 5/x)[(3x)^2 + (3x)(5/x) + (5/x)^2]$
$= (3x)^3 + (5/x)^3$
$= 27x^3 + 125/x^3$

(iii) We have, $(a/3 – 3b)(a^2/9 + ab + 9b^2)$
$= (a/3 – 3b)[(a/3)^2 + (a/3)(3b) + (3b)^2]$
$= (a/3)^3 - (3b)^3$
$= a^3/27 – 27b^3$

**3. Using suitable identity, evaluate**
**(i) $(104)^3$**
**(ii) $(97)^3$**
**Solution:**

Using identity: $(a \pm b)^3 = a^3 \pm b^3 \pm 3ab(a \pm b)$
(i) $(104)^3 = (100 + 4)^3$
$= (100)^3 + (4)^3 + 3 \times 100 \times 4(100 + 4)$
$= 1000000 + 64 + 1200 \times 104$
$= 1000000 + 64 + 124800$
$= 1124864$

(ii) $(97)^3 = (100 - 3)^3$
$= (100)^3 - (3)^3 - 3 \times 100 \times 3(100 - 3)$
$= 1000000 - 27 - 900 \times 97$
$= 1000000 - 27 - 87300$
$= 912673$

**4. Simply:**

$$\frac{(x^2 - y^2)^3 + (y^2 - z^2)^3 + (z^2 - x^2)^3}{(x - y)^3 + (y - z)^3 + (z - x)^3}$$

**Solution:**

We know that,
If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$
Now, if
$(x^2 – y^2) + (y^2 – z^2) + (z^2 – x^2) = 0$
Then, we have
$(x^2 – y^2)^3 + (y^2 – z^2)^3 + (z^2 – x^2)^3 = 3(x^2 – y^2)(y^2 – z^2)(z^2 – x^2)$ … (1)
Similarly, if
$x – y + y – z + z – x = 0$
Then,
$(x – y)^3 + (y – z)^3 + (z – x)^3 = 3(x – y)(y – z)(z – x)$ … (2)
Now,

$$\frac{\left(x^2-y^2\right)^3+\left(y^2-z^2\right)^3+\left(z^2-x^2\right)^3}{(x-y)^3+(y-z)^3+(z-x)^3}$$

$$= \frac{3\left(x^2-y^2\right)\left(y^2-z^2\right)\left(z^2-x^2\right)}{3(x-y)(y-z)(z-x)}$$ .....[From (1) and (2)]

= (x + y)(y + z)(z + x)

**5. Evaluate:**

(i) $\frac{0.8 \times 0.8 \times 0.8 + 0.5 \times 0.5 \times 0.5}{0.8 \times 0.8 - 0.8 \times 0.5 + 0.5 \times 0.5}$

(ii) $\frac{1.2 \times 1.2 + 1.2 \times 0.3 + 0.3 \times 0.3}{1.2 \times 1.2 \times 1.2 - 0.3 \times 0.3 \times 0.3}$

**Solution:**

(i) We have,

$$\frac{0.8 \times 0.8 \times 0.8 + 0.5 \times 0.5 \times 0.5}{0.8 \times 0.8 - 0.8 \times 0.5 + 0.5 \times .5}$$

Let's substitute 0.8 = a and 0.5 = b

So, now the given expression becomes

$$\frac{a \times a \times a + b \times b \times b}{a \times a - a \times b + b \times b}$$

$$= \frac{a^3 + b^3}{a^2 - ab + b^2}$$

$$= \frac{(a+b)\left(a^2 - ab + b^2\right)}{a^2 - ab + b^2}$$

= a + b

= 0.8 + 0.5

= 1.3

(ii) We have,

$$\frac{1.2 \times 1.2 + 1.2 \times 0.3 + 0.3 \times 0.3}{1.2 \times 1.2 \times 1.2 - 0.3 \times 0.3 \times 0.3}$$

Let's substitute 1.2 = a and 0.3 = b

So, now the given expression becomes

$$\frac{a \times a + a + b + b \times b}{a \times a \times a - b \times b \times b}$$

$$= \frac{a^2 + ab + b^2}{a^3 - b^3}$$

$$= \frac{a^2 + ab + b^2}{(a - b)(a^2 + ab + b^2)}$$

$$= \frac{1}{a - b}$$

$$= \frac{1}{1.2 - 0.3}$$

$$= \frac{1}{0.9}$$

$$= \frac{10}{9}$$

$$= 1\frac{1}{9}$$



**6. If a - 2b + 3c = 0; state the value of $a^3 - 8b^3 + 27c^3$.**
**Solution:**

Given, a - 2b + 3c = 0
Then,
$a^3 - 8b^3 + 27c^3 = a^3 + (-2b)^3 + (3c)^3 = 3(a)(-2b)(3c)$
$= -18abc$

**7. If x + 5y = 10; find the value of $x^3 + 125y^3 + 150xy - 1000$.**
**Solution:**

Given, x + 5y = 10
On cubing both sides, we get
$(x + 5y)^3 = 10^3$
$x^3 + (5y)^3 + 3(x)(5y)(x + 5y) = 1000$
$x^3 + (5y)^3 + 3(x)(5y)(10) = 1000$
$x^3 + (5y)^3 + 150xy = 1000$
Thus,
$x^3 + (5y)^3 + 150xy - 1000 = 0$

**8. If $x = 3 + 2\sqrt{2}$, find:**
**(i) $1/x$**
**(ii) $x - 1/x$**
**(iii) $(x - 1/x)^3$**
**(iv) $x^3 - 1/x^3$**
**Solution:**

We have, $x = 3 + 2\sqrt{2}$

(i) $1/x = 1/(3 + 2\sqrt{2})$
$= (3 - 2\sqrt{2})/ [(3 + 2\sqrt{2}) \times (3 - 2\sqrt{2})]$
s$= (3 - 2\sqrt{2})/ [3^2 - (2\sqrt{2})^2]$
$= (3 - 2\sqrt{2})/ (9 - 8)$
$= 3 - 2\sqrt{2}$

(ii) $x - 1/x = (3 + 2\sqrt{2}) - (3 - 2\sqrt{2})$ … [From (i)]
$= (3 + 2\sqrt{2} - 3 + 2\sqrt{2})$
$= 4\sqrt{2}$

(iii) $(x - 1/x)^3 = (4\sqrt{2})^3$ … [From (ii)]
$= (64 \times 2\sqrt{2})$
$= 128\sqrt{2}$

(iv) $(x^3 - 1/x^3) = (x - 1/x)^3 - 3(x - 1/x)$
$= 128\sqrt{2} - 3(4\sqrt{2})$ … [From (iii) and (ii)]
$= 128\sqrt{2} - 12\sqrt{2}$

**9. If $a + b = 11$ and $a^2 + b^2 = 65$; find $a^3 + b^3$.**
**Solution:**

Given, $a + b = 11$ and $a^2 + b^2 = 65$
Now, we know that
$(a + b)^2 = a^2 + b^2 + 2ab$
$(11)^2 = 65 + 2ab$
$121 = 65 + 2ab$
$2ab = 121 - 65$
$ab = (121 - 65)/2$
$= 56/2$
$= 28$
Considering the expansion $(a^3 + b^3)$
$(a^3 + b^3) = (a + b)(a^2 + b^2 - ab)$
$= (11)(65 - 28)$
$= 11 \times 37$
$= 407$
Thus, $a^3 + b^3 = 407$

**10. Prove that:**
**$x^2 + y^2 + z^2 - xy - yz - zx$ is always positive.**
**Solution:**

We have, $x^2 + y^2 + z^2 - xy - yz - zx$
$= 2(x^2 + y^2 + z^2 - xy - yz - zx)$
$= 2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx$
$= x^2 + x^2 + y^2 + y^2 + z^2 + z^2 - 2xy - 2yz - 2zx$
$= (x^2 + y^2 - 2xy) + (z^2 + x^2 - 2zx) + (y^2 + z^2 - 2yz)$
$= (x - y)^2 + (z - x)^2 + (y - z)^2$
As the square of any number is positive, the given equation is always positive.

**11. Find:**
**(i) (a + b)(a + b)**
**(ii) (a + b)(a + b)(a + b)**
**(iii) (a - b)(a - b)(a - b) by using the result of part (ii)**
**Solution:**

(i) We have, $(a + b)(a + b)$
$= (a + b)^2$
$= a \times a + a \times b + b \times a + b \times b$
$= a^2 + ab + ab + b^2$
$= a^2 + b^2 + 2ab$

(ii) We have, $(a + b)(a + b)(a + b)$
$= (a \times a + a \times b + b \times a + b \times b)(a + b)$
$= (a^2 + ab + ab + b^2)(a + b)$
$= (a^2 + b^2 + 2ab)(a + b)$
$= a^2 \times a + a^2 \times b + b^2 \times a + b^2 \times b + 2ab \times a + 2ab \times b$
$= a^3 + a^2 b + ab^2 + b^3 + 2a^2b + 2ab^2$
$= a^3 + b^3 + 3a^2b + 3ab^2$

(iii) We have, $(a - b)(a - b)(a - b)$
In result (ii), replacing b by -b, we get $(a - b)(a - b)(a - b)$
$= a^3 + (-b)^3 + 3a^2(-b) + 3a(-b)^2$
$= a^3 - b^3 - 3a^2b + 3ab^2$